EE 5654 - Digital Communications Spring 2005

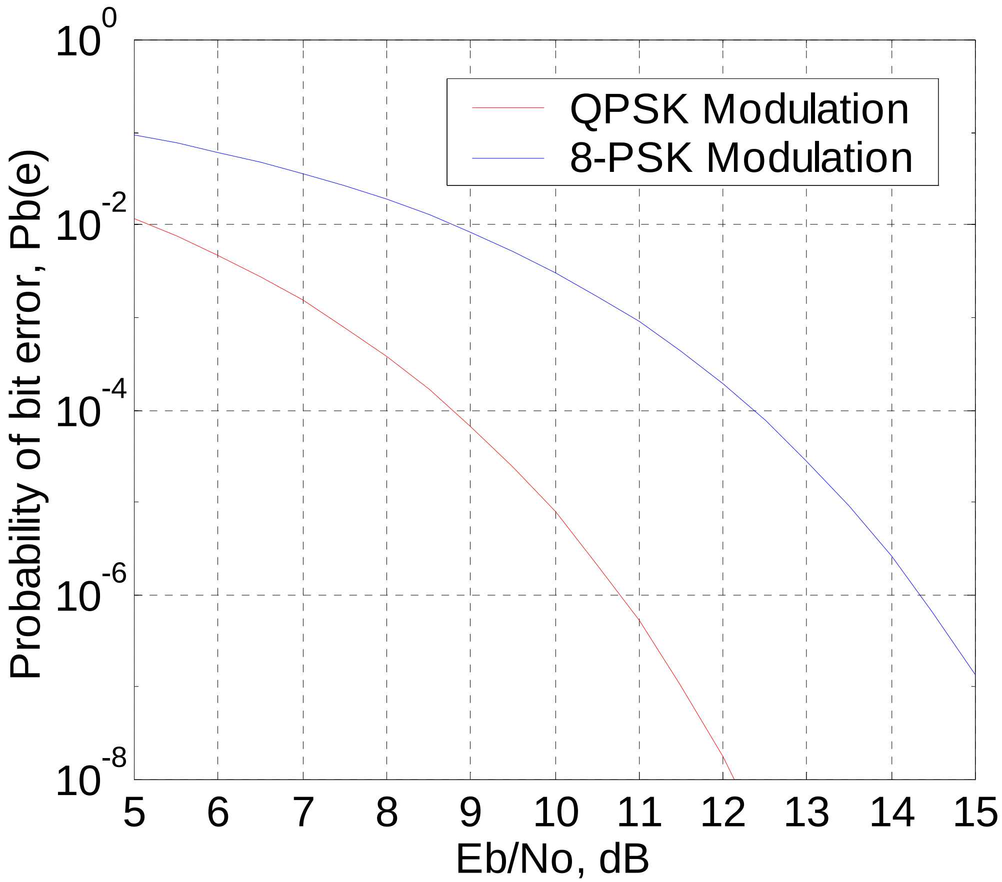
Instructor: R. Michael Buehrer

Lecture #27 –Trellis Coded Modulation

Bandwidth Efficiency and Coding

- Error correction coding increases energy efficiency at the expense of bandwidth efficiency
- We could compensate for coding by increasing the modulation scheme however energy efficiency of the modulation scheme degrades
- Example: Uncoded QPSK could be replaced by 8-PSK and r=2/3 conv. coding
  - Both have same bandwidth efficiency
  - However, a very powerful code must be used since 8-PSK loses 3-4dB in performance from QPSK

QPSK vs 8-PSK



Convolutional code must make up for 3-4dB loss before any gain is achieved

Solution:Trellis Coded Modulation

- The energy efficiency of modulation schemes depends on the Euclidean distance between points.
- The energy efficiency of binary codes depends on the Hamming distance between code words or sequences.
- We can combine the demodulation and decoding operations to some extent by soft-decision decoding
  - However, the design of the modulation and coding schemes are separate and thus not optimal.
- TCM is motivated by the combined design of coding and modulation (maximizing the Euclidean distance between sequences of modulation symbols)

Trellis Coded Modulation (TCM) vs Convolutional Coding

- **Conventional error-correcting codes:**
  - Block codes, convolutional codes.
  - Insert extra symbols to add redundancy to signal.
  - Require either expanded bandwidth or lower data rate.
- **Trellis Coded Modulation [Ungerboeck, 1982]:**
  - Add redundancy by expanding size of signal set.
  - Combine coding and modulation into a single operation.
  - No loss in rate or bandwidth expansion required.
- **Applications:**
  - Telephone modems, microwave, satellite

Description of Trellis Codes

Let $S = \{s_1, \ldots, s_Q\}$ be some signal constellation. Then the trellis code is specified as follows:

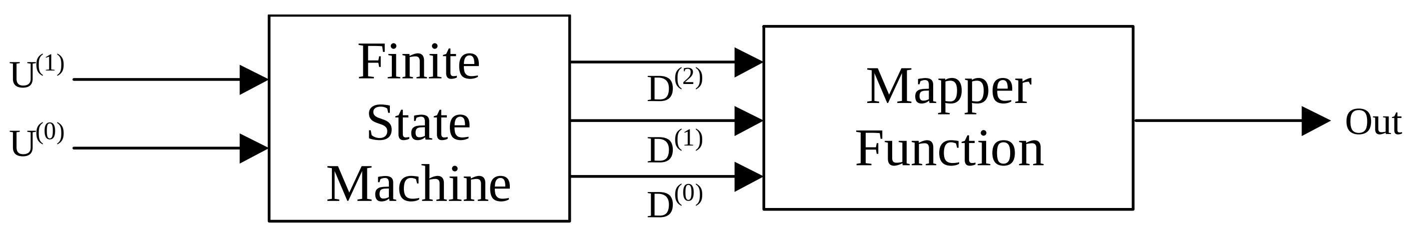
- The transmitted signal $s_{p_i} \in S$ is a function of the current state and current input bits
- The next state is a function of the current state and current input bits
- p_i = path followed by encoder

  $p = \{p_i, i = -\infty, \ldots, \infty\}$

  $C = \{p : p \text{ is a valid path through the trellis}\}$
- Allowable paths through trellis maximize Euclidean distance

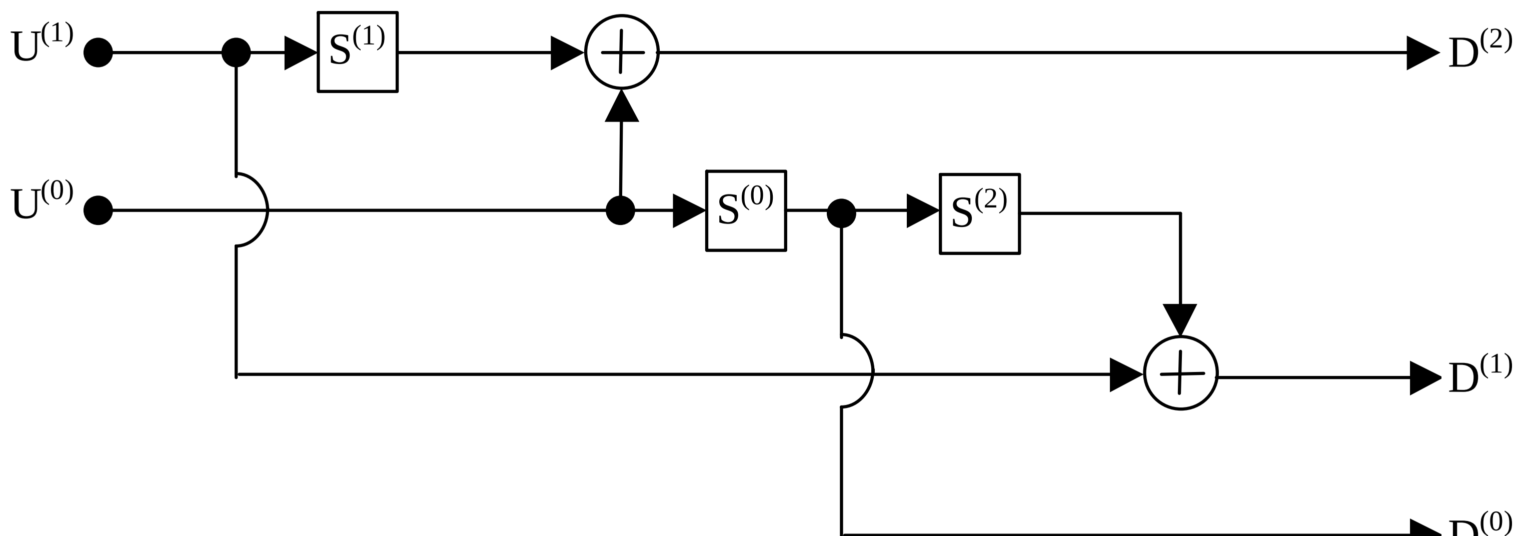
Encoder/Modulator

- One way to view TCM is as two separate functions - a Finite State Machine (FSM) and a Mapper Function.
- The FSM is a r = m/(m+1) convolutional encoder.
- The Mapper Function is a modulation format which maps the output of the FSM to signal points.



Encoder/Modulator

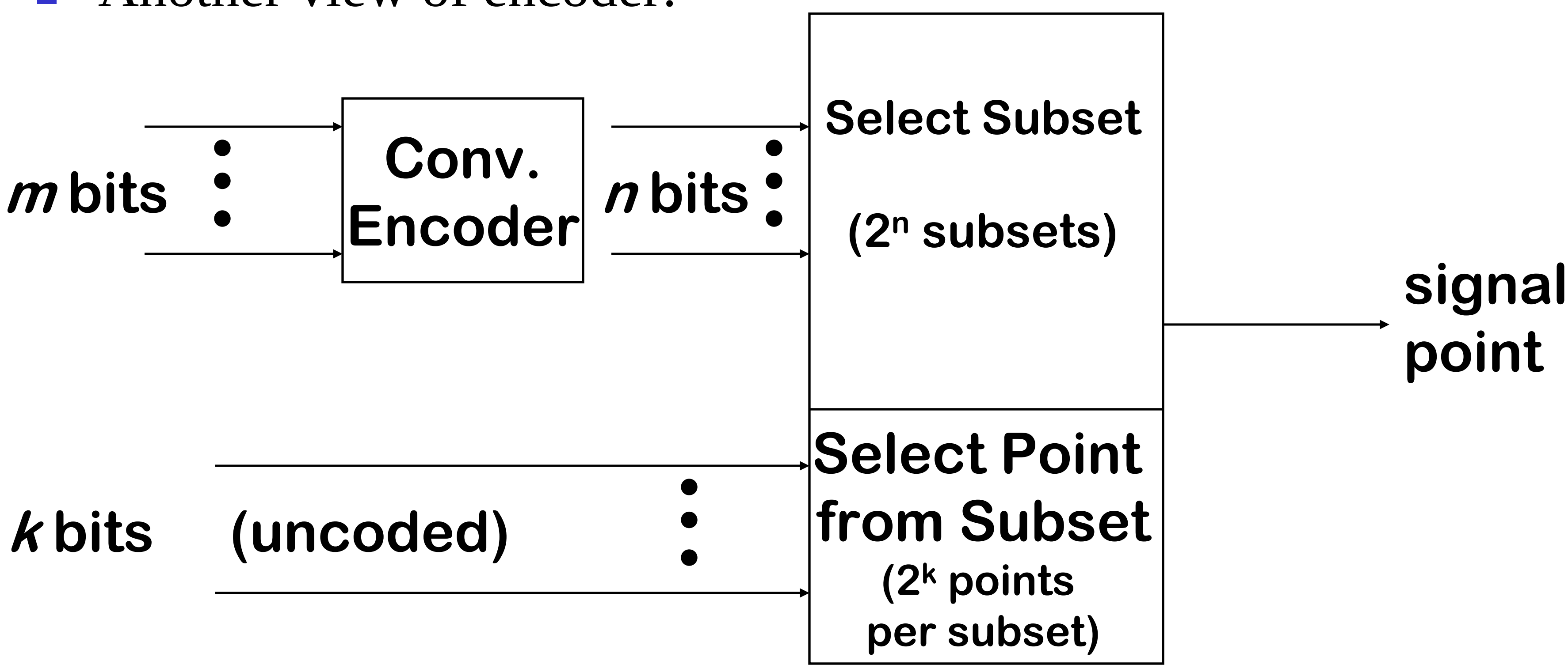
- Encoder Block Diagram of Finite State Machine



- Convolutional Encoder (r = 2/3)
  - Feed-forward implementation.
  - Can also have systematic recursive implementations
  - Identical in nature to convolutional coding

Encoder/Modulator

- Another view of encoder:

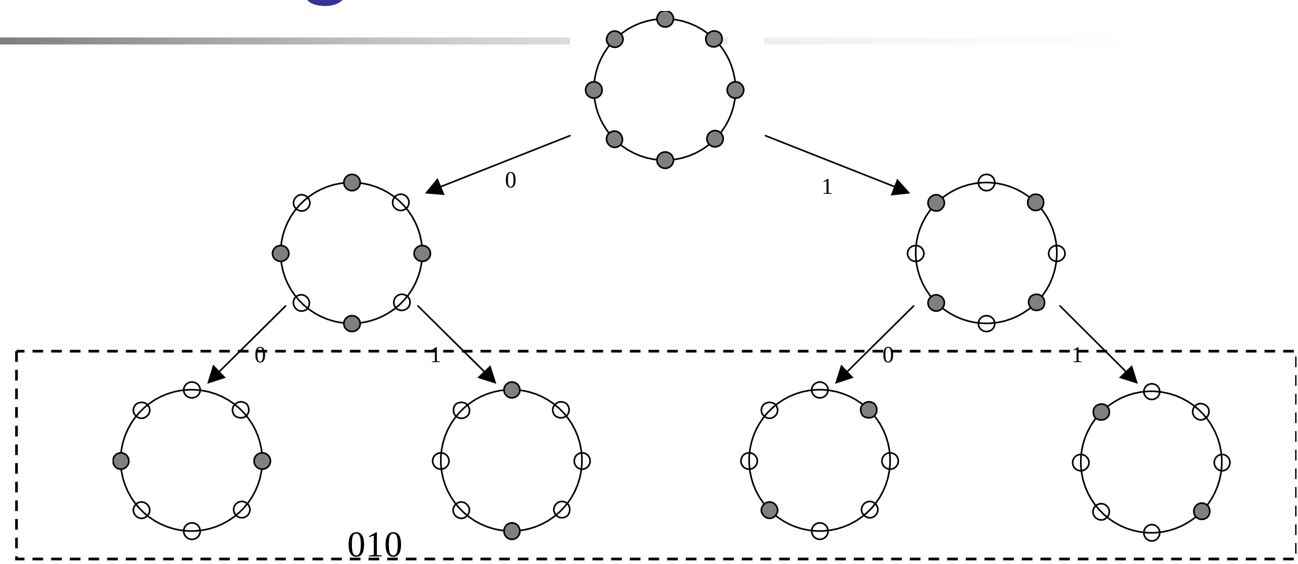


k+m **input bits per symbol**

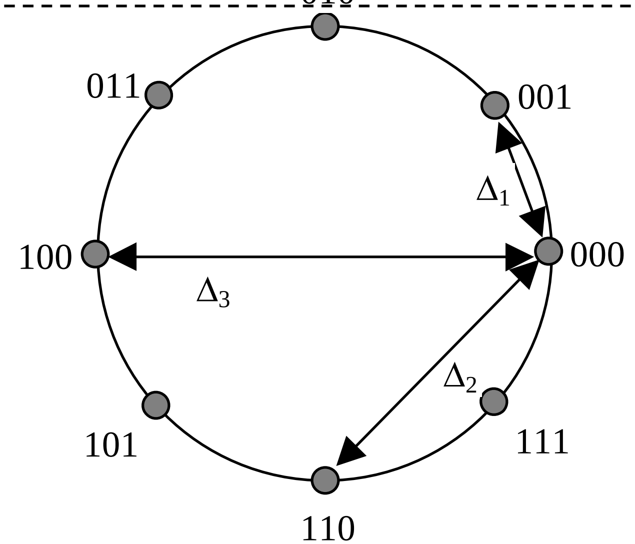
Ungerboeck's Set Partitioning Rules

- Ungerboeck devised rules to maximize the Euclidean distance between paths through the trellis.
- For 8-PSK
  - Parallel Transitions are associated with the signals with maximum distance of 2 between them.
  - Four transitions originating or merging in one state are labeled with distance of at least 1.414 between them.
  - All 8-PSK signals are used in the Trellis Diagram with equal frequency.
- The rules can be easily generalized for other modulation sets

Set Partitioning



Final Subsets
Parallel transitions since two symbols per subset



$$\Delta_1 = \sqrt{(2-\sqrt{2})E_S}$$

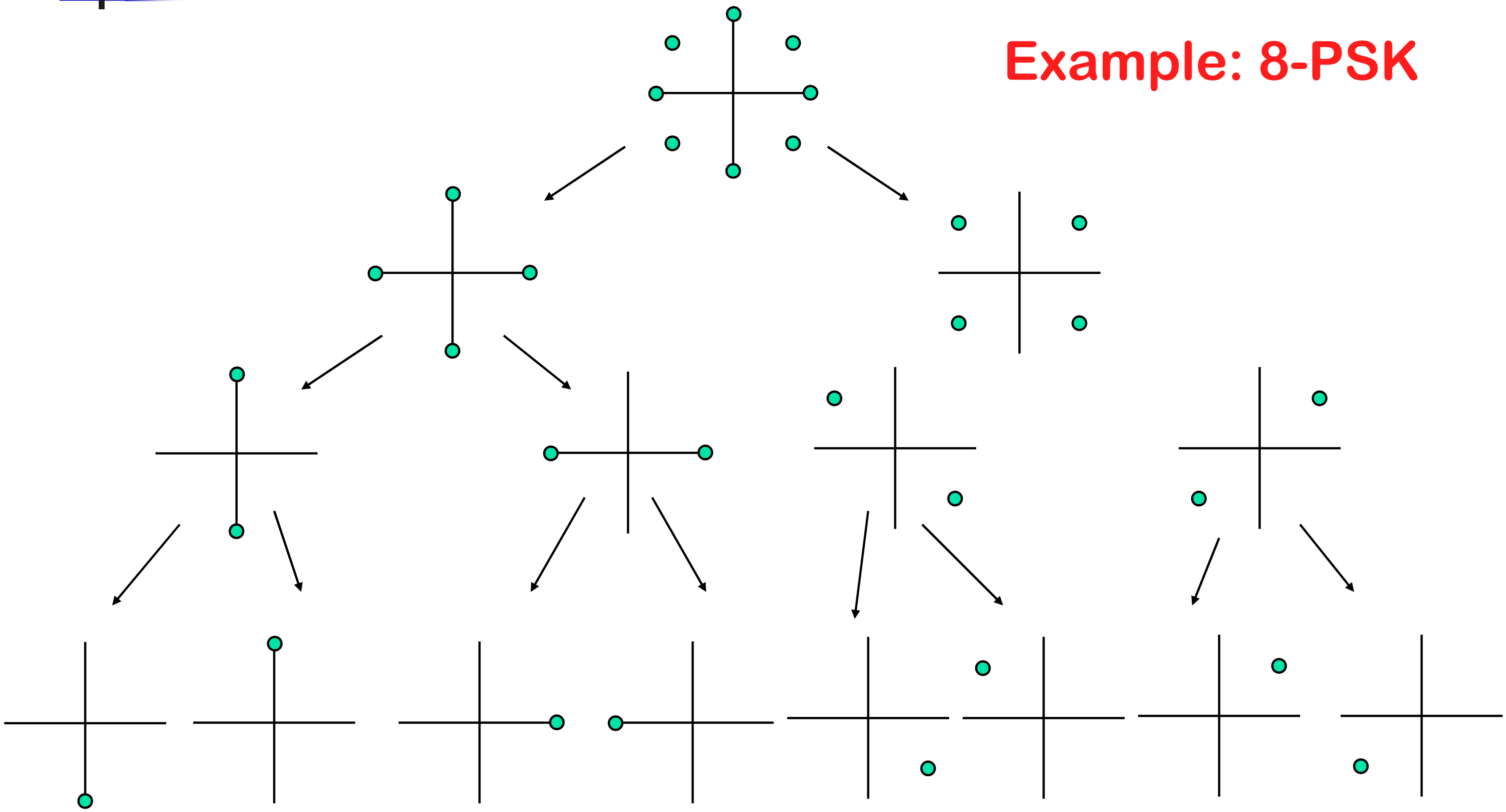
$$\Delta_2 = \sqrt{2E_S}$$

$$\Delta_3 = 2\sqrt{E_S}$$

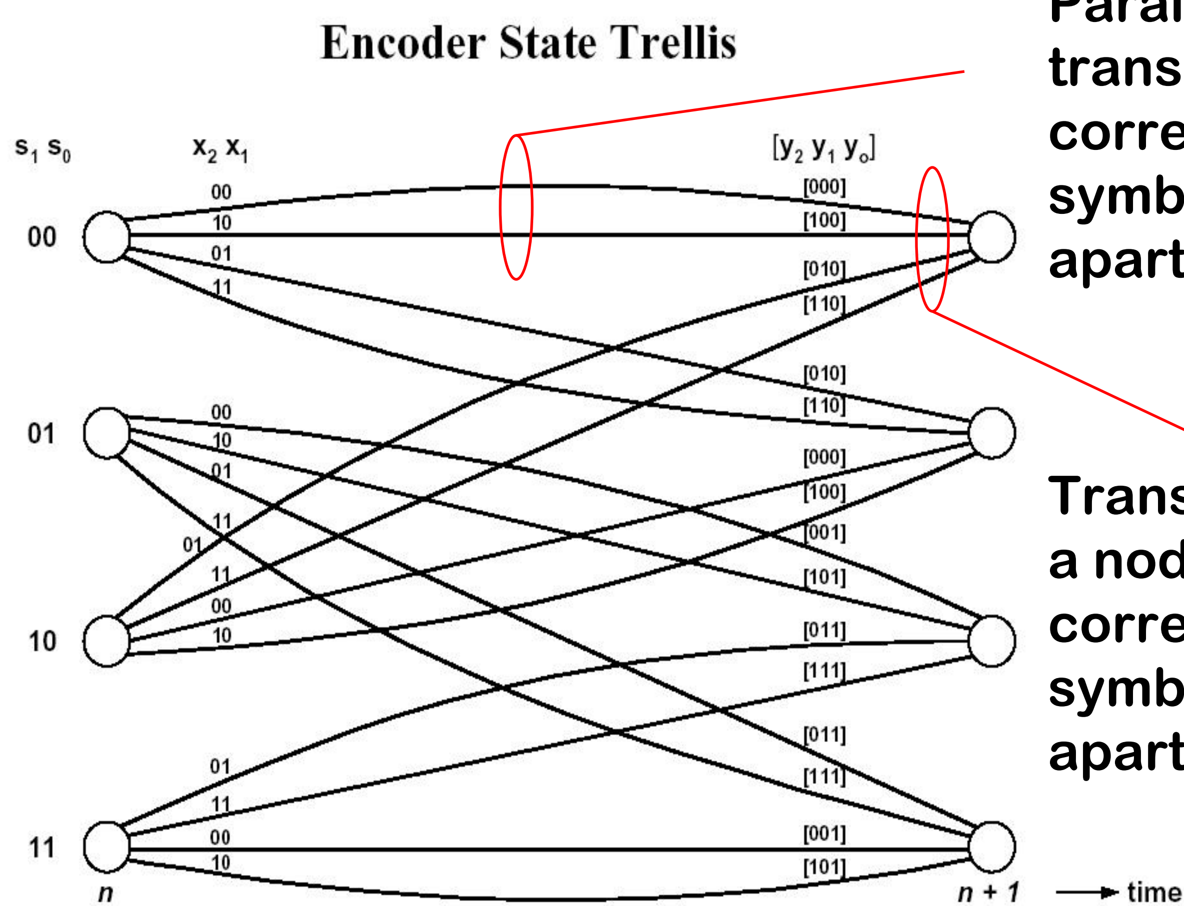
- Partitioning of 8-PSK into subsets with increasing minimum distance.

Set Partitioning – Eight subsets

Example: 8-PSK



Trellis Example – Four States

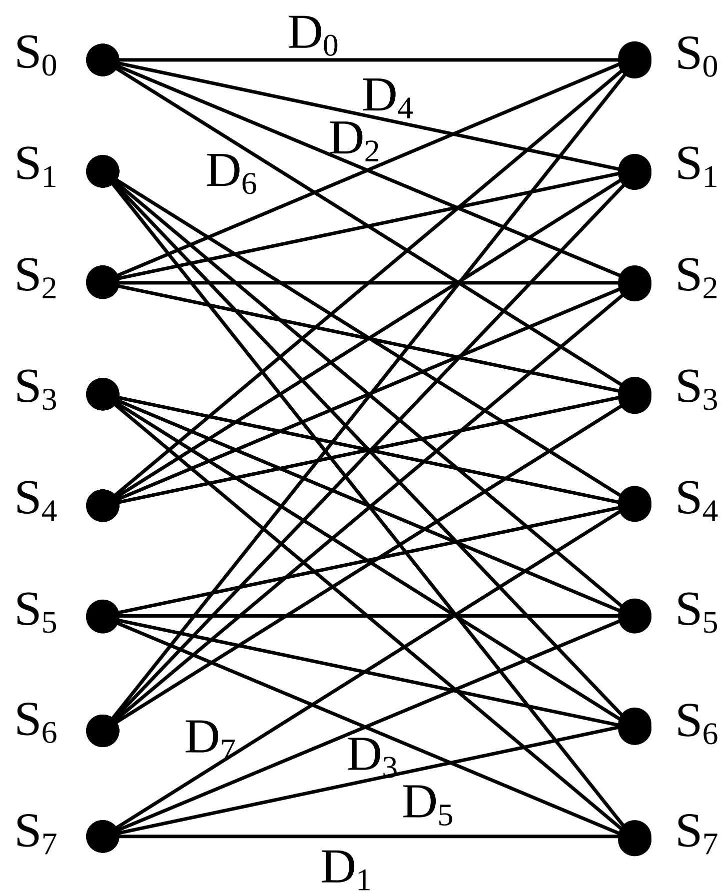


Parallel transitions correspond to symbols 180° apart

Transitions into a node correspond to symbols 90° apart

Trellis Example – Eight States

Input:	U_0	U_1	U_2	U_3
Output:	D_0	D_4	D_2	D_6
	D_1	D_5	D_3	D_7
	D_4	D_0	D_6	D_2
	D_5	D_1	D_7	D_3
	D_2	D_6	D_0	D_4
	D_3	D_7	D_1	D_5
	D_6	D_2	D_4	D_0
	D_7	D_3	D_5	D_1

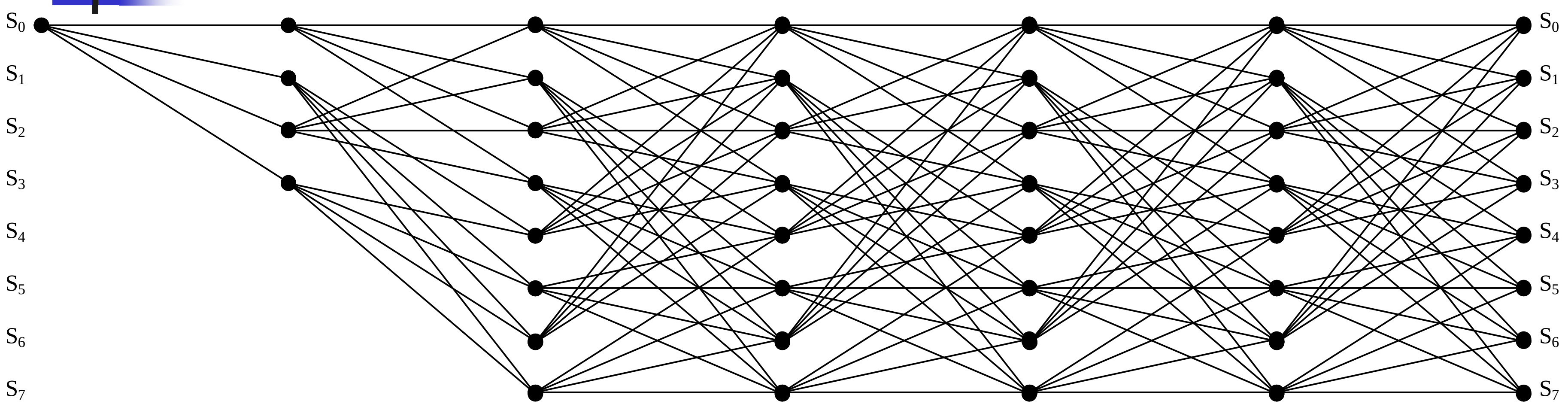


- **No parallel transitions due to larger set of states**
- **One symbol per subset**

U_0 = 00
U_1 = 01
U_2 = 10
U_3 = 11

D_i = symbol *i* from 8-PSK set

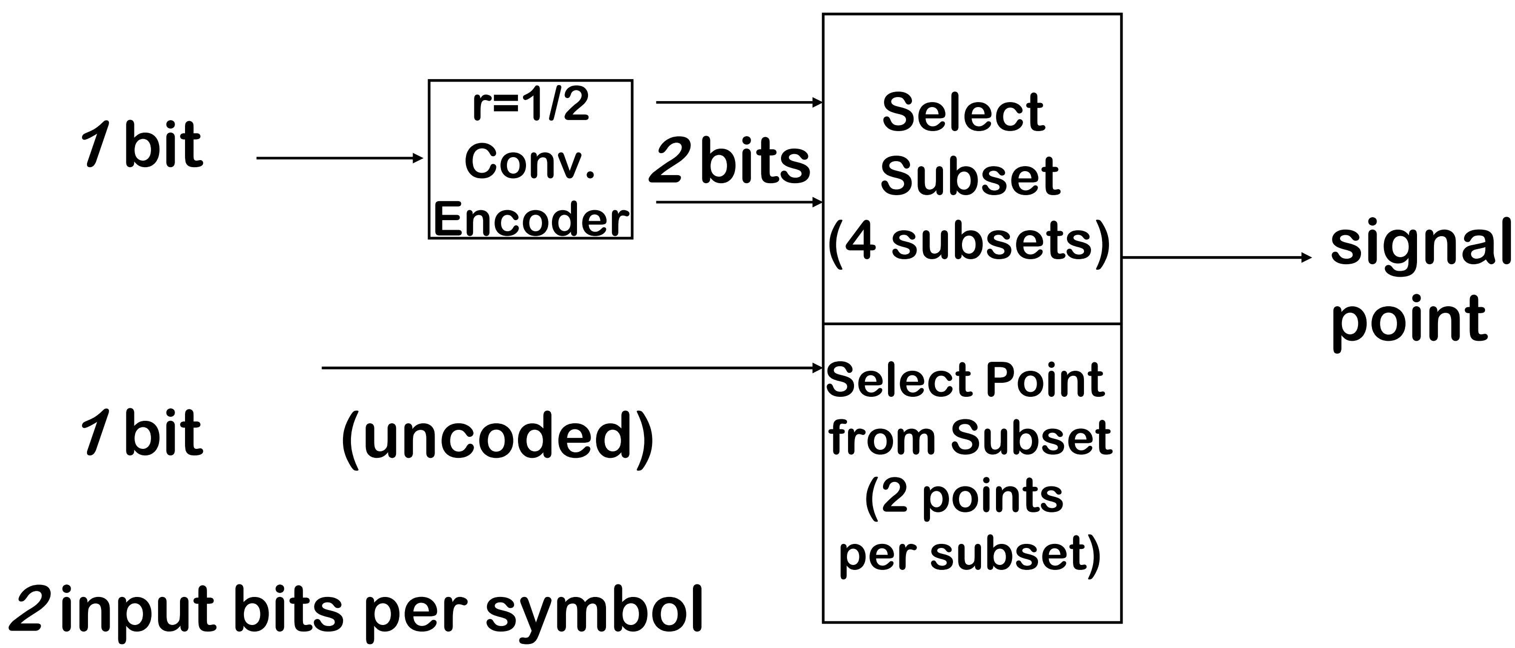
Encoder/Modulator



- When the FSM and the Mapper Function are combined, each path through the trellis is associated with a unique sequence.
- However, these sequences are not binary digits – they are modulated symbols.
- This is where the term, Trellis Coded Modulation, originates.

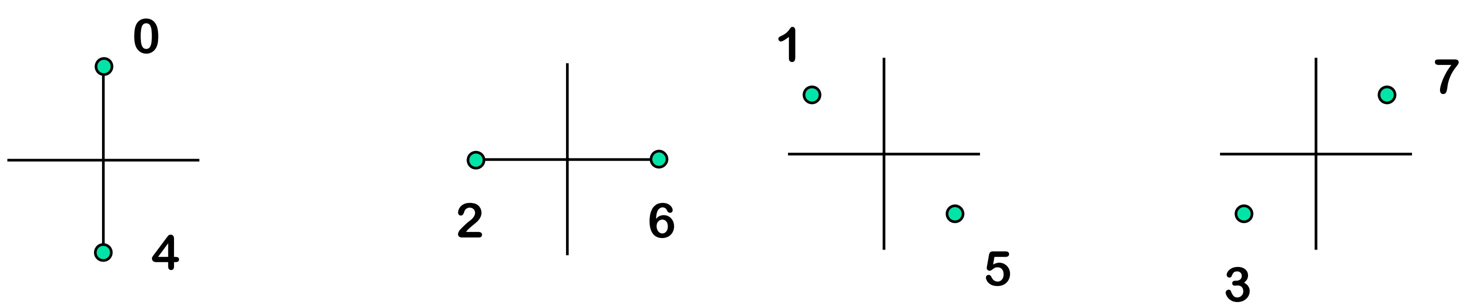
Example: QPSK → 8-PSK TCM

- Suppose we require 2bps/Hz
- We could use uncoded QPSK, or we could use 8-PSK TCM which converts 2 information bits into one of 8 possible symbols using trellis coding



QPSK vs. 8-PSK TCM

- Four subsets: subset determined by the convolutional encoder
- Two symbols per subset: symbol determined by uncoded bit

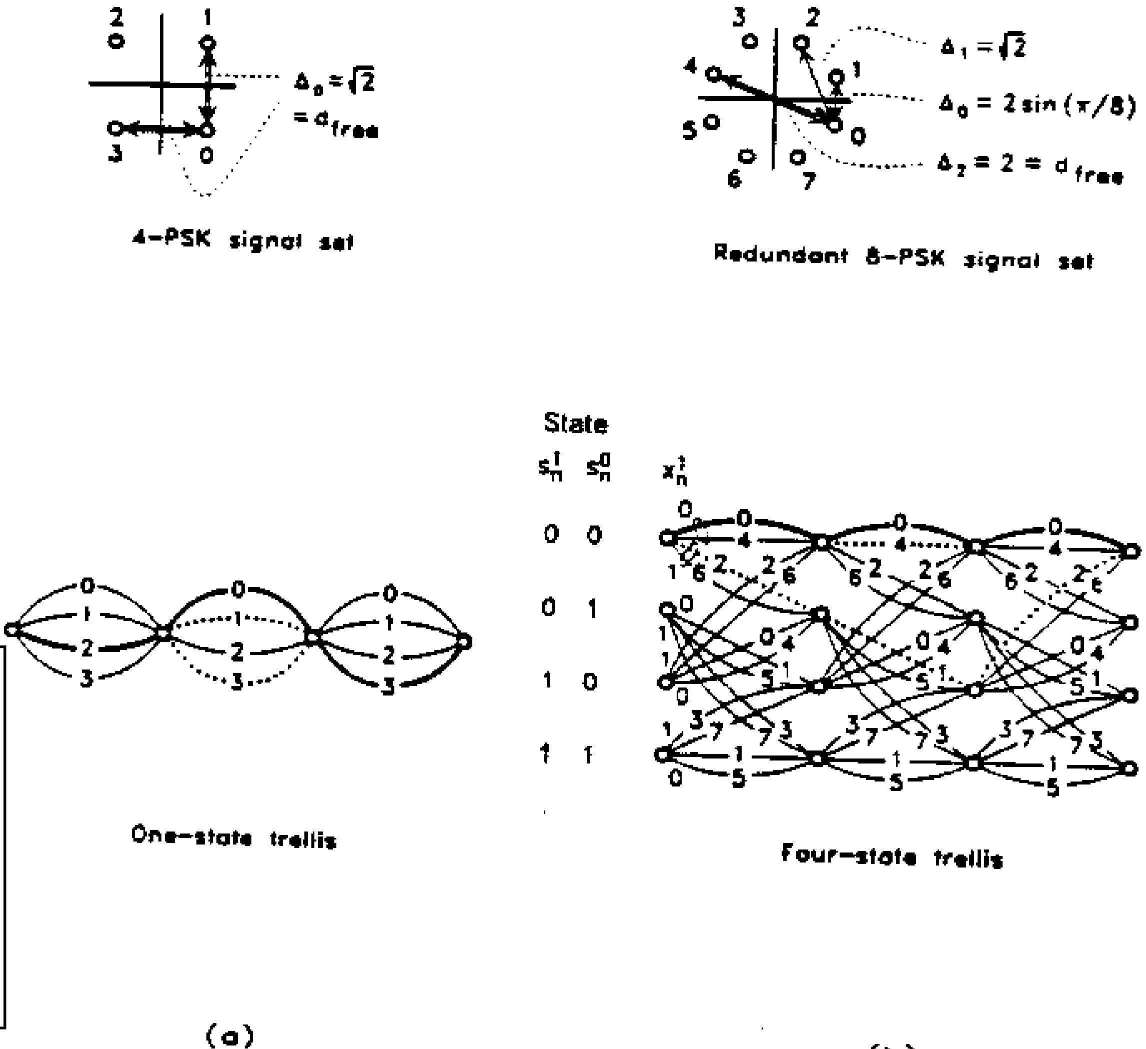


- Symbols from same subset represent parallel paths in the trellis

QPSK vs. Trellis Coded 8PSK

QPSK

TCM 8-PSK



Uncoded: no memory, thus single state 4 possible symbols, thus 4 parallel transitions

Four states and 2 possible tx symbols per state, thus 2 parallel transitions

Fig. 2. (a) Uncoded four-phase modulation (4-PSK), (b) Four-state trellis-coded eight-phase modulation (8-PSK).

Decoding Trellis Coded Modulation

- Due to the convolutional code, there exists a trellis which describes the transitions between states for the codes.
- However, due to uncoded bits choosing one of 2^k modulated symbols for each transition, there are 2^k parallel branches for all transitions.
- Decoding takes place by first choosing one of the parallel branches which is closest in Euclidean distance to the received symbol (*subset decoding*).
- The second step is to simply run the Viterbi algorithm (preferably soft decision) on the resulting trellis.

Performance of Trellis Codes

- **Rate:** The rate of the trellis code is the number of bits transmitted per modulation symbol (can be > 1).
- **Complexity**: proportional to the number of states
- **Euclidean Distance Between Signal Points:**

$$x, y \in S$$

$$d^2(x,y) = \|x - y\|^2$$

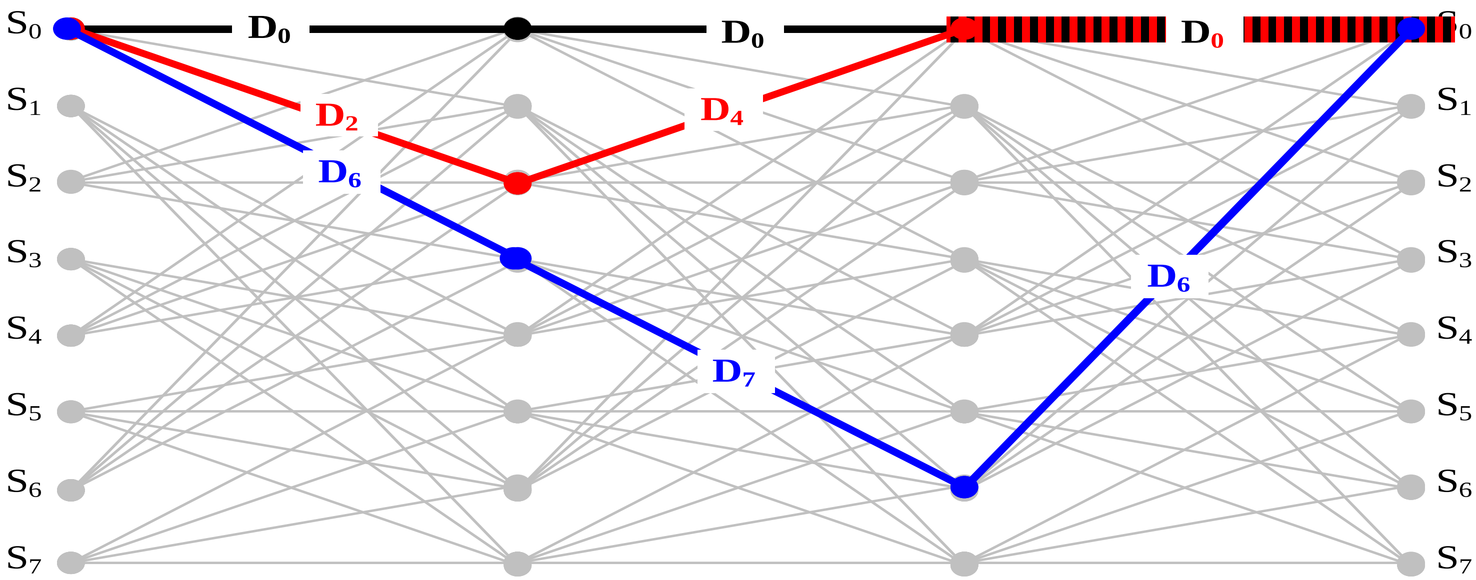
- **Euclidean Distance Between Paths:**

$$p, \tilde{p} \in C$$

$$d^2_{p,\tilde{p}} = \sum_{i=-\infty}^{\infty} d^2\left(c_{p_i}, c_{\tilde{p}_i}\right)$$

Decoder/Demodulator - Example

- Each possible path through the trellis is separated from the other paths by some Euclidean distance.



$$d_{red} = \sqrt{\Delta_2^{\,2} + \Delta_3^{\,2}} = \sqrt{2E_s + 4E_s} = 2.45\sqrt{E_s}$$

$$d_{blue} = \sqrt{\Delta_2^{\,2} + \Delta_1^{\,2} + \Delta_2^{\,2}} = \sqrt{2E_s + 0.59E_s + 2E_s} = 2.14\sqrt{E_s}$$

Performance of Trellis Codes (continued)

- **Free Euclidean Distance:**

$$d_{\text{free}}^2 = \min_{\{p,\tilde{p} \in C : p \neq \tilde{p}\}} d_{p,\tilde{p}}^2$$

- **Coding Gain:**

$$G_c = 10\log_{10}\left(d_{\text{free}} / d_{\text{uncoded}}\right)^2$$

- **Error Event Probability:**

$$P_e \approx a_{d_{\text{free}}} Q\left(\frac{d_{\text{free}}}{\sqrt{2N_0}}\right)$$

Performance Comparison of Uncoded QPSK with 4 State 8-PSK Trellis Code

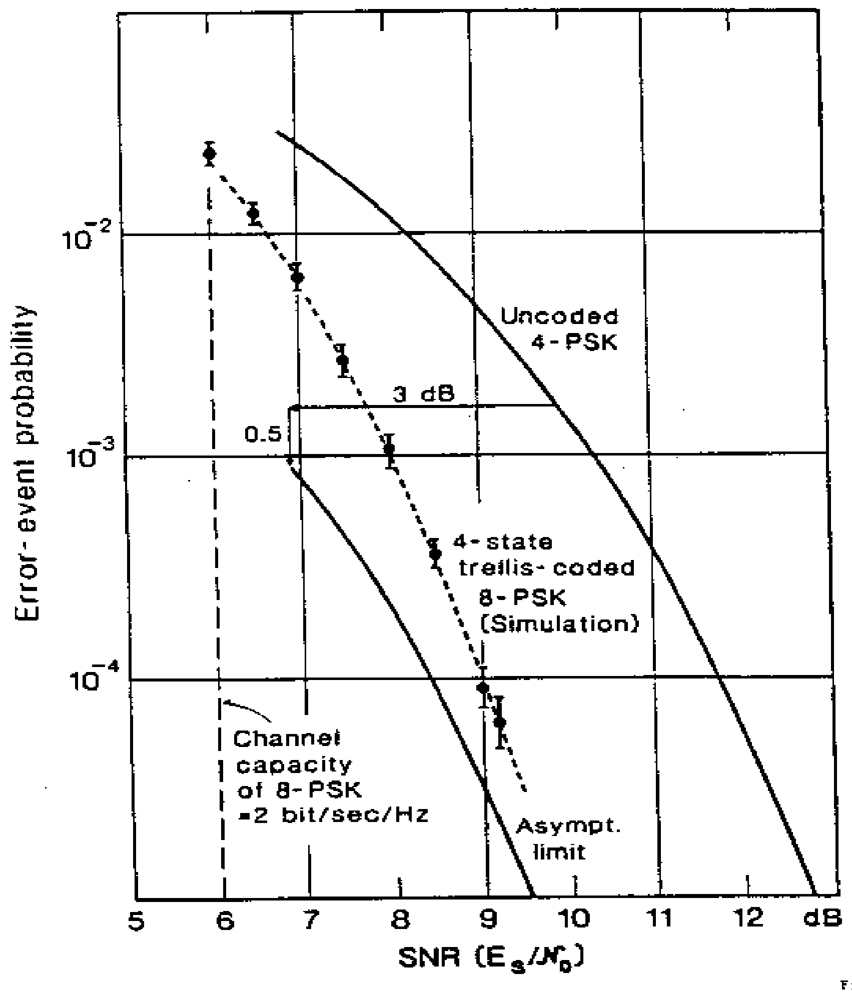


Fig. 1. Error-event probability versus signal-to-noise ratio for uncoded 4-PSK and four-state coded 8-PSK.

Performance

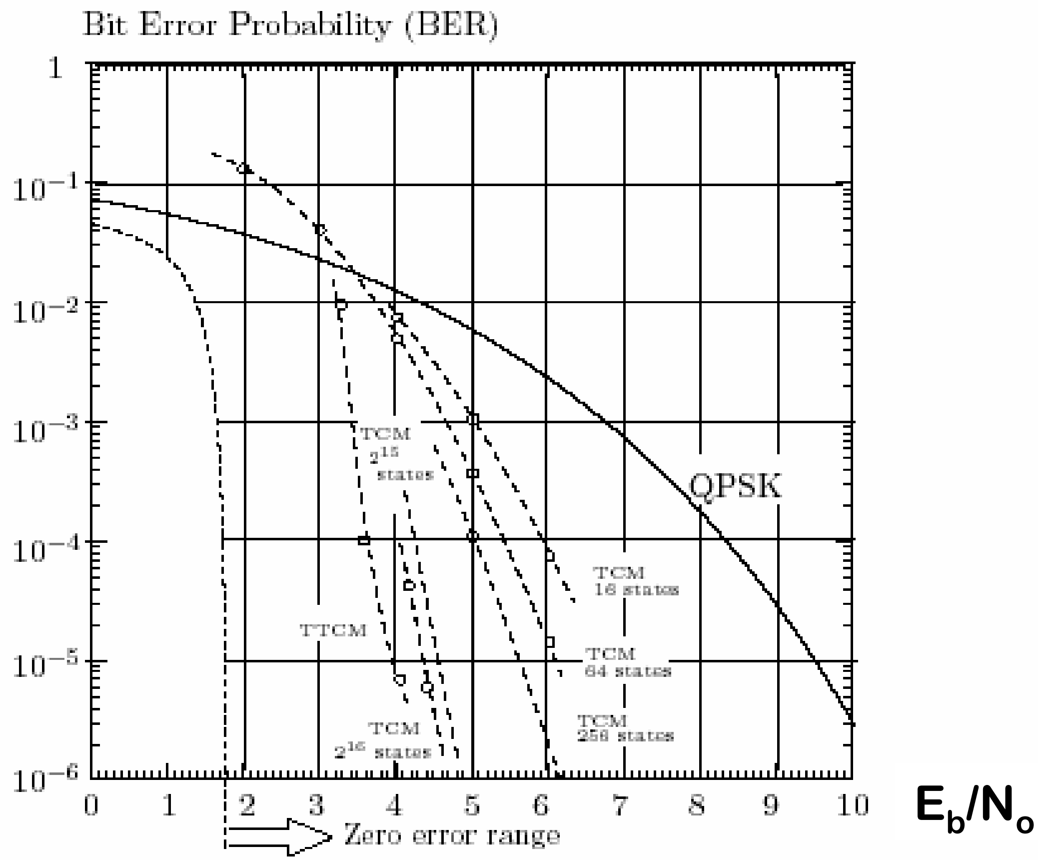
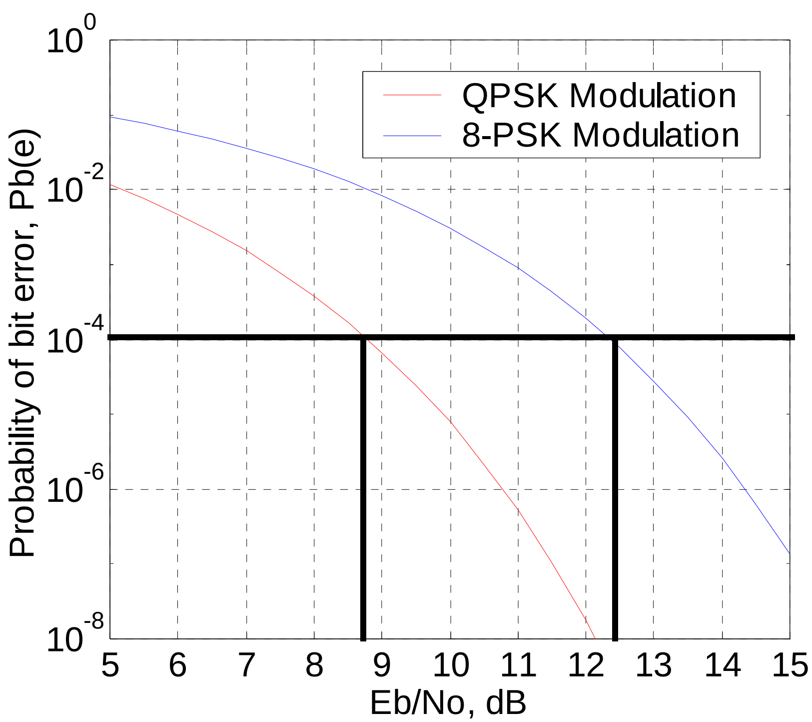
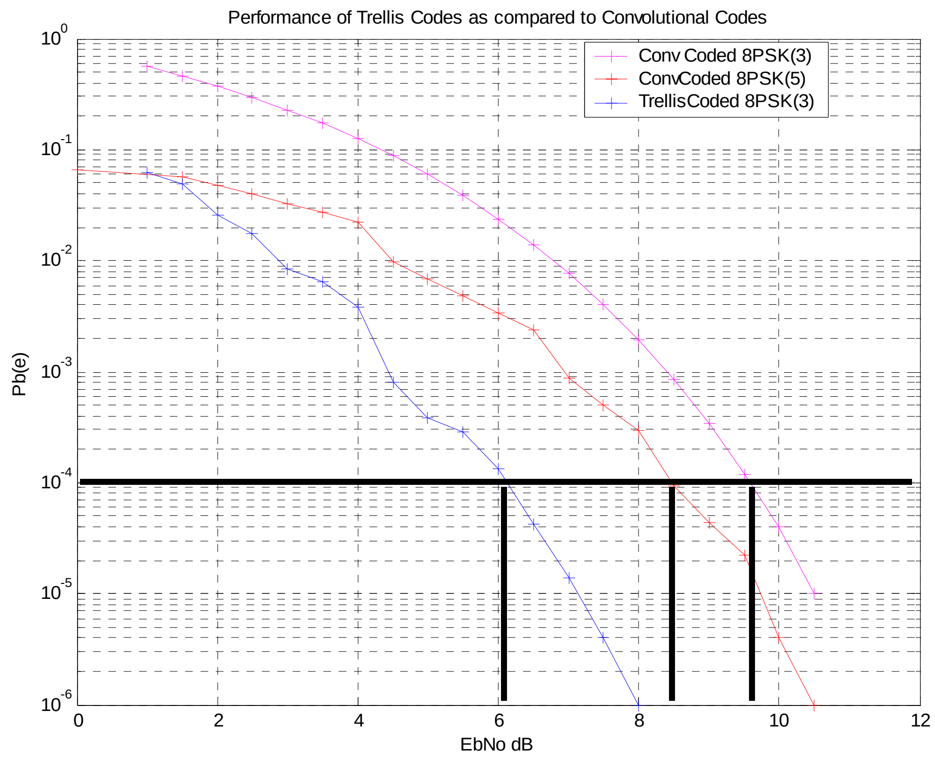


Figure 1.2: Bit error probability of Quadrature Phase-Shift Keying (QPSK) and selected 8-PSK trellis-coded modulation (TCM), trellis-turbo coded (TTCM), and block turbo coded (BTC) systems as a function of the normalized signal-to-noise ratio.

Performance





References on Trellis Coding

- G. Ungerboeck, “Channel coding with multilevel/phase signals,” *IEEE Trans. on Information Theory*, pp. 55-67, Jan. 1982
- G. Ungerboeck, “Trellis coded modulation with redundant signal sets - Parts I&II” *IEEE Communications Magazine*, pp. 5-21, Feb. 1987
- Shuzo Kato, Masahiro Morikura, Shuji Kubota, “Implementation of Coded Modems,” *IEEE Comm. Magazine*, pp. 88-97, Dec. 1991.

Applications of Trellis Coding

- Voice Band Telephone Modems (V.32 standards employ 128 QAM and 32 QAM)
- Terrestrial Microwave Systems (256 QAM)
- Mobile Satellite (8 PSK) (Can tolerate slow fading)
- Few current applications to mobile radio (because of rapid fading)
- One version of Space-Time Codes are trellis codes, constructed across multiple antenna elements