EE 5654 - Digital Communications Spring 2005

Lecture #17 - OFDM

Instructor: R. Michael Buehrer

Introduction

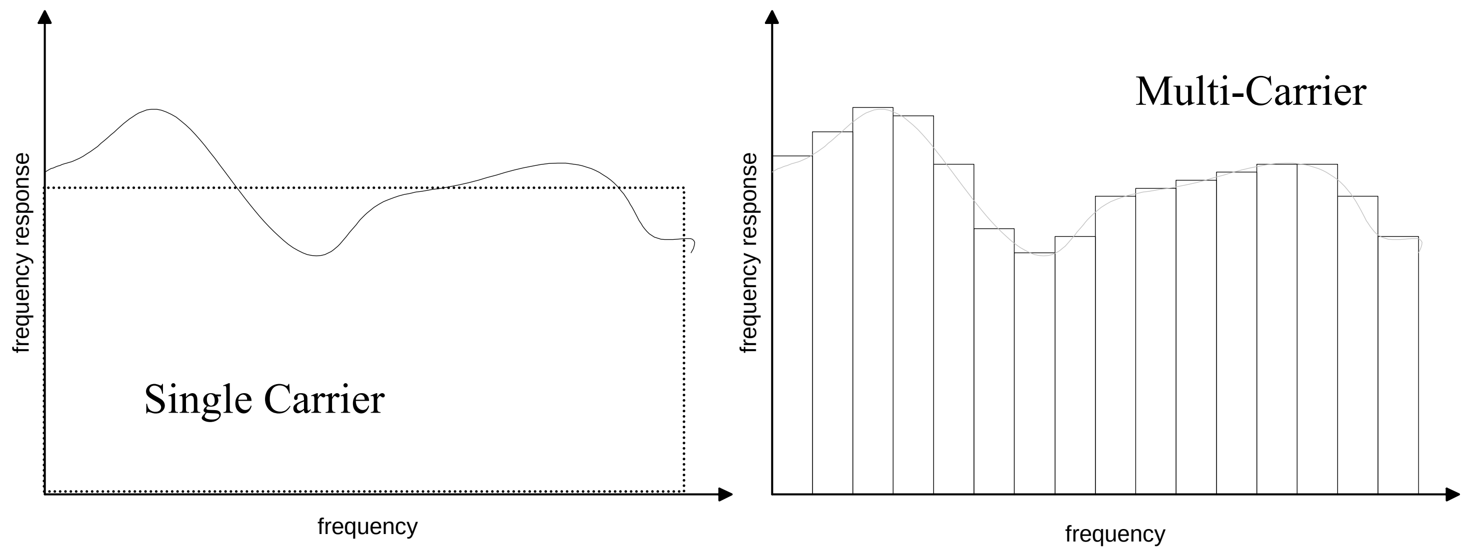
- In the previous three lectures we discussed techniques to handle the impact of frequency selective fading
  - Frequency selective fading is a major impediment to high data rate communications
- A modulation approach which is resistant to multipath fading is Orthogonal Frequency Division Multiplexing or OFDM
- Basic idea: Convert a single high rate signal which experiences frequency selective fading into a large number of parallel low rate signals each of which experiences flat fading
- Exploits
  - the minimum theoretical spacing of orthogonal carriers
  - Fast Fourier Transform

Multi-carrier modulation

- Typical mobile radio channel is a fading channel that is *flat* or *frequency selective*
- For high bandwidth applications channel is frequency selective and delay spread dictates throughput
- Multicarrier modulation is a technique where multiple low data rate carriers are combined by a transmitter to form a composite high data rate transmission
- In a classic multi-carrier system, the available spectrum is split into several non-overlapping frequency sub channels. The individual data elements are modulated into these sub channels and are thus frequency multiplexed

Multi-carrier Modulation

- Increases the symbol time by modulating into narrow sub-channels. Increase in symbol time makes it more robust to delay spread effects



Channel frequency responses for a single carrier and multicarrier system. In the multicarrier system each sub channel only undergoes slight distortion

Orthogonal Frequency Division Multiplexing (OFDM) - Introduction

- In classic multicarrier system guard bands have to be inserted resulting in poor spectral efficiency
- A more efficient approach is to allow the spectra of individual subcarriers to overlap
- Problem: If individual subcarriers are overlapping isn't there interference between carriers?
- Answer: No! If subcarrier tones are separated by the inverse of the signaling symbol duration, independent separation of frequency multiplexed tones is possible.
  - This ensures that the spectra of individual sub channels are zeros at other subcarrier frequencies.

This is the basic principle of OFDM

OFDM carrier

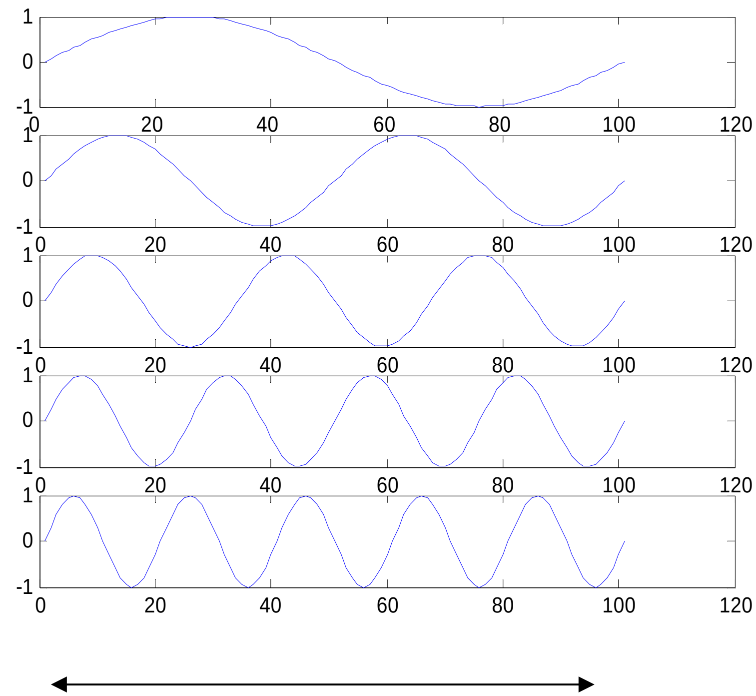
- Orthogonal waveforms are generated by using signals that have integer number of cycles in the duration T_{sym}
- The base-band equivalents of the orthogonal subcarriers satisfy the following relationship where *k* and *i* are subcarrier indices

$$f = 1/T_{sym}$$

$$\text{Real Part} \int_0^{T_{sym}} \cos(2\pi kft).\cos(2\pi ift) = \begin{cases} T_{sym}/2 & (k = i) \\ 0 & (k \neq i) \end{cases}$$

$$\text{Imaginary Part} \int_0^{T_{sym}} \sin(2\pi kft).\sin(2\pi ift) = \begin{cases} T_{sym}/2 & (k = i) \\ 0 & (k \neq i) \end{cases}$$

$$\text{and} \int_0^{T_{sym}} \sin(2\pi kft).\cos(2\pi ift) = 0$$



T_{sym}

Subcarriers in OFDM

OFDM Carrier

- The base-band information on the *k*th subcarrier can be written as

$$\underbrace{\left(x_k + jy_k\right)}_{\text{data symbol}}\underbrace{\left\{\cos(2\pi kft) + j\sin(2\pi kft)\right\}}_{k\text{th carrier}}$$

- The OFDM signal is the sum of all the signals in each of its subcarriers which can be written as (usually implemented using IFFT)

$$s(t) = \sum_{k=0}^{N-1}\left(x_k + jy_k\right)\left\{\cos(2\pi kft) + j\sin(2\pi kft)\right\}$$

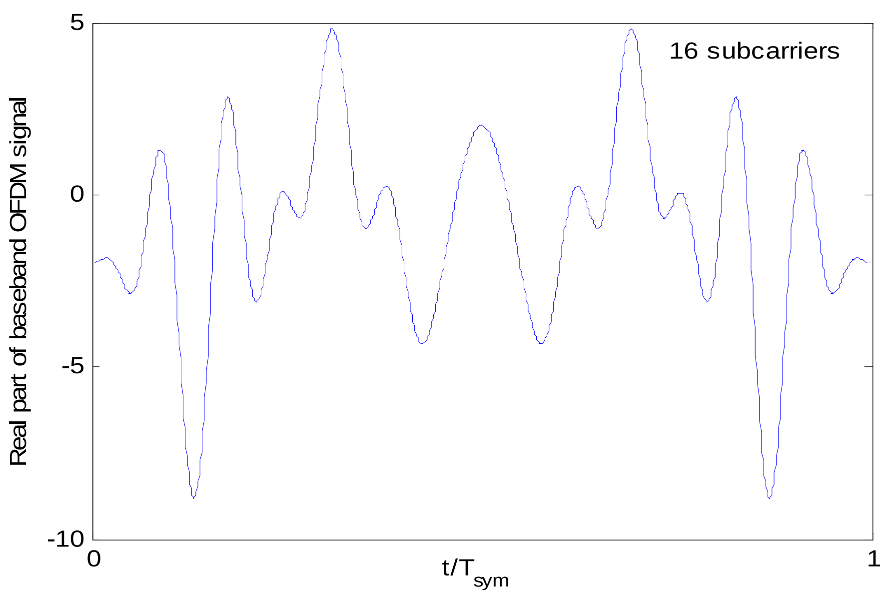
- The individual modulated symbols at the receiver are recovered using the FFT. The *k*th output from the FFT is:

$$Z_k = \int_0^{T_{sym}} s(t)\left\{\cos(2\pi kft) - j\sin(2\pi kft)\right\}dt = \sum_{n=0}^{N-1}\left\{\begin{array}{l}\displaystyle\int_0^{T_{sym}} \cos(2\pi kft)\left(x_k + jy_k\right)\left\{\cos(2\pi kft) + j\sin(2\pi kft)\right\}dt + \\ \displaystyle j\int_0^{T_{sym}} \sin(2\pi nft)\left(x_k + jy_k\right)\left\{\cos(2\pi kft) + j\sin(2\pi kft)\right\}dt\end{array}\right\}$$

$$= x_k + jy_k$$

OFDM

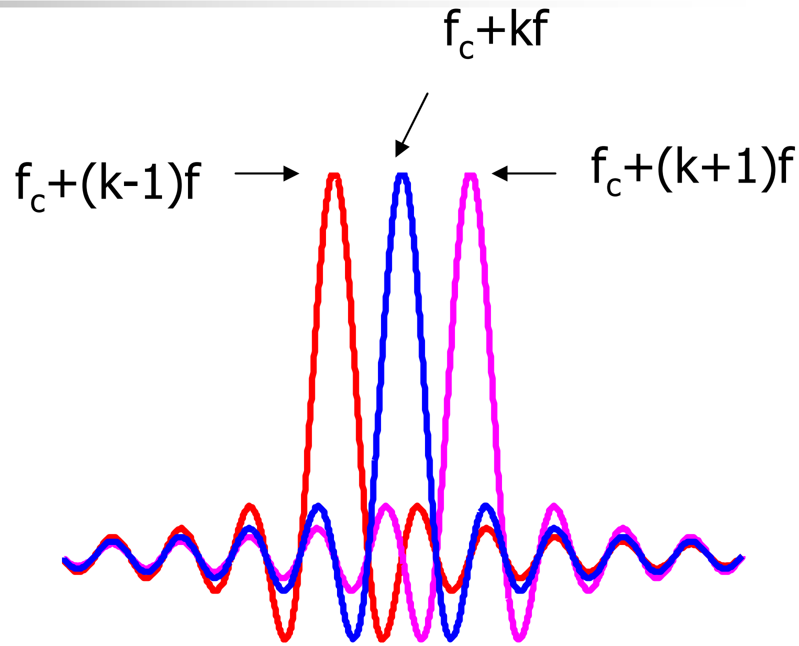
- N serial input data elements separated by $\Delta t = 1/f_s$ (f_s is symbol rate)
- Symbol duration increased to $T = N\Delta t$ (helps in time dispersive channels)
- Subcarrier separation $\Delta f = 1/(N\Delta t)$
- Sub-bands overlap but are still *orthogonal*



example time domain waveform

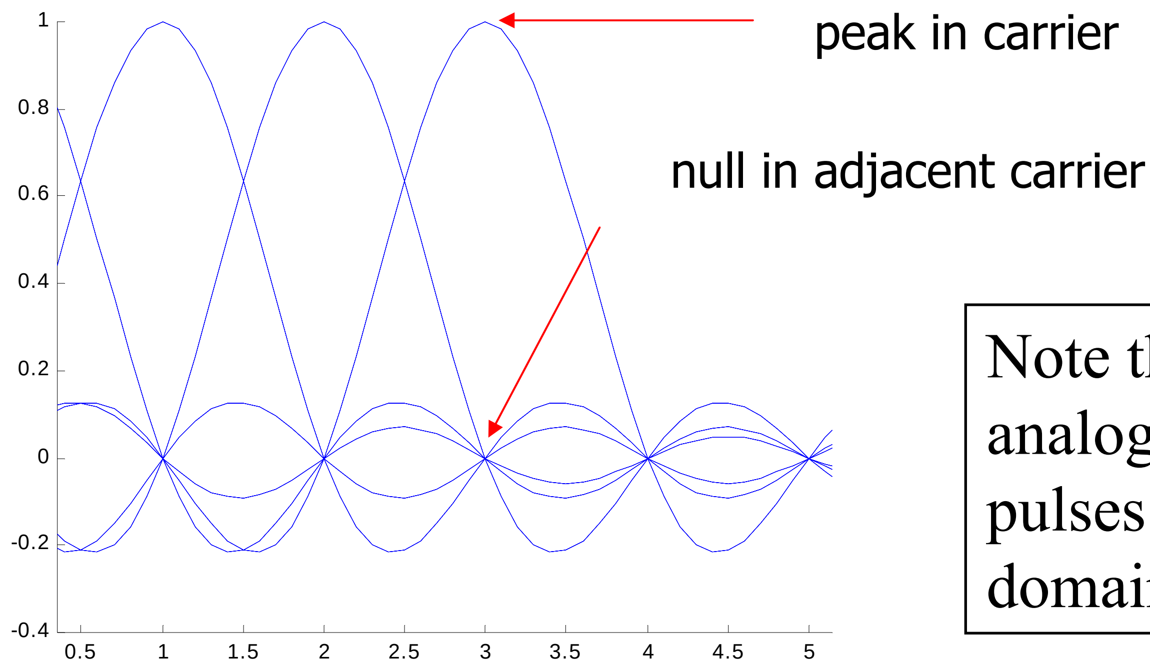
OFDM Spectrum

- The individual spectra of the subcarriers are sinc functions
  - Time gated sinusoids
- Zero crossings occur at every integer multiple of *f* and hence no Inter-Carrier Interference occurs in the frequency domain
- Note the analogy with time-domain sinc pulses



Spectral Efficiency

- For N sub-carriers, the bandwidth of conventional FDM is $2N/T$ while that of OFDM is $(N+1)/T$. By allowing the sub-carrier spectra to overlap, OFDM improves the spectral efficiency.

- 

Note that this is analogous to sinc pulses in the time-domain

Advantages of OFDM

- For a given delay spread, implementation complexity is much lower than that of a single carrier with an equalizer.

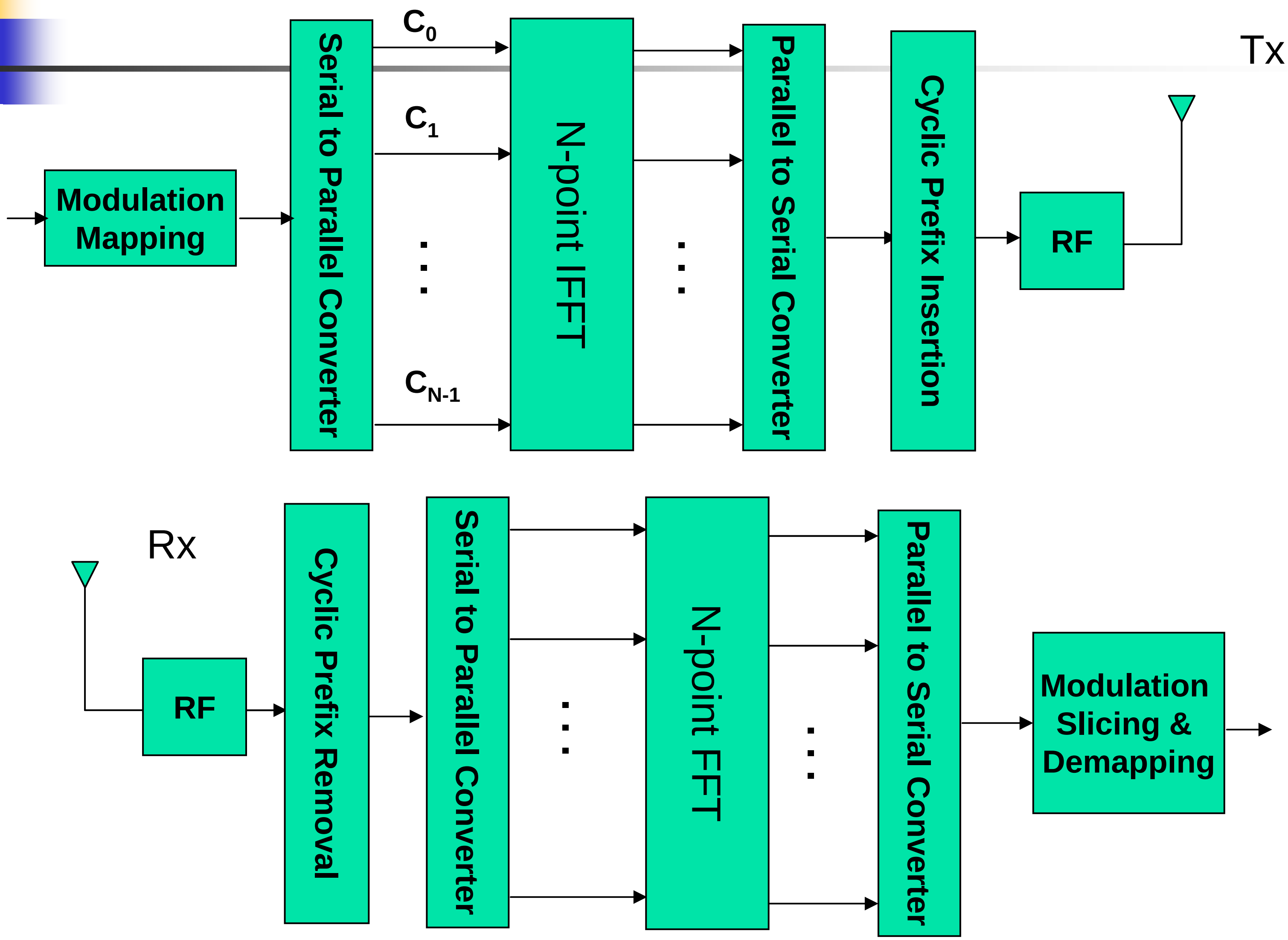
  Note: Channel delay spread causes inter-symbol interference (ISI). ISI causes irreducible error floor, hence limits maximum data rate. Symbol duration of each sub-carrier in OFDM is N times longer than that of single carrier system. Hence, OFDM is more robust to delay spread.
- Equalization in OFDM is just one tap multiplication for each sub-carrier while in SC system the equalization complexity can be prohibitively high especially for large delay spread channel.
  - This low complexity equalization is one of the main advantages of OFDM.
- Robust against narrowband interference (impulse noise) owing to frequency diversity.
- Adaptive bit loading/ modulation, power distribution across sub-carriers
- Maximize capacity according to sub-channel responses

Allows much larger data rates without equalization than with single carrier modulation

Disadvantages of OFDM

- High Peak-to-Average Power Ratio (PAPR)
  - Being a sum of several sinusoids, the time-domain OFDM signal has a high PAPR which can cause nonlinear distortion of the signal at the transmit amplifier. (In-band and out-of-band distortion)
    - Can be avoided with strictly linear amplifiers, but they are very inefficient
  - Having high PAPR is one of the main problems in OFDM.
- High Sensitivity to Frequency Offset Errors
  - Frequency offset destroys orthogonality among sub-carriers
  - High Doppler will also cause this problem
  - Another main problem of OFDM

Basic Transceiver Block Diagram



Received signal model

The output of the IFFT block can be written as

$$s(n) = \sum_{k=0}^{N-1} \left(x_k + jy_k \right) e^{j2\pi nk/N} = \sum_{k=0}^{N-1} d_k e^{j2\pi nk/N}$$

The received OFDM signal can be written as

$$r(n) = \sum_{l=0}^{L} h(n,l) s(n-l) + w(n)$$

where *L* is the number of multipaths, *h(n,l)* represents the time varying nature of the channel coefficients for each path, *l*, w(n) is the additive white Gaussian noise and *s(n)* represents the output of the IFFT at the transmitter.

Received signal model

After some manipulation, we can represent the signal in frequency domain

$$r(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k H_k(n) \exp(j2\pi nk / N) + w(n) \qquad 0 \le n \le (N-1)$$

where $H_k(n)$ is the Fourier transform of the channel impulse response at time '*n*'.

At the receiver after the FFT process the received signal can be written as

$$Y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(n) \exp(-j2\pi nk / N) = d_k H_k + \alpha_k + W_k$$

where α_k represents the Inter-Carrier Interference term while W_k represents the noise term

Compact Representation

- In vector form the expression for the received signal vector after the removal of the guard time can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{d} + \mathbf{w}$$

- where **d** is the vector containing the individual modulated symbols and $\mathbf{H}$ is the matrix containing the channel coefficients. The $\mathbf{H}$ matrix is given by,

$$\mathbf{H} = \frac{1}{\sqrt{N}} \begin{bmatrix} H_0(0) & H_1(0) & \cdots & H_{N-1}(0) \\ H_0(1) & H_1(1)e^{j2\pi/N} & \cdots & H_{N-1}(1)e^{j2\pi(N-1)/N} \\ \vdots & \vdots & \ddots & \\ H_0(N-1) & H_1(N-1)e^{j2\pi(N-1)/N} & \cdots & H_{N-1}(N-1)e^{j2\pi(N-1)(N-1)/N} \end{bmatrix}$$

Compact Representation (cont.)

At receiver:

1. After sampling, remove the cyclic prefix
2. Take FFT
3. Compensate for the channel

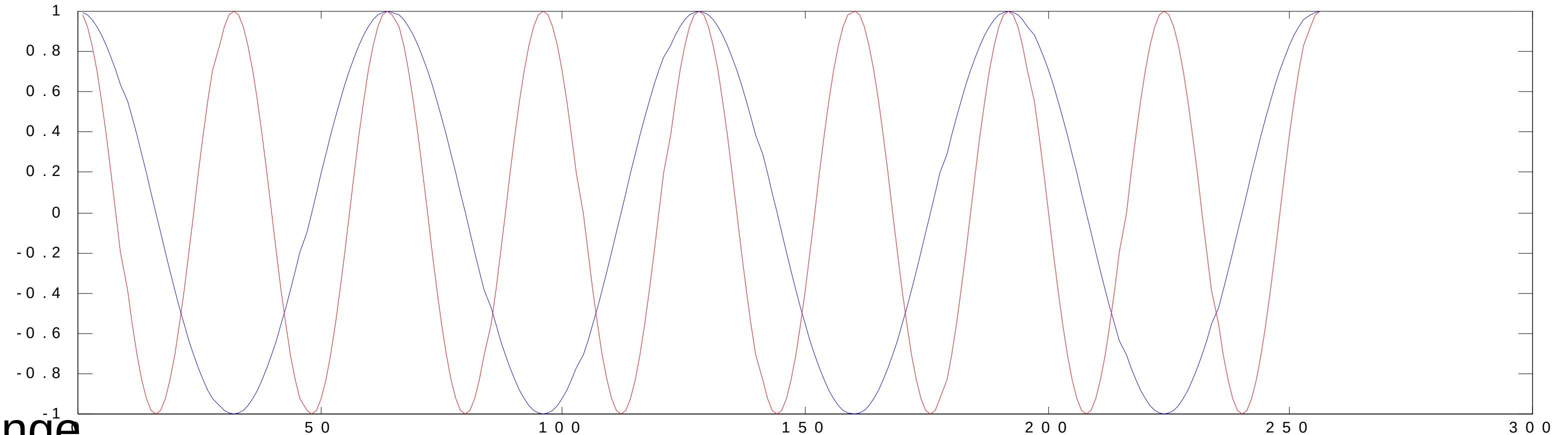
Can be incorporated into a single matrix operation $\mathbf{H}$

The FFT Block essentially multiplies by the conjugate of the received signal vector

$$\mathbf{z} = \mathbf{H}^H \mathbf{r} = \mathbf{H}^H \mathbf{H} \mathbf{d} + \mathbf{H}^H \mathbf{w}$$

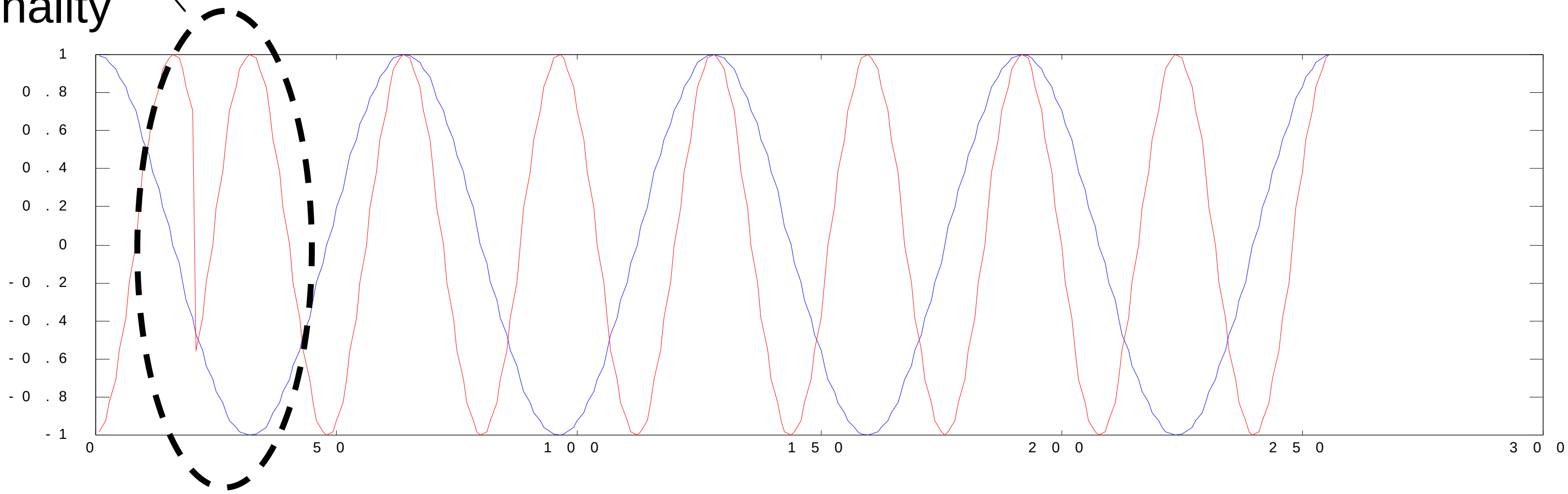
Multipath Delay

- When two versions of the signal arrive at the receiver with different delays, the two paths will not be orthogonal.
- Sign changes in consecutive bits will destroy the orthogonality between carriers
- This can be remedied by extending the length of the symbol using a cyclic prefix
- The cyclic prefix is disregarded at the receiver maintaining orthogonality at the expense of slight time inefficiency



Sign change destroys orthogonality

Carriers 4 and 8 out of 256 with no delay difference



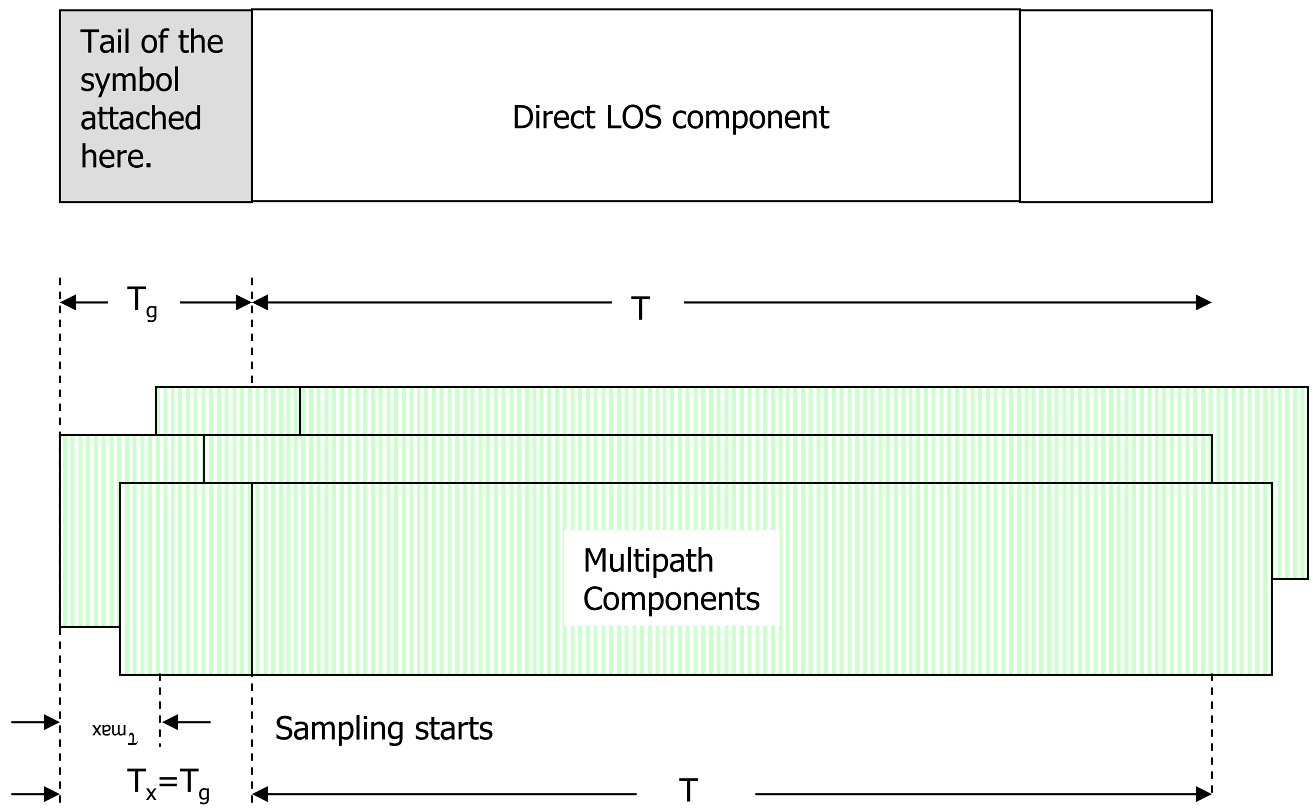
Carriers 4 and 8 out of 256 with 20 sample delay difference

Cyclic Prefix in OFDM

- By dividing the input data stream into *N* subcarriers, the symbol duration is made *N* times larger, which reduces the multipath delay spread relative to the symbol time, by the same factor.
- However, ISI can still occur if delay spread is large.
- To remove ISI completely, a guard time is inserted in each OFDM symbol. This guard time is always chosen to be larger than the maximum delay spread due to the channel.
- The OFDM symbol is cyclically extended in the guard time. This ensures that the delayed replicas always have an integer number of cycles during the FFT interval
- The guard time T_g should be always greater than the worst case delay spread (ζ_{max}) of the channel.

Cyclic Prefix – Contd.

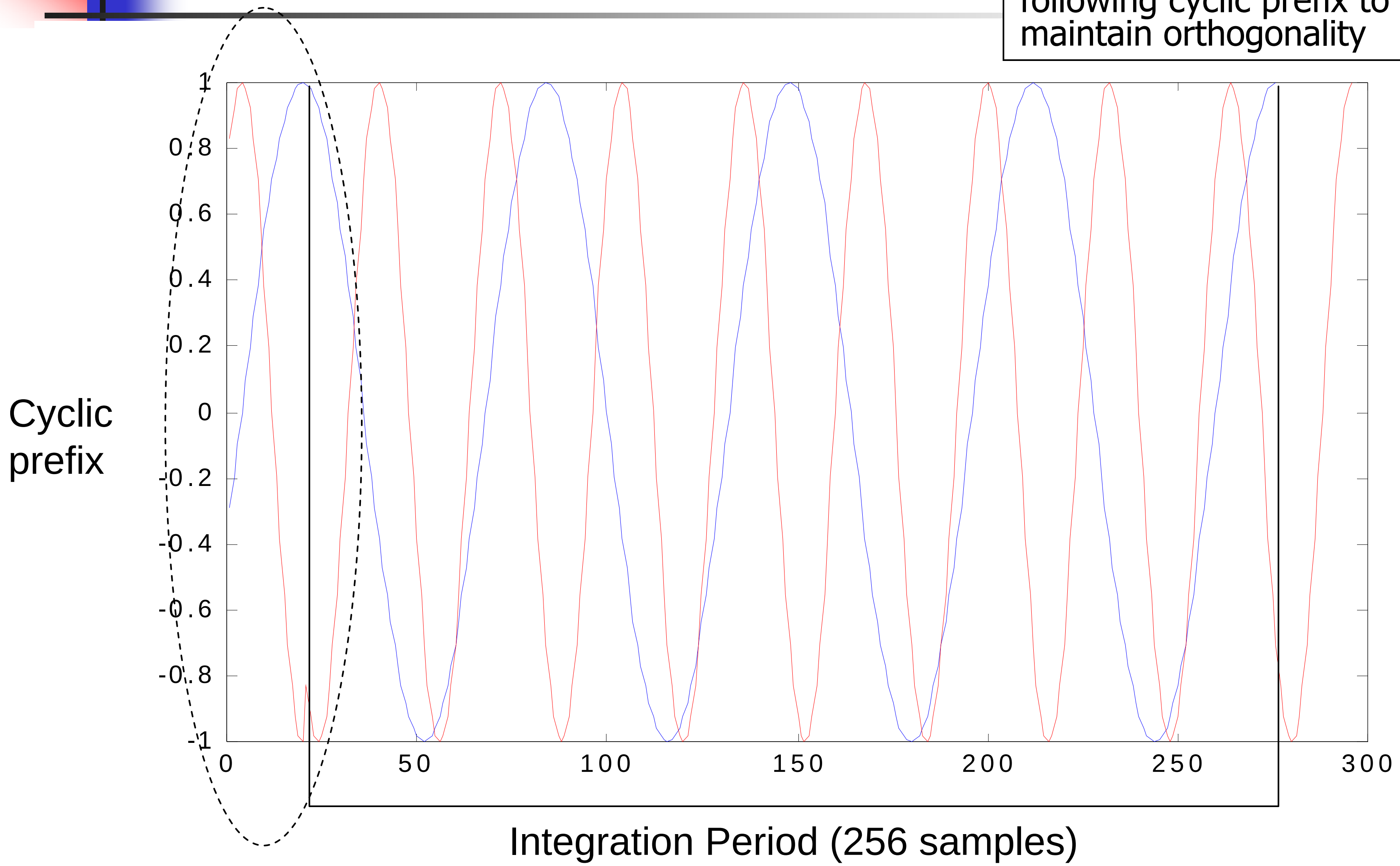
During the FFT interval, the OFDM receiver sees a sum of pure sine waves which does not destroy the orthogonality between the subcarriers.



Cyclic Prefix

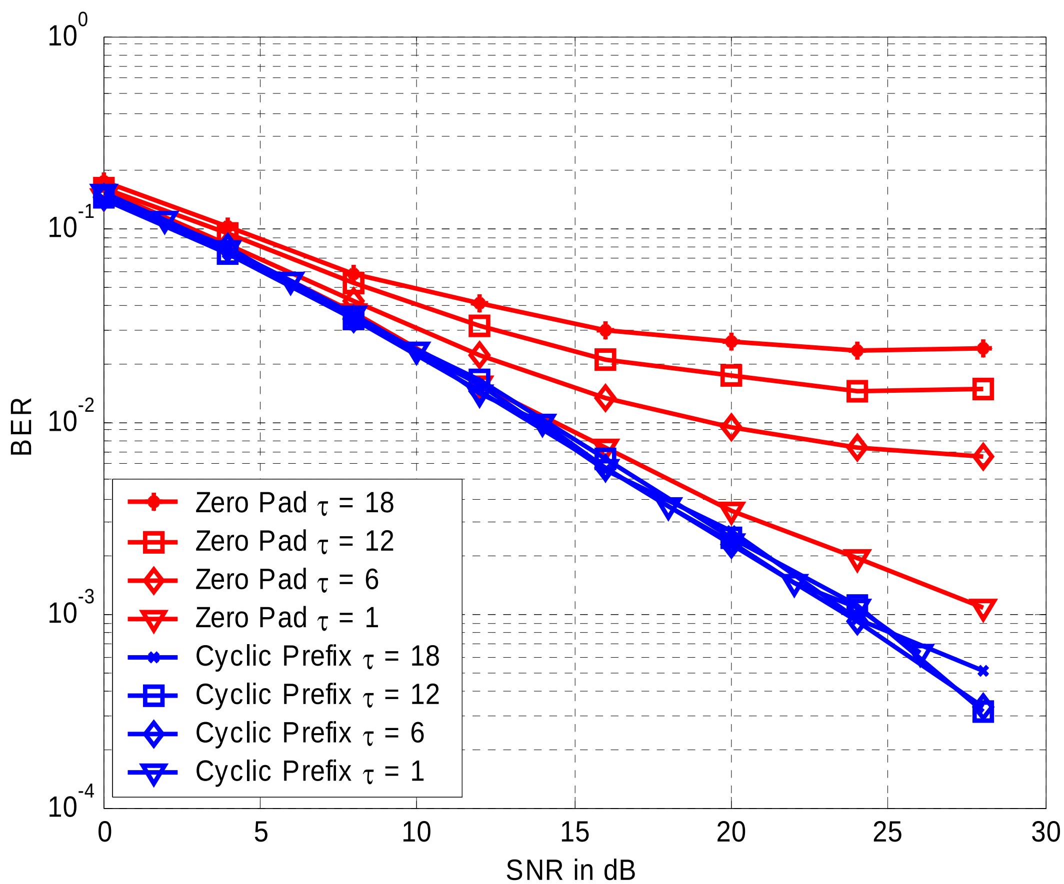
Steps:

- Strip off the cyclic prefix which contains ISI
- Integrate over one period following cyclic prefix to maintain orthogonality



Performance results

- OFDM makes a frequency selective channel a flat fading channel on each carrier
- Insertion of Cyclic Prefix eliminates the inter-carrier interference

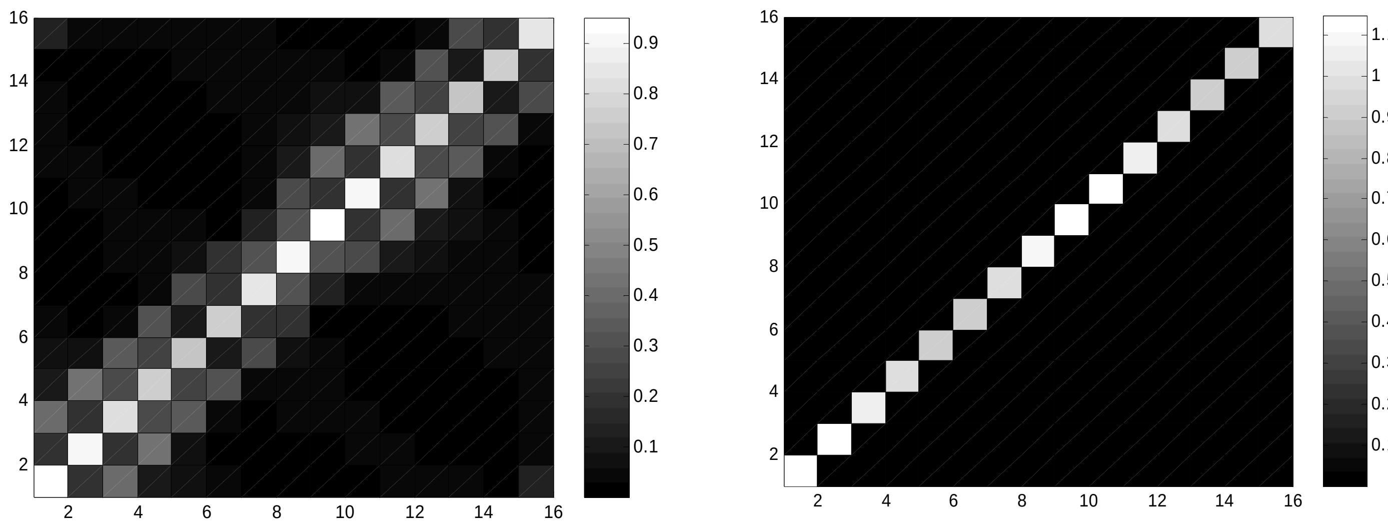


Impact of Doppler Spread

- The carriers only remain orthogonal provided that the channel is constant over the duration of the OFDM symbol
- This becomes more unlikely since the OFDM symbol is N times longer than the original symbol
- Doppler effects cause the signal to change in time and possibly over a symbol duration causing inter-carrier interference (symbols are no longer orthogonal)
- In the frequency domain we can envision a frequency shift or frequency smearing

The effect of Doppler

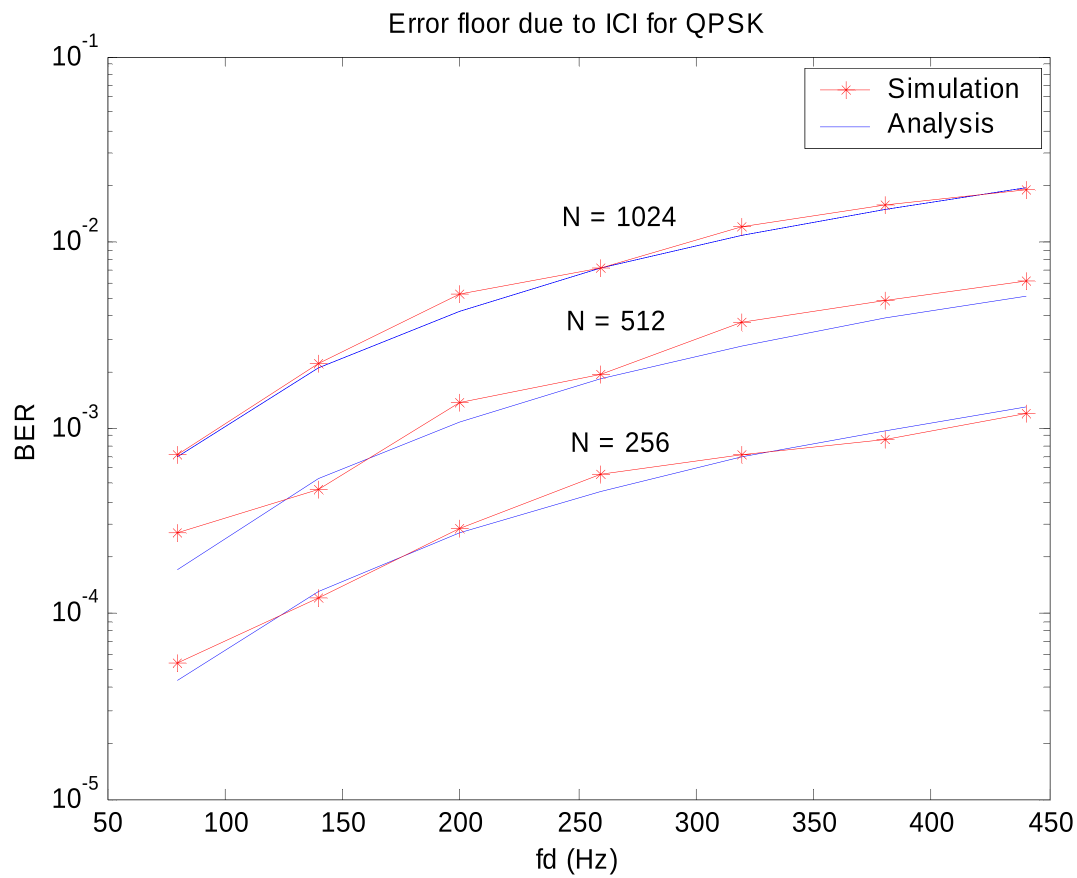
- We want the output of he FFT, $\mathbf{H}^H\mathbf{H}$ matrix to be identity. Time variations in channel distorts this. (This results in inter-Carrier Interference)



Plot of the $\mathbf{H}^H\mathbf{H}$ matrix after FFT detection for OFDM systems with 16 carriers. The channel is frequency selective and time variant. **(a)** $f_dT = 0.1$ **(b)** $f_dT = 0.01$

The plots are obtained by averaging over 20 simulation runs.

Effect of Doppler-induced Inter-Carrier Interference



- Effect of ICI for various number of subcarriers. SNR = 50 dB

- Time varying channels disrupt the orthogonality between subcarriers (spread in frequency)
- ICI it is modeled as a Gaussian Random process during analysis
- ICI increases with the number of subcarriers
- ICI can be reduced by proper detection technique like MMSE instead of a simple FFT

Detection techniques

FFT Detection $\mathbf{z} = \mathbf{G}\mathbf{r}$ $\mathbf{G} = \mathbf{H}^{H}$

LS Detection $\mathbf{z} = \mathbf{G}\mathbf{y}$ $\mathbf{G} = (\mathbf{H}^{H}\mathbf{H})^{-1}\mathbf{H}^{H}$

MMSE Detection $\mathbf{z} = \mathbf{G}^{H}\mathbf{y}$ $\mathbf{G}^{H} = \mathbf{H}^{H}(\mathbf{H}\mathbf{H}^{H} + \sigma^{2}I_{N})^{-1}$

MMSE with Successive Detection $z_k = \mathbf{g}_k{}^{\mathbf{H}}\mathbf{y}$

Choose *(k+1)*th column of the equalizer matrix such that the condition is satisfied

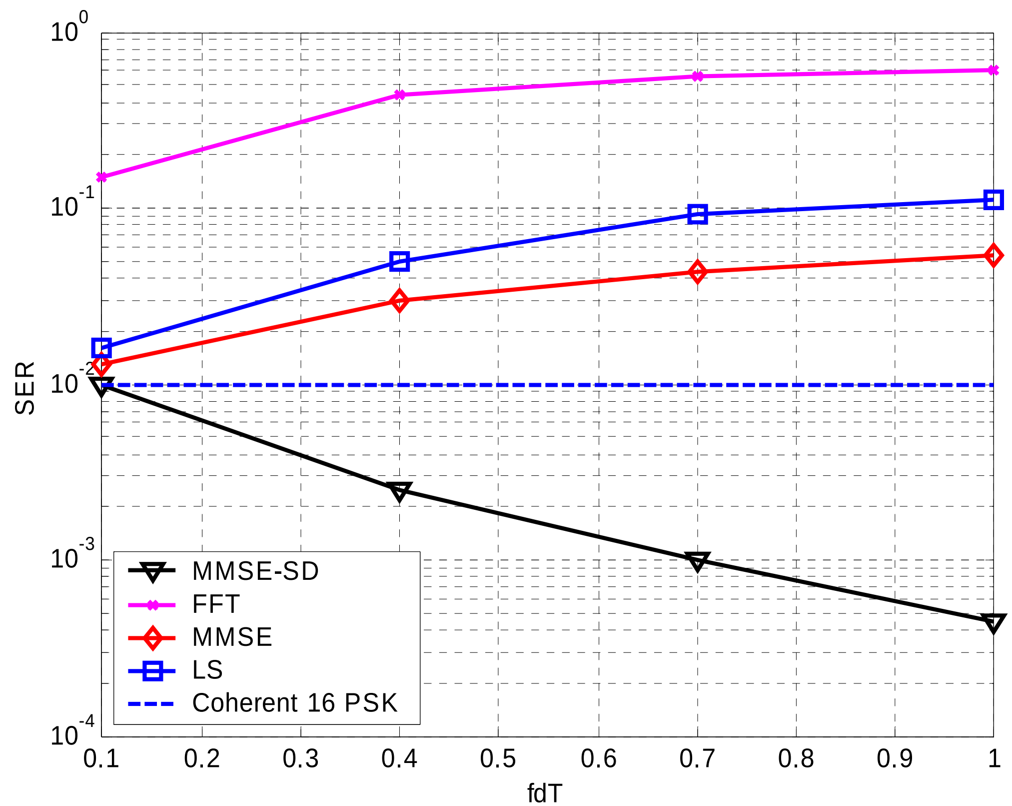
$$\arg\max \mathrm{SINR}_{k} = \frac{\left|\langle \mathbf{g_k}, \mathbf{h_k} \rangle\right|^{2}}{\sum_{m, m \neq k} \left|\langle \mathbf{g_k}, \mathbf{h}_{m} \rangle\right|^{2} + \sigma^{2} \left\|\mathbf{g_k}\right\|^{2}}$$

and update *k*th column vector of *H* is replaced with zeros and above procedure is repeated till all subcarriers are detected.

$$\mathbf{y}_{new} = \mathbf{y}_{old} - \mathbf{h}_k \hat{d}_k$$

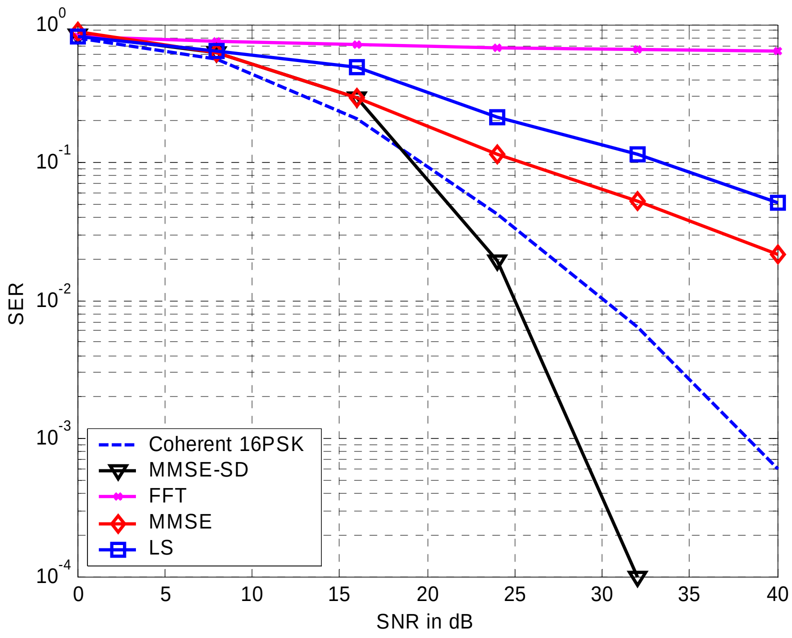
Detection techniques vs. Doppler Spread

- Performance of FFT detection the worst – Matched filter
- MMSE and LS performance is better than simple FFT detection
- MMSE with SD performs the best with the performance improving with increase in normalized Doppler. SD exploits time diversity provided by the channel.

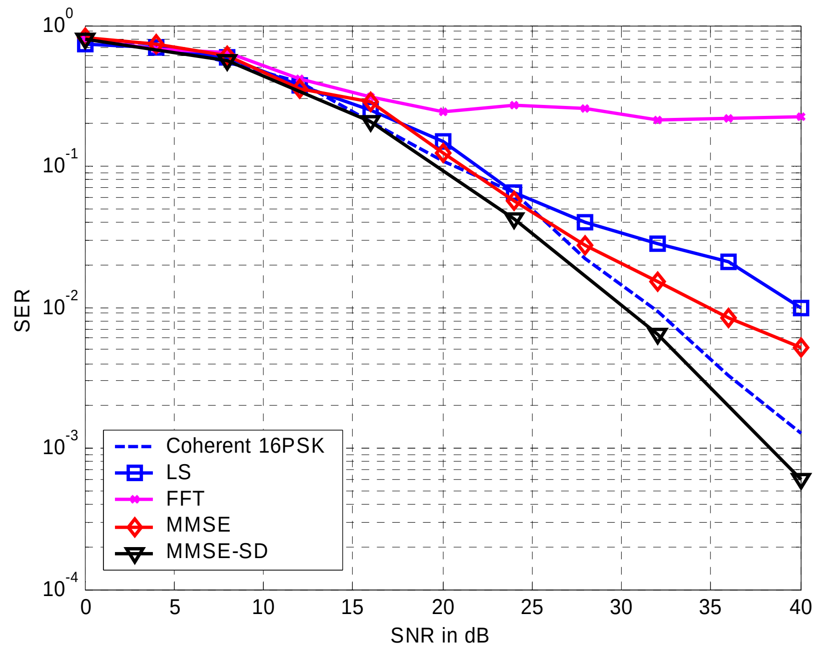


Performance using 16PSK modulation for varying $f_d T$. SNR= 30dB

Detection techniques- contd.

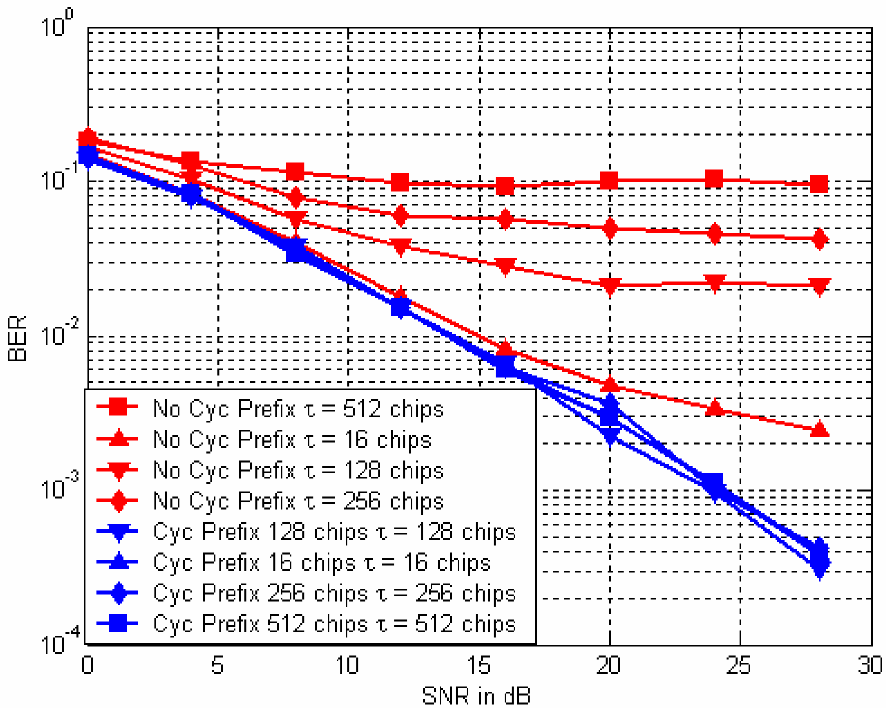


Performance of different detection techniques
$f_d T = 0.1$, N = 1024



Performance of different detection techniques
$f_d T = 0.01$, N = 1024

Usefulness of Cyclic Prefix



- Effect of Cyclic Prefix on MC-CDMA for various Channel delay spreads (τ)

- Effect of Cyclic prefix is to make the channel look like circular convolution
- Cyclic Prefix length greater than or equal to the maximum delay spread
- Performance in frequency selective fading same as flat fading after inclusion of Cyclic Prefix

OFDM Applications

- Wireline
  - Asymmetric Digital Subscriber Loop (ADSL)

- Wireless
  - Digital Audio Broadcasting (DAB)
  - Digital Video Broadcasting-Terrestrial (DVB-T)
  - Integrated Services Digital Broadcasting-Terrestrial (ISDB-T)
  - Wireless LAN (IEEE 802.11(a), HiperLAN/2, MMAC)
  - Wireless MAN (IEEE 802.16 a/b)

References

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- Z.Wang, G.Giannakis, "Wireless Multicarrier Communications," *IEEE Signal Processing Magazine,* vol 17. May 2000, pp. 29-48
- Leonard J.Cimini, "Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing," *IEEE Trans. Commun.,* vol. 33 No 4, July 1985, pp. 665-675