EE 5654 - Digital Communications Spring 2005

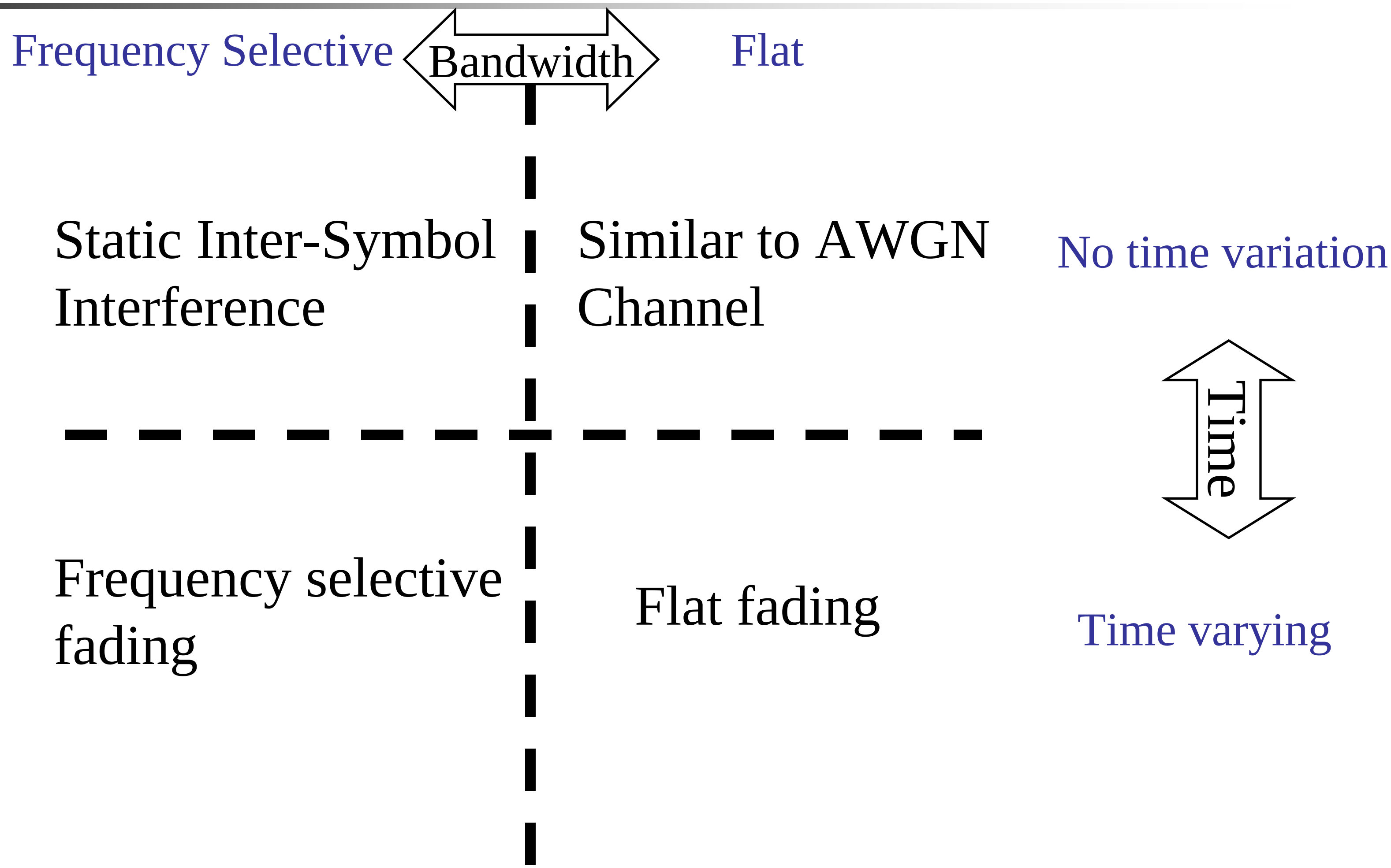
Instructors: R. Michael Buehrer

Lecture #14 – Introduction to Equalization

Multipath

- Previously we discussed the impact of *flat time-varying* mulitpath channels
  - Flat fading causes fluctuations in the signal power but does not distort the pulse.
  - This can be combated through diversity
- Today we will discuss *frequency selective* mulitpath channels cause pulse distortion
  - This must be combated through equalization
  - We will focus on the static (non-time varying) case first

Multipath Effects

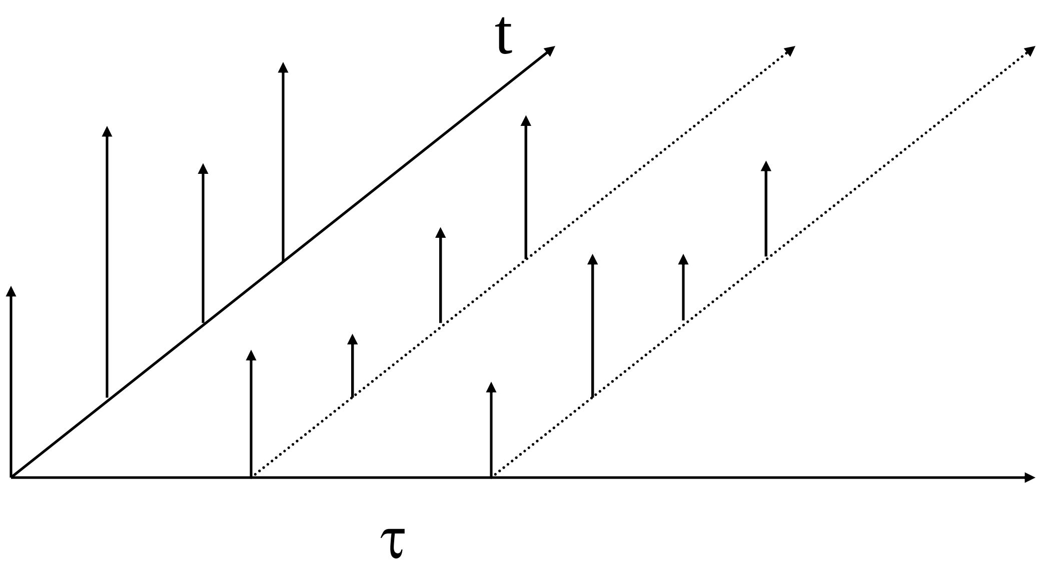


Inter-symbol Interference

- We previously studied the influence of pulse shape on bandwidth
- By lengthening the pulse we can reduce bandwidth but may introduce inter-symbol interference (ISI) if the pulses aren't chosen properly
- Viewed another way, limiting the bandwidth of a pulse will smear the pulse in time and cause pulses to interfere with succeeding pulses (ISI)
- This bandwidth limiting can be caused by the transmitter or receiver but should be avoided with proper pulse design
- ISI can also be introduced by the channel

Time-Varying Channel Impulse Response

In general the multipath channel is time-varying linear filter

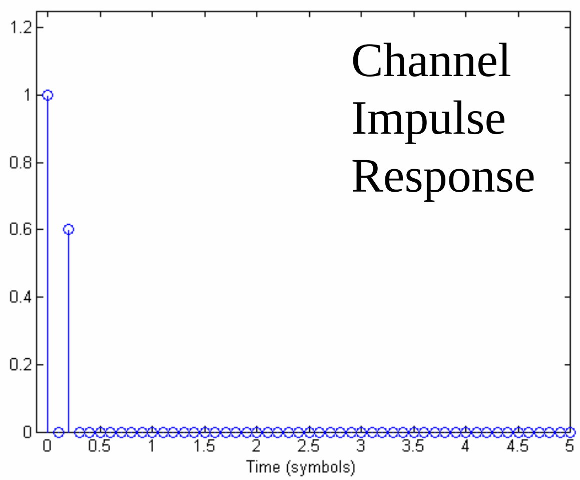


At any time t the channel is a linear filter represented by the taps at times τ_1, τ_2, τ_3.... The values for each tap vary with time. The temporal variation is due to movement of mobile transmitter/receiver.

Tap Delay Line Model of Multipath Channel

- Each 'tap' in the filter varies in time and may be the sum of many unresolvable paths
  - Filter also called the impulse response of the channel
- If there is only one tap (or all of the taps are close relative to the symbol duration) the signal only experiences fading and there is no frequency distortion
  - This is called 'flat' fading
- If there is more than one tap in the channel impulse response (i.e., the multpath delays are large relative to the symbol duration), the signal experiences both fading and intersymbol interference (ISI) which causes frequency distortion
  - This is called frequency selective fading
- Frequency selective fading requires the use of an 'equalizer' for adequate performance

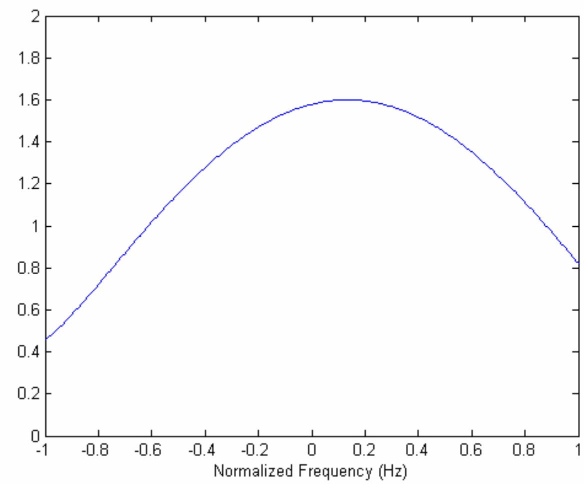
Flat vs. Frequency Selective



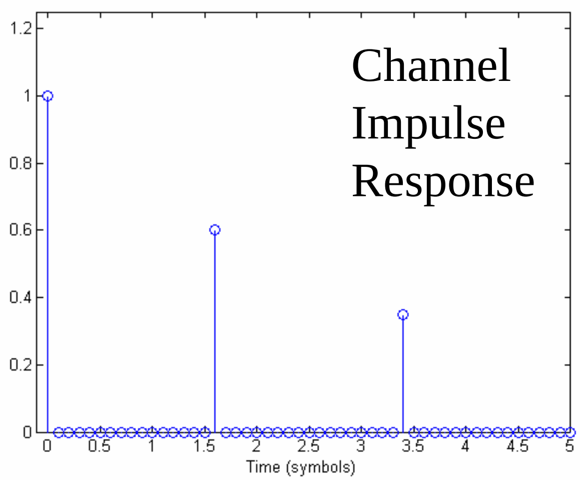
Flat

No ISI

Fourier Transform



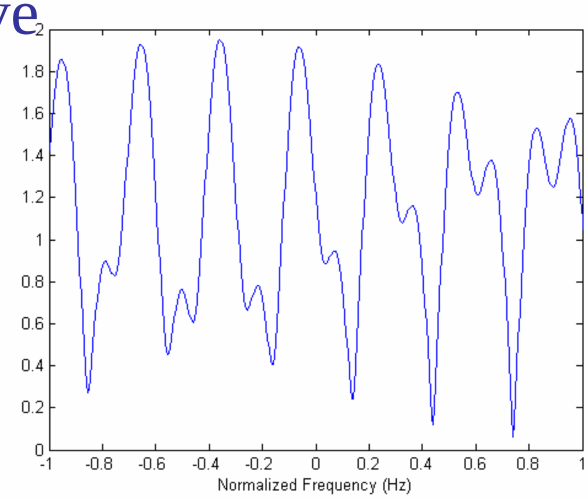
Channel Transfer Function



Frequency Selective

ISI

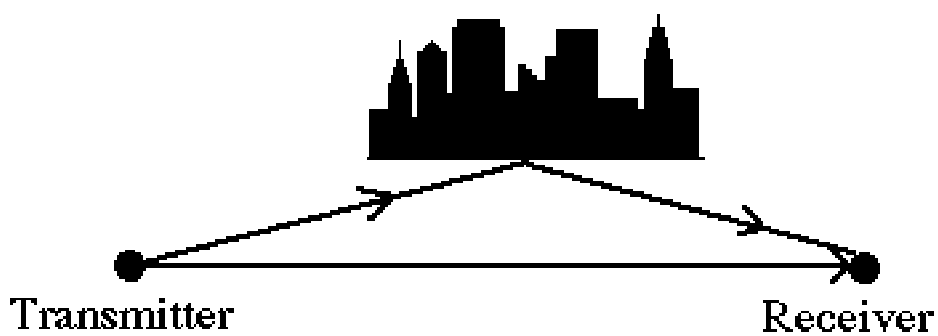
Fourier Transform



Channel Transfer Function

Multipath channel

- ISI can be introduced by transmission through a multipath channel *h(τ)*. Previously we assumed that the delay spread of the channel was much smaller than the symbol duration. If it is not, ISI results

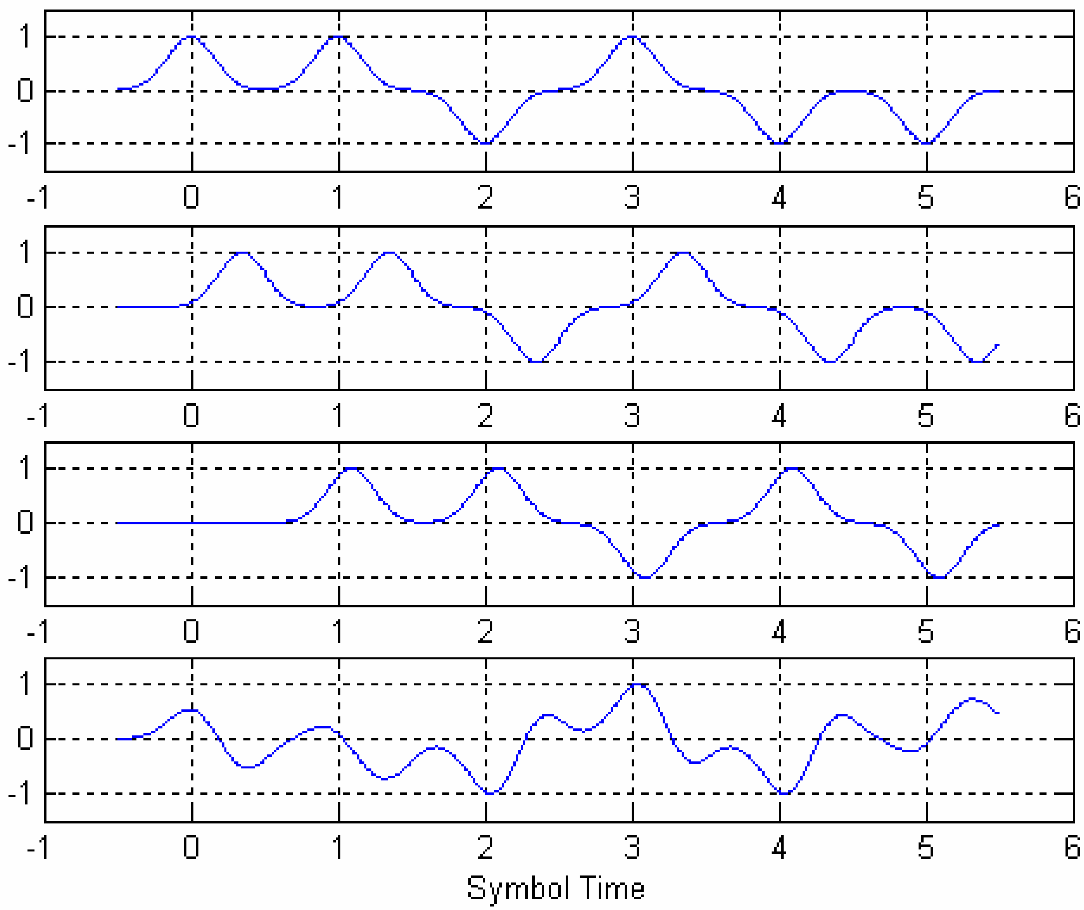


$$s(t) = \sum_{n=0}^{\infty} \underbrace{I_n}_{\substack{\textit{information}\\ \textit{symbol}}} \underbrace{p(t-nT)}_{\substack{\textit{transmit}\\ \textit{pulse}}}$$

$$r(t) = \sum_{n=0}^{\infty} I_n g(t-nT) + n(t)$$

$$\underbrace{g(t)}_{\text{output pulse}} = \int_{-\infty}^{\infty} p(\tau) \underbrace{h(t-\tau)}_{\text{IR of channel}} d\tau$$

Multipath Example



Path 1

+

Path 2

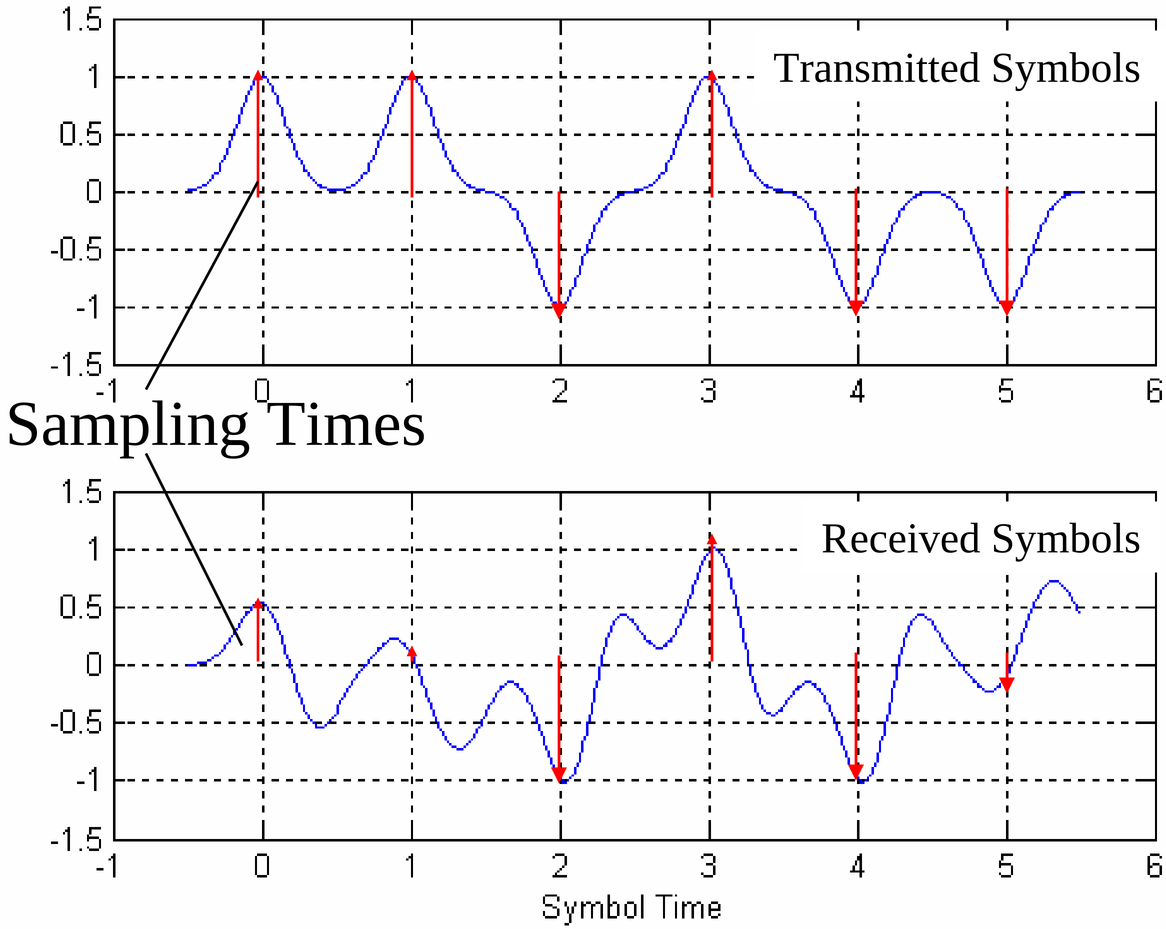
+

Path 3

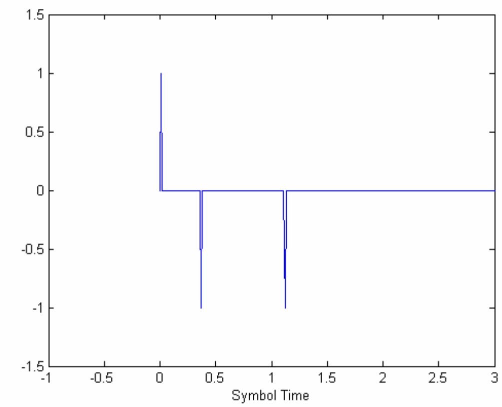
=

Received Signal

Inter-Symbol Interference



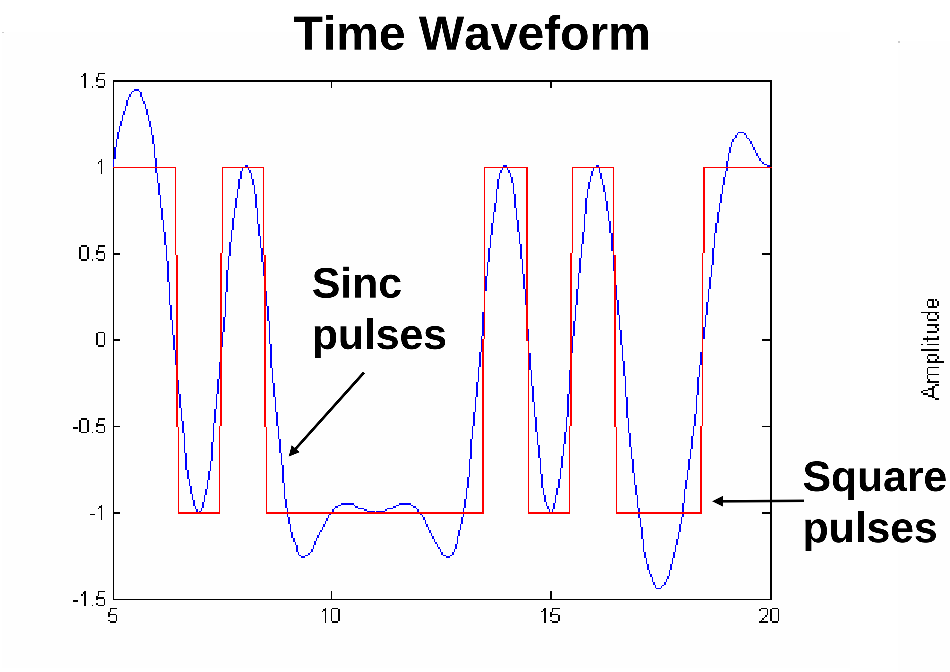
Channel Impulse Response

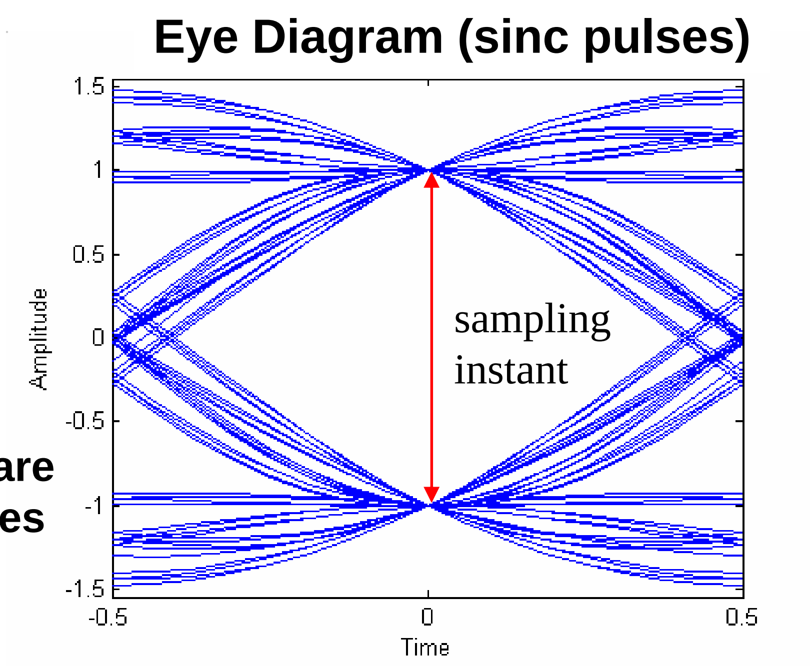


- Multipath Causes Inter-symbol Interference

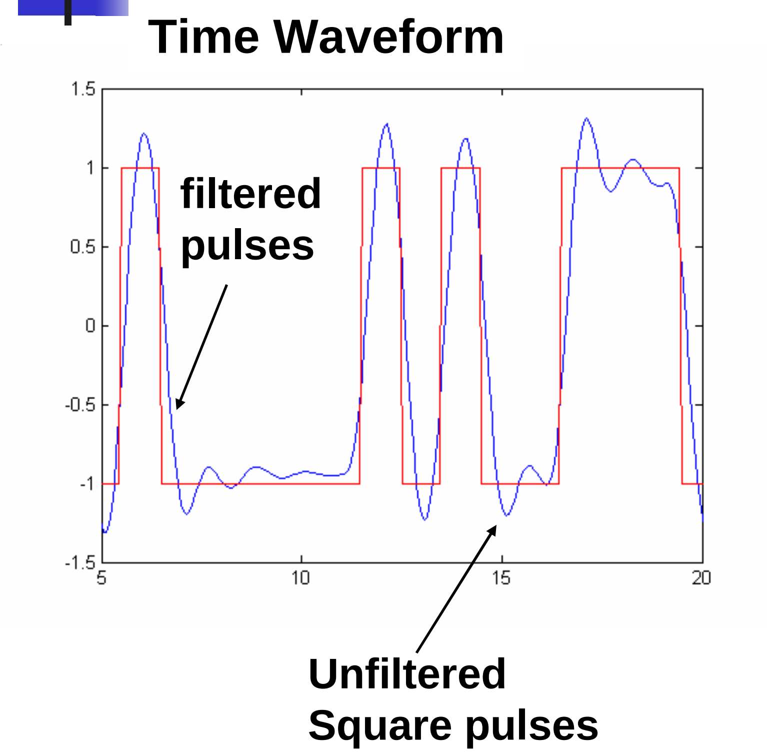
Eye-Diagrams

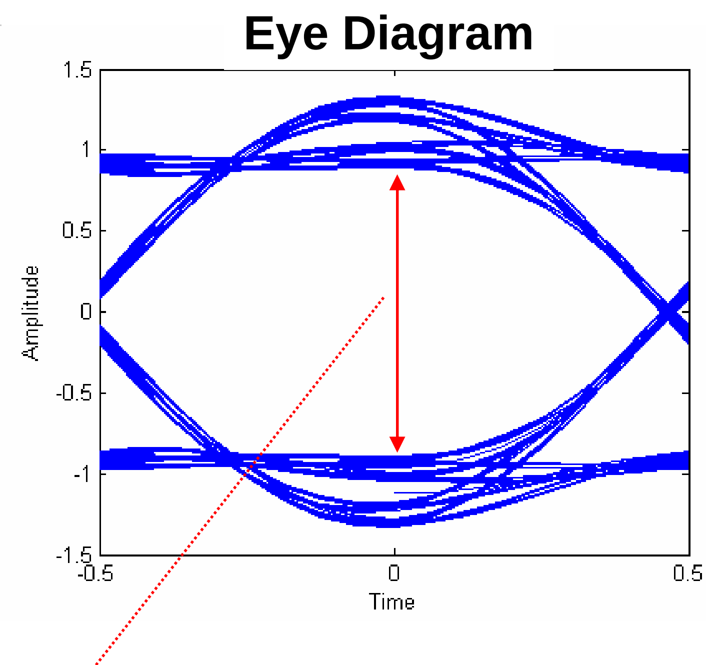
- Eye-diagram is a plot of consecutive pulses on top of each other normalized to the symbol time
- Value at the sampling instant should be equal to the symbol (bit) value
- Large "eye" means little ISI





Example: Band-limited Square Pulses



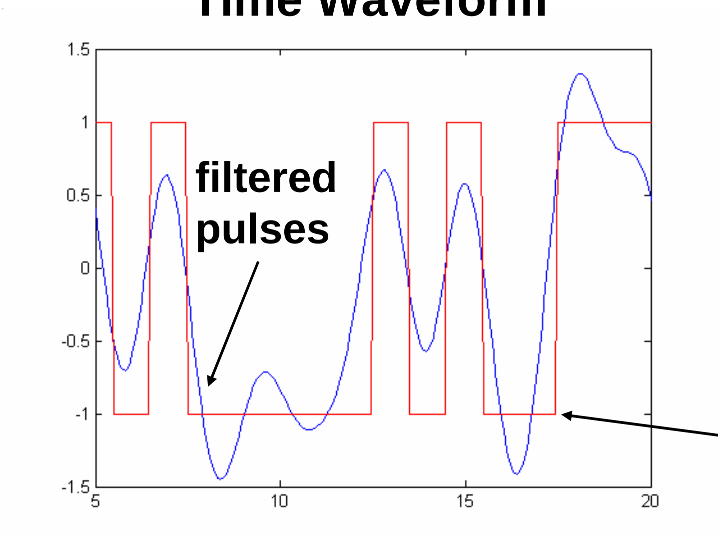


- Smaller “eye”
- Greater susceptibility to noise
- Moderate susceptibility to sampling error

Moderate band-limiting (filtering) due to channel

Example: Band-limited Square Pulses

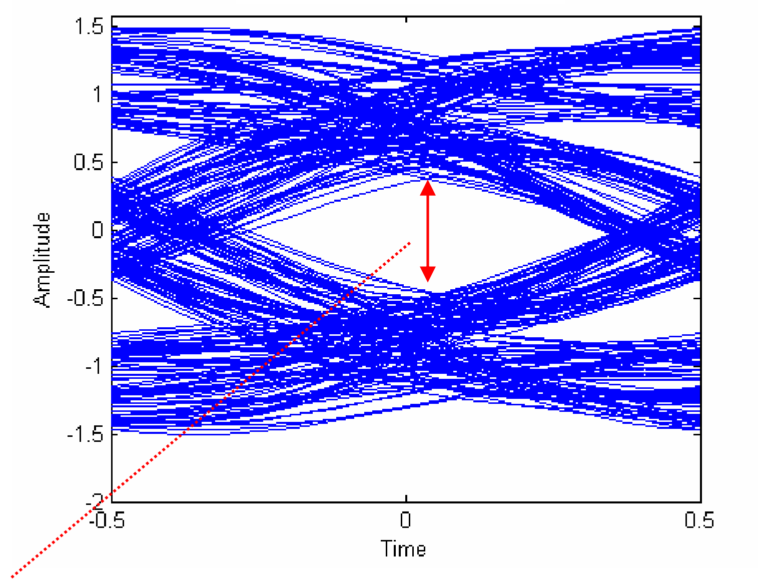
Time Waveform



Square pulses sent but channel limits bandwidth

Original Square pulses

Eye Diagram

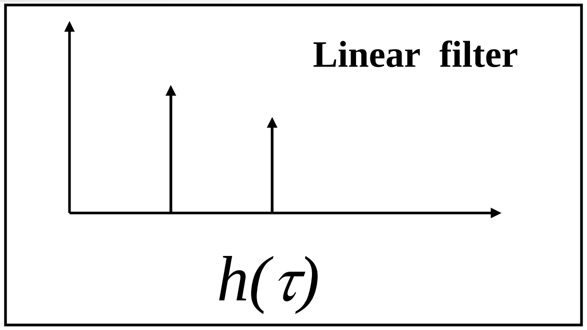


Severe band-limiting (filtering) due to channel

- Small "eye"
- Great susceptibility to noise and sampling error

Multipath Channels

Assuming constant channel $h(\tau)$:



If there is only one tap (flat fading):

$$r\left(kT\right) = I_k g(0) + n(kT)$$

If there are multiple resolvable taps (frequency selective fading), the channel response $g(t)$ will have non-zero values beyond a symbol duration:

$$r\left(kT\right) = \sum_{n=0}^{\infty} I_n g(kT - nT) + n(kT)$$

$$= \underbrace{I_k g\left(0\right)}_{\substack{desired \\ symbol}} + \underbrace{\sum_{\substack{n=0 \\ n\neq k}}^{\infty} I_n g\left(\left[k-n\right]T\right)}_{ISI} + \underbrace{n(kT)}_{\substack{thermal \\ noise}}$$

Intersymbol interference (ISI) will severely degrade performance

Nyquist's Criteria for Zero ISI

- Recall that for pulse shaping we chose pulses to insure that

$$h_e(kT_s) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

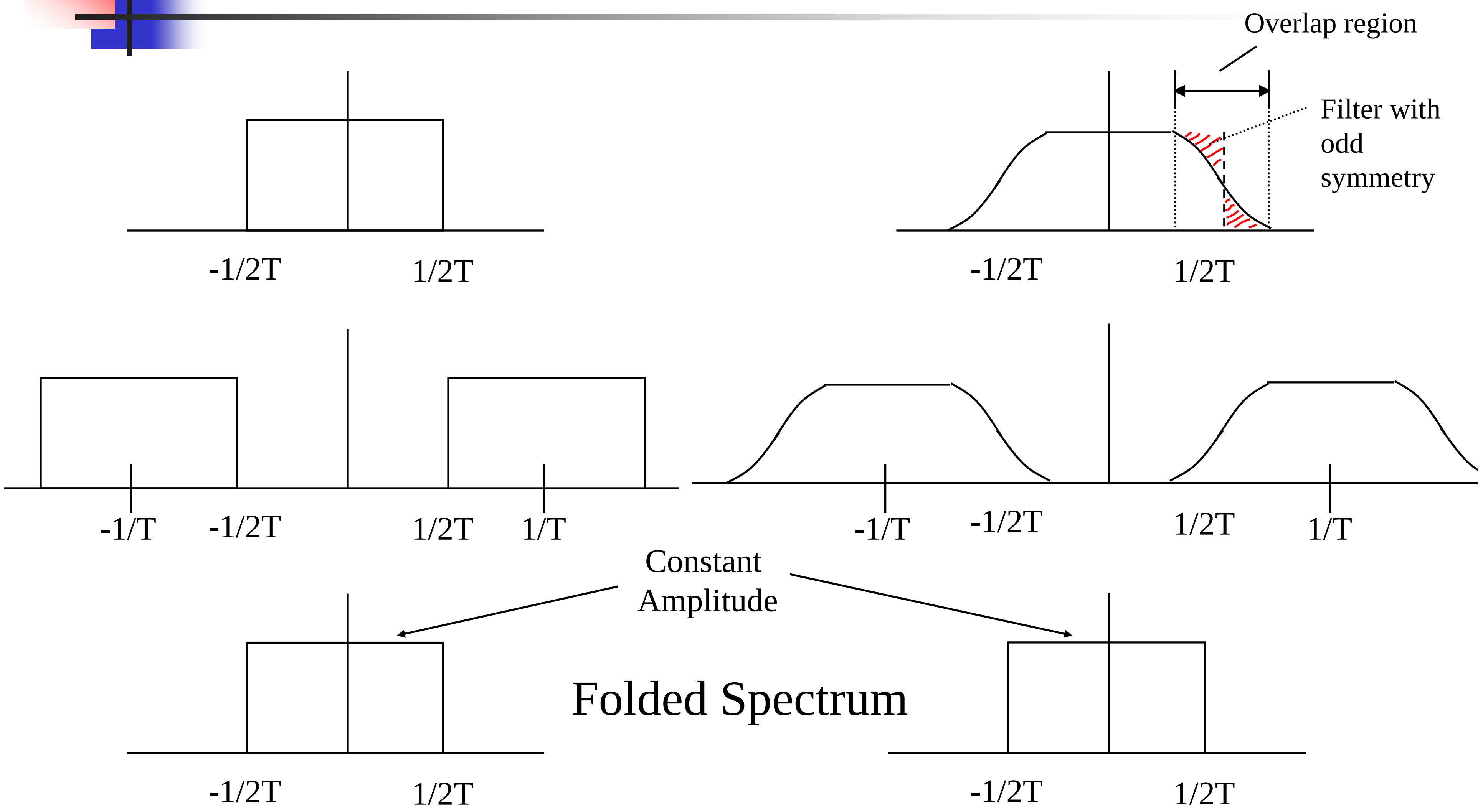
  - where k is an integer and T_s is one symbol duration
- This is equivalent to having a transfer function

$$H_n(f) = \begin{cases} \Pi\left(\dfrac{f}{2f_o}\right) + Y(f) & |f| < 2f_o \\ 0 & else \end{cases}$$

where $f_o = R_s/2$ (i.e., ½ the symbol rate) and $Y(f)$ is a real function that is even symmetric about $f=0$ and odd symmetric about $f=f_o$.

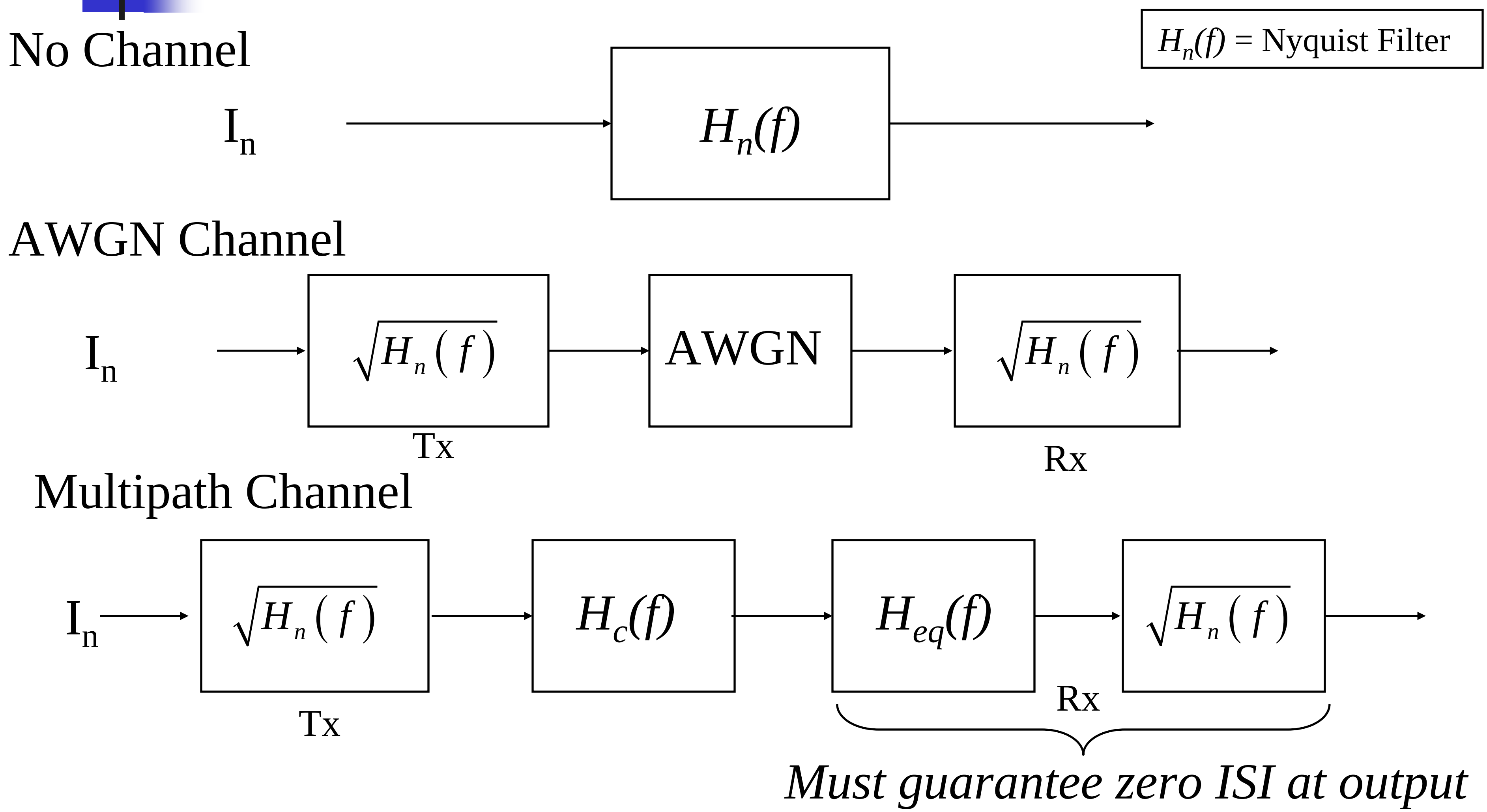
$$\boxed{\begin{array}{ll} Y(-f) = Y(f) & |f| < 2f_o \\ Y(-f+f_o) = -Y(f+f_o) & |f| < f_o \end{array}}$$

Nyquist Filters

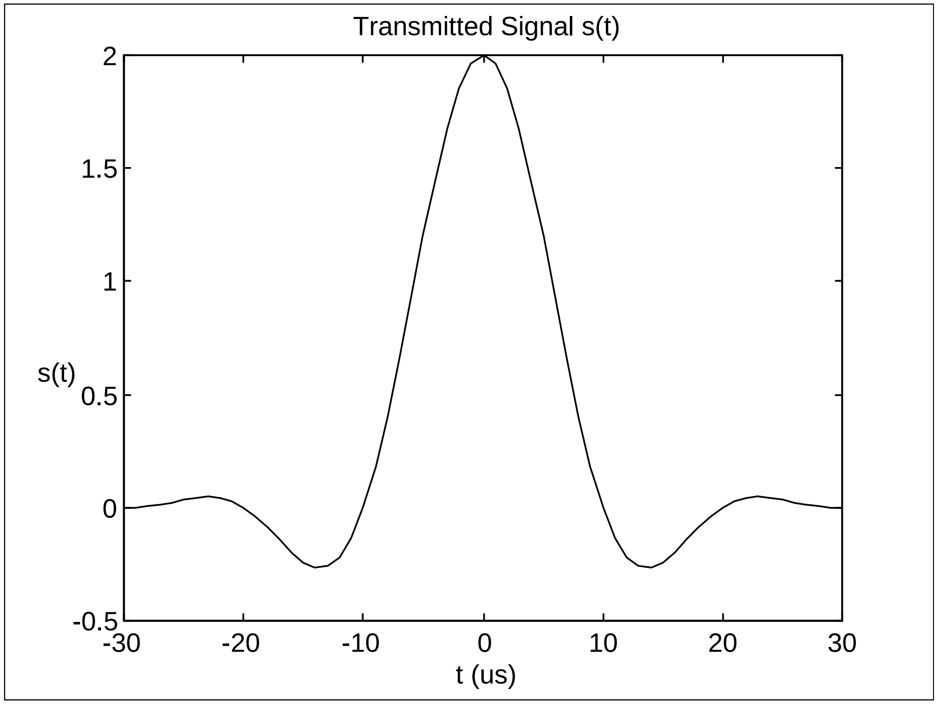


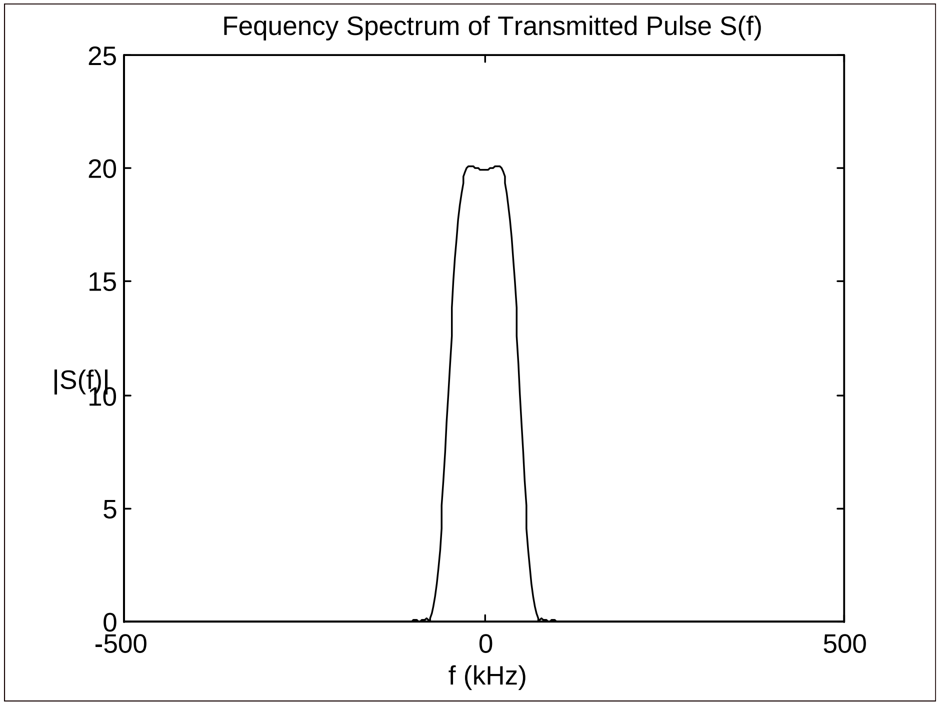
Folded Spectrum

How do we eliminate ISI ?



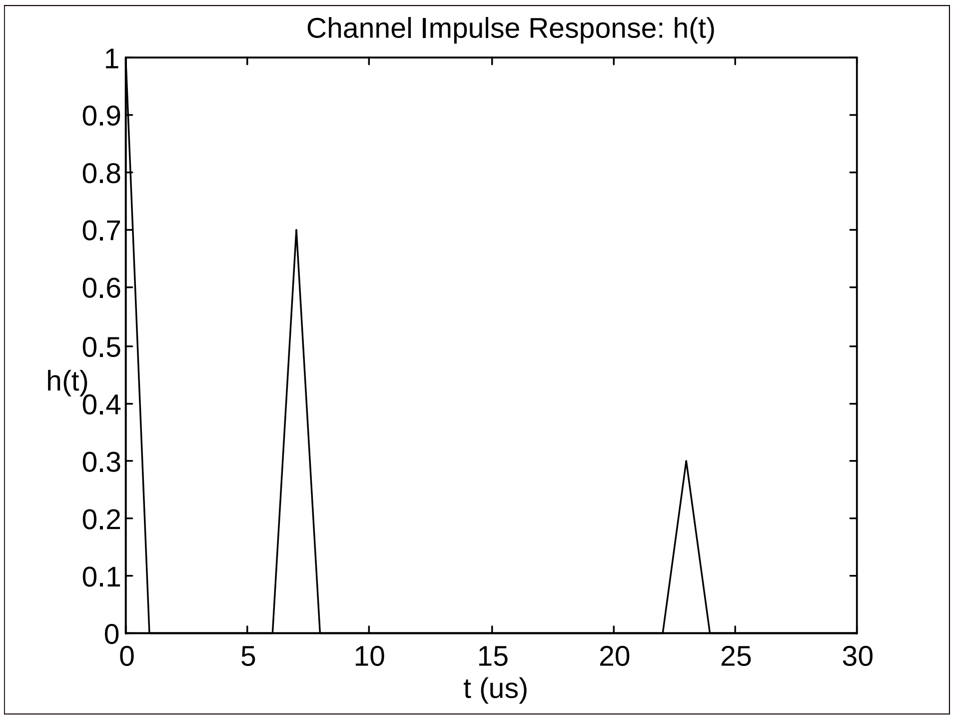
Example of Equalization: Transmitted Signal Pulse (Data Rate = 100 kbits/sec)

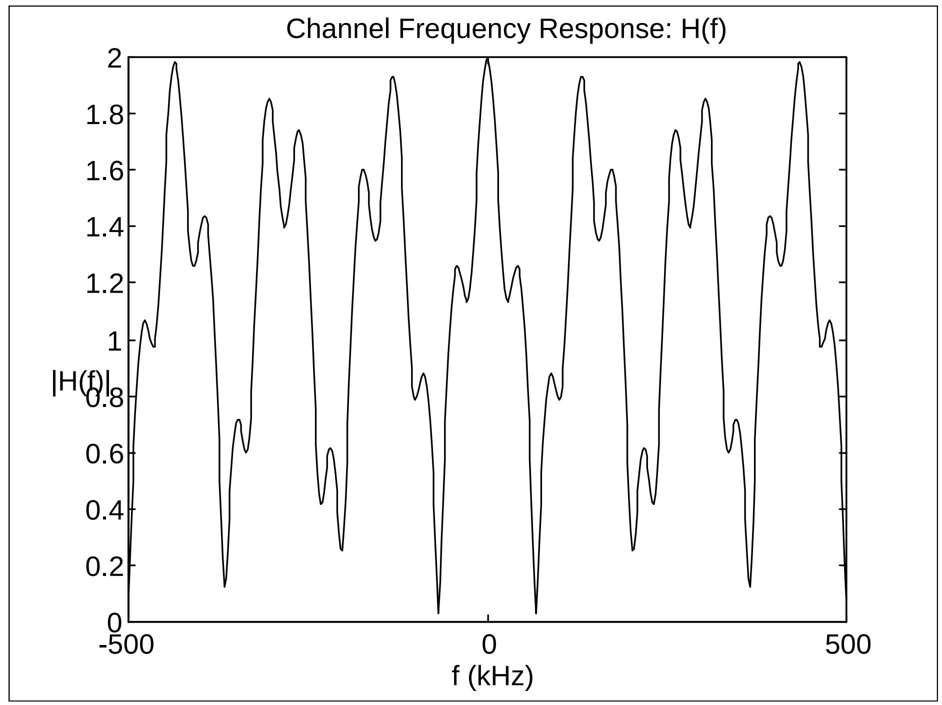




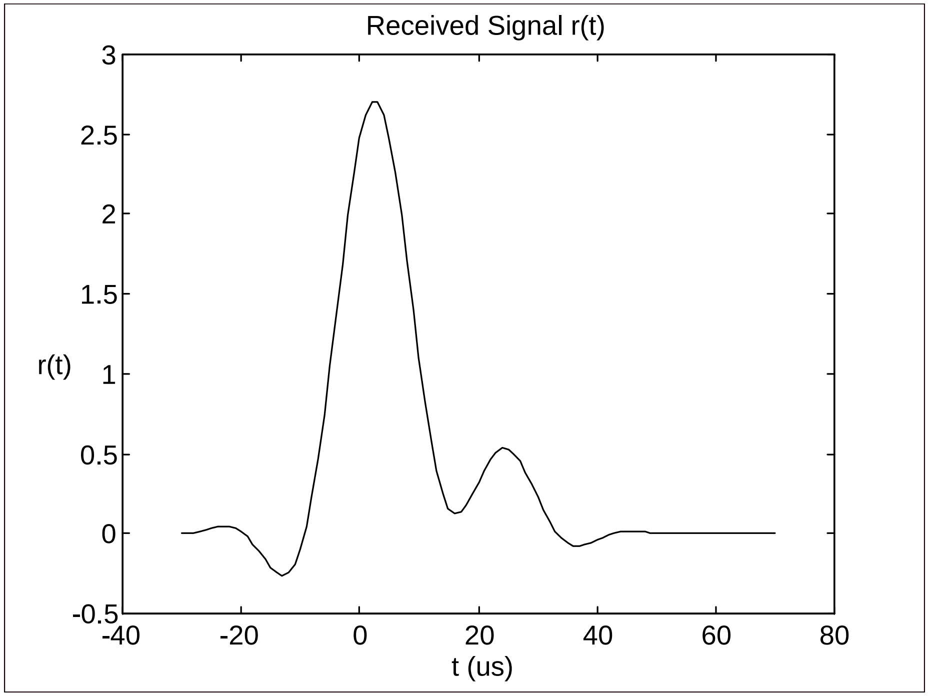
Channel Impulse Response

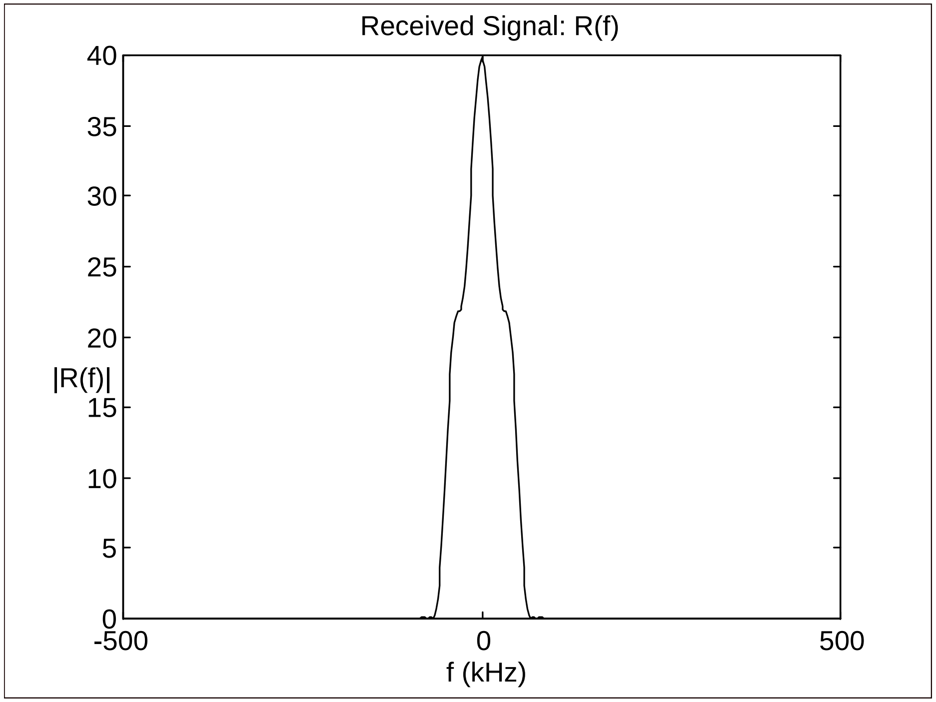
$$h(\tau) = 1\delta(\tau) + 0.7\delta(\tau - 7\mu s) + 0.3\delta(\tau - 23\mu s)$$



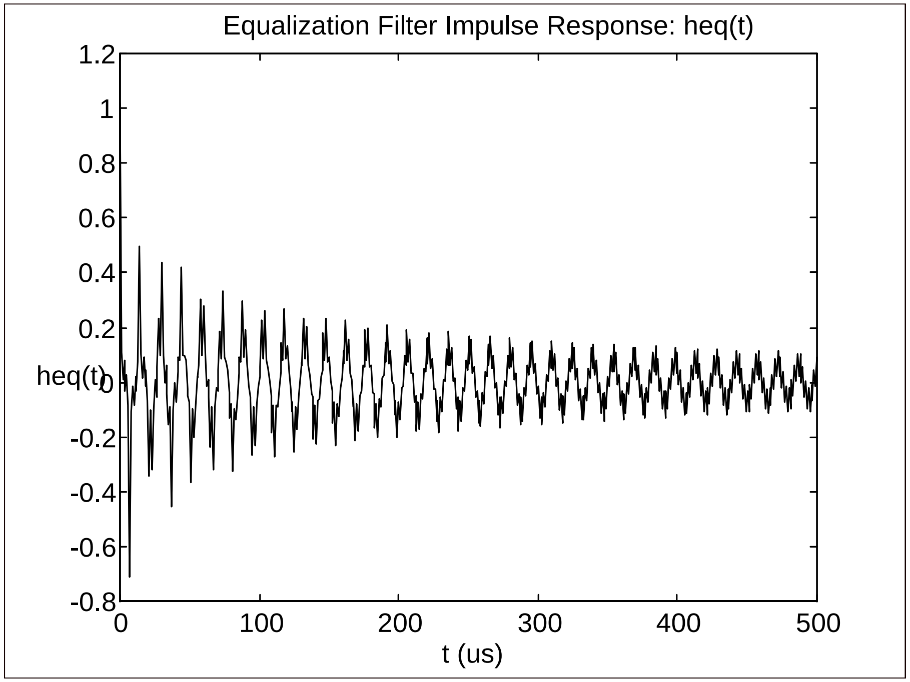


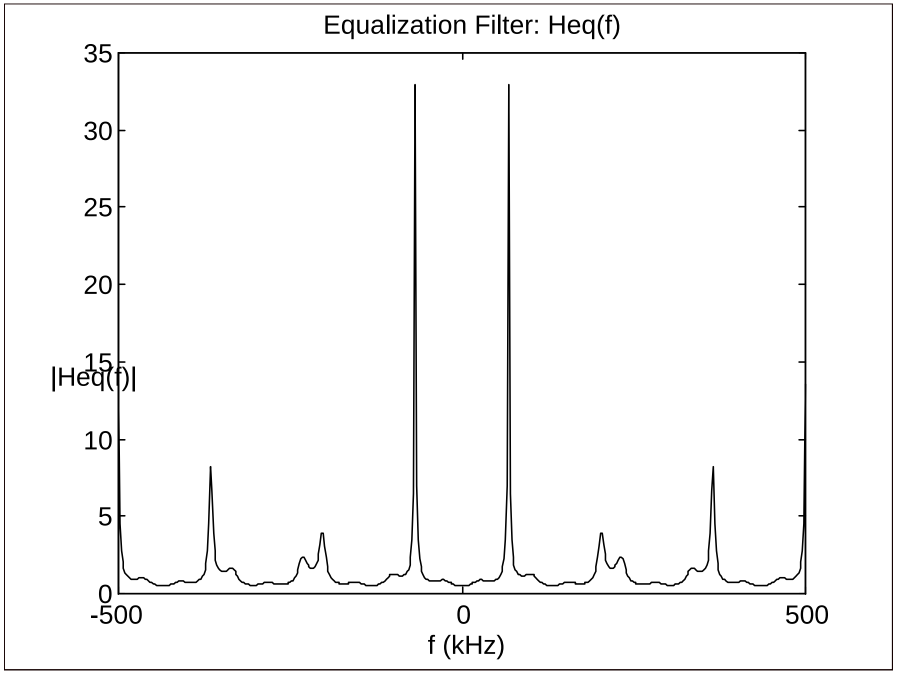
Received Signal: r(t) = s(t)*h(t)



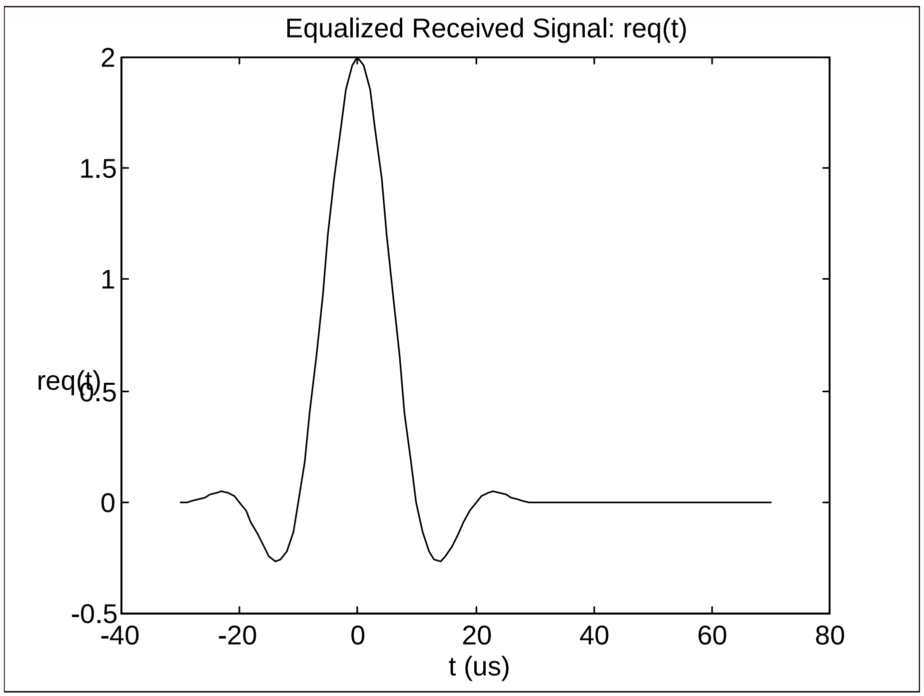


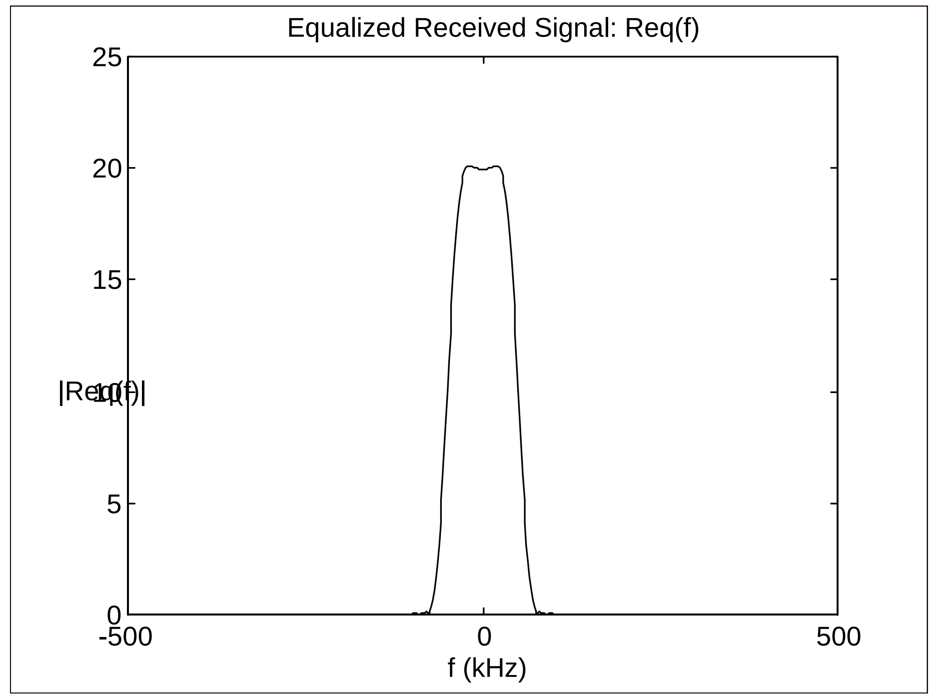
Equalization Filter:





Equalized Received Signal





Complicating factors

- The structure of the inverse filter can become very complicated to implement
- The multipath channel structure is not always known and sometimes must be estimated.
- The channel changes in real time so equalization must be adaptive.
- Further, completely eliminating the ISI may not provide the best bit error rate in the presence of AWGN
  - The minimum BER approach is not necessarily to “invert” the channel
  - Consider the noise gain in bands where the equalizer has peaks

Classes of Equalizer Structures

- Maximum Likelihood Sequence Estimation
  - Optimal equalizer in maximum likelihood sense
  - Viterbi Algorithm
- Linear Equalizers
  - Zero-forcing
  - MMSE
- Decision Feedback
  - Similar to interference cancellation
- We will discuss these structures next class

Model of Received Signal with Correlated Noise

- A matched filter receiver will base it's decision on the decision statistic

$$y_n = \int_{-\infty}^{\infty} r(t) \underbrace{g^*(t - nT)}_{\text{channel response}} dt$$

$$y_k = \sum_n I_n x_{k-n} + v_k ,$$

where

$$I_n = data$$

$$x_n = \int_{-\infty}^{\infty} g^*(t) g(t + nT) dt$$

$$v_k = \int_{-\infty}^{\infty} n(t) g^*(t - kT) dt$$

Maximum Likelihood Receiver for this Model

Matched Filter Output

$$y_k = \sum_n I_n x_{k-n} + \nu_k$$

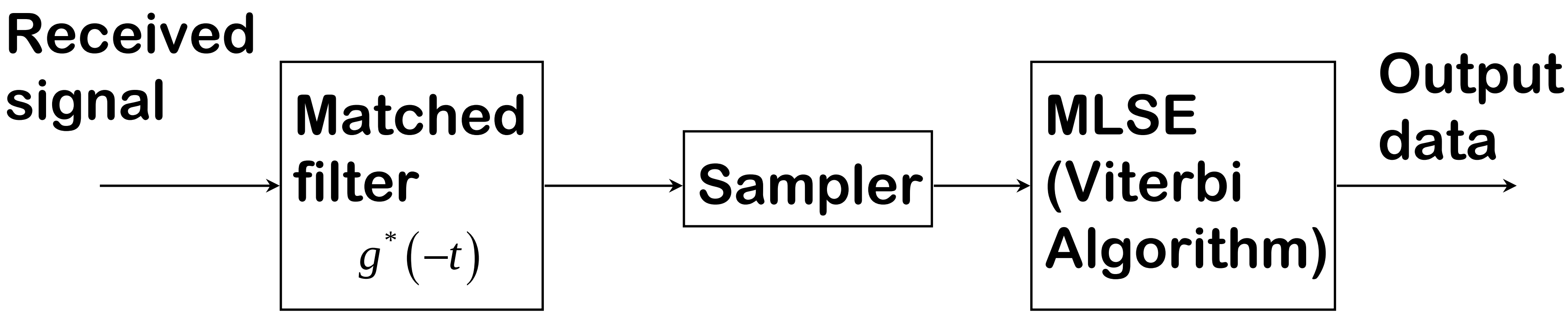
Maximizing the conditional probability function is equivalent to choosing the data sequence which maximizes:

$$2\operatorname{Re}\left(\sum_n I_n y_n\right) - \underbrace{\sum_n \sum_m I_n^* I_m x_{n-m}}_{\substack{ISI \\ terms}}$$

This metric can be computed recursively via the Viterbi Algorithm

- **There is a similarity between the ISI channel and a convolutional code which we will discuss later**

Maximum Likelihood Sequence Estimator (MLSE)



Problems with MLSE

- Still requires channel estimates (i.e., we need to know the structure of the underlying channel filter) to compute metrics
- Complex for practical systems. For binary signaling with ISI of length L, Viterbi algorithm must compute 2^L metrics.
- Linear equalization techniques lead to more manageable complexity
- Basic structure: Linear Transversal Filter

Matched Filtering

- Recall that the received signal is

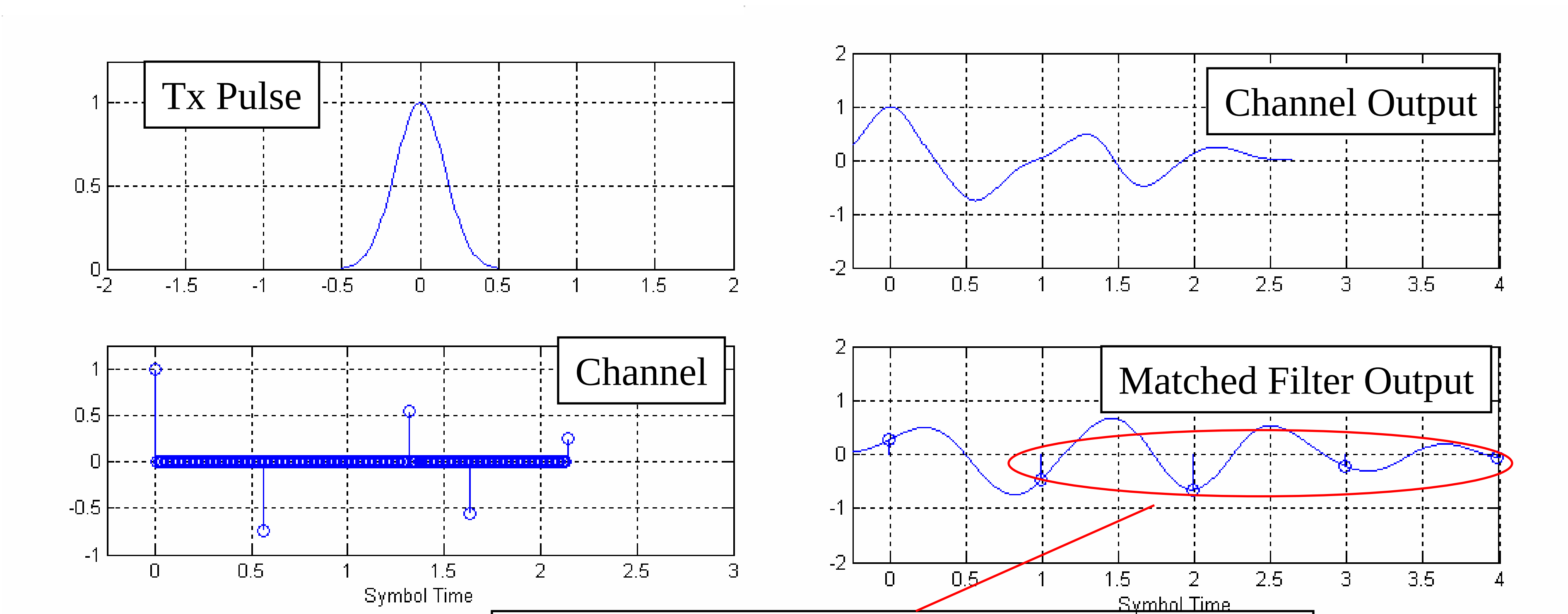
$$r(t) = \sum_{n=0}^{\infty} I_n g(t - nT) + n(t)$$

where

$$\underbrace{g(t)}_{\text{output pulse}} = \int_{-\infty}^{\infty} p(\tau)\, \underbrace{h(t-\tau)}_{\text{IR of channel}}\, d\tau$$

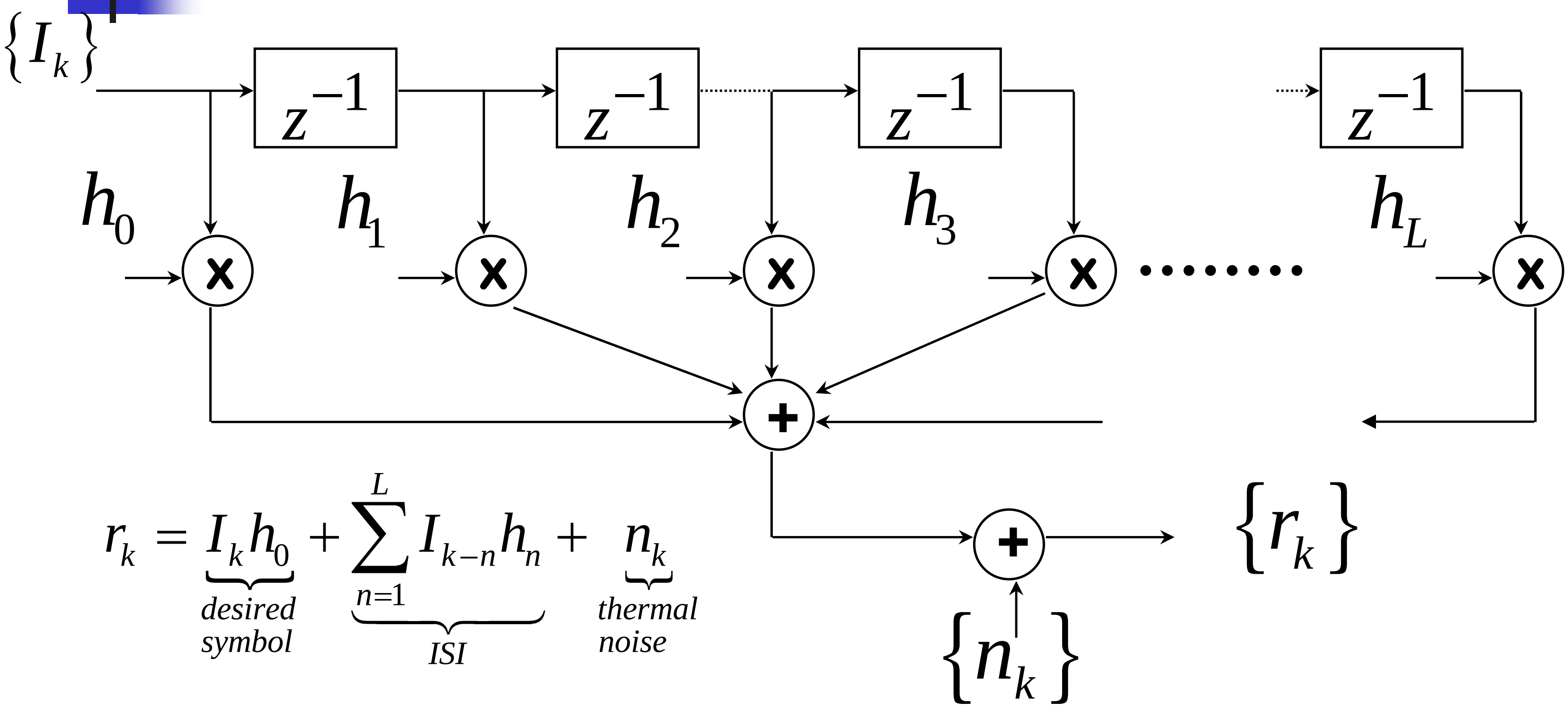
- Thus, to maximize SNR (before removing ISI) we should pass the received signal through a filter matched to *g(t)*
- However, for ease of the discussion we will ignore the pulse distortion and concentrate on the ISI. This means that we will assume that the channel is made up of discrete multipath components with spacing *1/T.*

Example



For the discrete tap model, we will simply model the resulting discrete ISI terms in the sampled matched filter output

Tapped Delay Model of Channel with ISI (assuming L paths)



$$r_k = \underbrace{I_k h_0}_{\text{desired symbol}} + \underbrace{\sum_{n=1}^{L} I_{k-n} h_n}_{\text{ISI}} + \underbrace{n_k}_{\text{thermal noise}}$$

We will assume that the channel impulse response has taps spaced at *1/T* and then examine the output of the pulse matched filter. This eliminates correlated noise, pulse distortion and other complicating factors.

Key References on Equalizers

- John G. Proakis, Digital Communications, 4th ed., McGraw-Hill, 2001 (Chapters 10 and 11)
- Shahid U. H. Qureshi, "Adaptive Equalization," *Proceedings of the IEEE*, Vol. 73, No. 9, pp. 1349, Sept. 1985.
- Much of the lectures over the next few classes is drawn from Proakis although some of the notation may be slightly different