EE 5654 - Digital Communications Spring 2005

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Lecture #28 –Examples

Example 1

As a communications engineer for SpaceCom, Inc. you are designing a modulation and coding scheme for a satellite communications link. The system requires a 200kbps data rate in a 200kHz channel. The RF engineers inform you that based on filtering limitations, the system will use a root raised cosine pulse shaping scheme (i.e., to achieve an overall pulse shape which is raised cosine) with alpha = 0.5. The carrier frequency is 1.8GHz. A matched filter receiver is to be used. The following modulation and coding schemes are to be considered: QPSK, 8-PSK, (n=63, k=36, t=5) BCH code, (n=63, k=24, t=7) BCH code, (n=63, k=16, t=11) BCH code and no coding. The received power is -42dBm and the noise power spectral density is -105dBm/Hz. The desired bit error rate is 10^{-5}. What modulation/coding scheme gives the best performance while meeting the specifications?

Example 1 (solution)

- First we must check the spectral efficiency (in terms of bits/symbol) of the various combinations. These can be easily shown to be

	No Coding	*(63,36)*	*(63,24)*	*(63,16)*
QPSK	*2*	*1*	*0.75*	*0.5*
8-PSK	*3*	*1.5*	*1.125*	*0.75*

- The required spectral efficiency can be easily determined from

$$B = R_s(1+r) = \frac{R_b(1+r)}{\zeta}$$

$$\begin{aligned}\zeta &= \frac{R_b(1+r)}{B} \\ &= \frac{200kbps(1.5)}{200kHz} \\ &= 1.5b/symbol\end{aligned}$$

Example 1 (solution)

- Thus, only the uncoded cases and 8-PSK with the ½ rate code could be used from a spectral efficiency point of view.
- The performance will depend on the E_b/N_o received and the scheme we choose.
- The E_b/N_o can be calculated as

$$\left(\frac{E_b}{N_o}\right)_{\text{info}} = \frac{P_r T_b}{N_o} = \frac{P_r}{N_o R_b}$$

$$\left(\frac{E_b}{N_o}\right)_{\text{info}} = -42dBm - 10\log\{200000Hz\} - (-105dBm/Hz)$$

$$= 10dB$$

Example 1 (solution)

- The performance of the three can be shown to be

	Performance
QPSK	$Q\left(\sqrt{\frac{2E_b}{N_o}}\right)=Q\left(\sqrt{20}\right)=4*10^{-6}$
8-PSK	$\frac{2}{3}Q\left(\sqrt{\frac{2E_b}{N_o}k\sin^2\left(\frac{\pi}{M}\right)}\right)=Q\left(\sqrt{20*3*\sin^2\left(\frac{\pi}{8}\right)}\right)=1*10^{-3}$
8-PSK with ½ rate coding	$P_b=\frac{1}{2}\left\{\binom{n}{t+1}p^{t+1}(1-p)^{n-t-1}+\binom{n}{t+2}p^{t+2}(1-p)^{n-t-2}\right\}=\binom{63}{6}p^6(1-p)^{57}=1*10^{-5}$ $p=\frac{2}{3}Q\left(\sqrt{\frac{2E_b}{N_o}k*r*\sin^2\left(\frac{\pi}{M}\right)}\right)=Q\left(\sqrt{20*3*\frac{36}{63}\sin^2\left(\frac{\pi}{8}\right)}\right)=8.8*10^{-3}$

- *Thus, both coded 8-PSK and uncoded QPSK meet the requirement. However, uncoded QPSK gives slightly better performance and is much less complex and thus should be preferred.*

Example 2

- You are part of a design team that is determining the communications link parameters for a 200kbps wireless satellite system. The channel is well modeled by AWGN.
- (a) It was decided that QPSK modulation would be used. What E_b/N_o value is needed for $2*10^{-5}$ if a matched filter is used at the receiver with a raised cosine pulse shape (r = 1)?
- (b) It was determined that maximum E_b/N_o that can be achieved is 8dB. The only coder available is a ½ rate convolutional code with $d_{free} = 7$ (K=5). Knowing that the transfer function coefficients (for determining BER) are $c_7 = 4$, $c_8 = 12$, $c_9 = 20$, $c_{10} = 72$ can we achieve the desired performance with the available code using hard decision decoding?
- (c) Your co-worker Wally just informed you that the bandwidth constraint is 200kHz. Change the modulation such that we can meet this constraint with our convolutional code. Can this new system meet the performance requirements?
- (d) A bright Virginia Tech intern determined that a higher constraint length code (K=9) could be used that has $d_{free} = 12$. Will this new code achieve the desired performance with hard decision decoding when used with the modulation chosen in (c)? (The transfer function coefficients for determining BER are $c_{12} = 33$, $c_{13} = 0$, $c_{14} = 281$, $c_{15} = 0$)

Example 2 (solution)

- (a) For QPSK in AWGN

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

$$2*10^{-5} = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

$$\frac{E_b}{N_o} = 8.4 \quad (9.3dB)$$

- (b) The BER for hard decision decoding can be bounded by

$$P_b \le \sum_{d=d_{\text{free}}}^{\infty} c_d \cdot \left(\sqrt{4p(1-p)}\right)^d$$

$$p = Q\left(\sqrt{\frac{2E_b}{N_o} r}\right)$$

$$p = Q\left(\sqrt{2*10^{0.8}\frac{1}{2}}\right)$$

$$= 0.006$$

Example 2 (solution)

- Consequently *D = 0.15 and*

$$P_b < \sum_{k=7}^{10} c_k 0.15^k$$
$$= 1.4e - 5$$

- Yes, we can achieve the desired performance.

Solution (cont.)

- (c) With 16-QAM the BER is

$$p = Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_o}r}\right)$$

$$= Q\left(\sqrt{\frac{3*4}{15}10^{0.8}\frac{1}{2}}\right)$$

$$= 0.056$$

- This results in D = 0.46 and P_b = 0.091. Thus we cannot achieve the desired performance.

Example 2 (cont.)

- (d)

$$P_b < \sum_{k=12}^{15} c_k 0.46^k$$
$$= 0.008$$

- *No. The new system cannot achieve the desired error rate.*

Example 3

- You are in charge of a modem design project at Fred and Brothers Aerospace Corp. Your task is to find the most efficient design for the modulation and coding. Specifically, your team has developed Raised Cosine pulse filters with r = 0.33. Further, you have 100kHz of bandwidth in which you must transmit 150kbps. You may choose any modulation scheme you wish, but must choose from BCH codes. Design the modem to meet your requirements with the lowest required E_b/N_o. Provide plots of various options to verify your design.

Solution (1)

- We must deliver 150kbps in a bandwidth of 100kHz using RC pulses with r = 0.33. Thus, we have the following relationship between bandwidth and symbol rate

$$B = (1+\alpha)R_s$$
$$100kHz = 1.33R_s$$

- Further, the data rate is limited by the symbol rate, modulation scheme and code rate that is used:

$$R_b = R_s kr$$
$$150kbps = R_s kr$$

Solution (2)

- Combining the two equations results in the requirement $100kHz = 1.33\frac{150kbps}{kr}$

$$kr = 2$$

- In other words, we must obtain 2b/s/Hz regardless of coding and modulation scheme. To satisfy this requirement, we could simply use QPSK uncoded. We would like to try to improve performance by adding coding. The options with PSK are listed in Table 1. The bit error performance of PSK is given by:

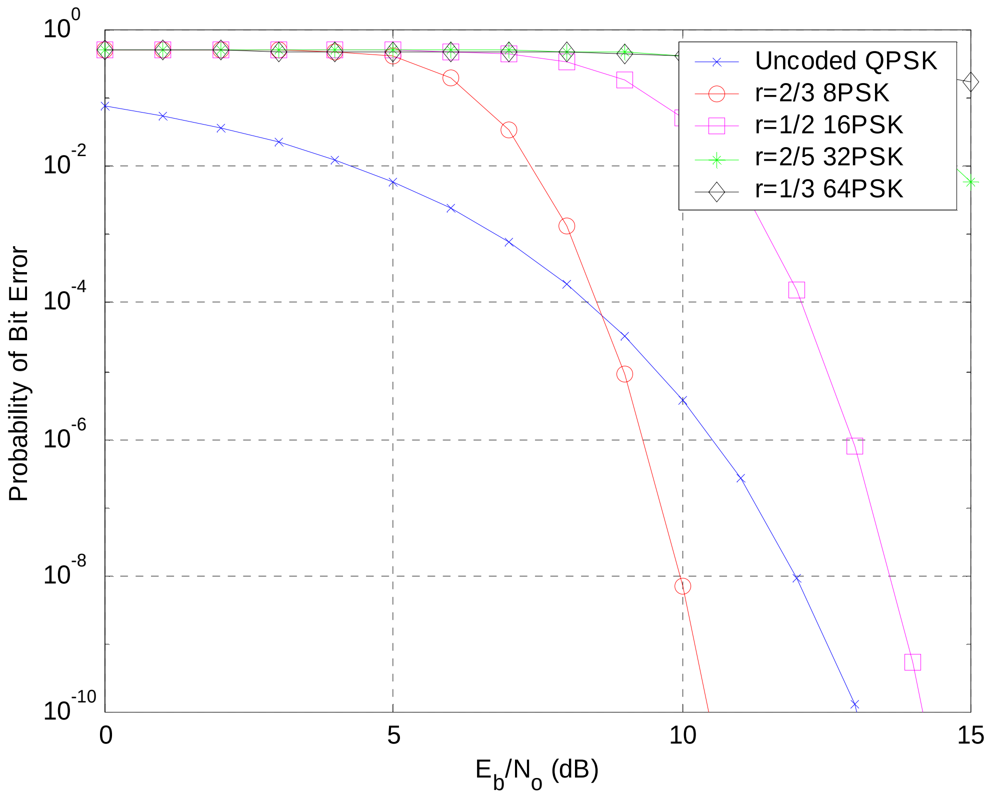
$$P_b = \frac{2}{k} Q\left(\sqrt{\frac{2kE_b}{N_o} r} \sin\left(\frac{\pi}{M}\right)\right)$$

- while the coded error performance for hard decision decoding of the BCH code is

$$P_M = 1 - \sum_{m=0}^{t} \binom{n}{m} P_b^{\,m} \left(1 - P_b\right)^{n-m}$$

Solution (3)

- We choose to use n=255 since it will provide the best performance (of those given in the text) and choose k to match the rate specified in the table. The resulting performance of each option with PSK is given in the Figure below.



Modulation	Coding	*k*	*r*	*kr*
QPSK	None	2	1	2
8-PSK	BCH (255,171)	3	~2/3	2
16-PSK	BCH (255,131)	4	~1/2	2
32-PSK	BCH (255,107)	5	~2/5	2
64-PSK	BCH (255,87)	6	~1/3	2

Solution (4)

- We can see that in order to accommodate coding we must increase the order of the modulation scheme. However, PSK degrades quickly as modulation order increases due to the $\sin(\pi/M)$ term in the bit error probability. Also, increasing the code rate reduces the energy per information symbol due to the term r in the bit error probability. Thus, the coding gains typically cannot keep up with that degradation (particularly when using the BCH code with hard decisions). The best option will depend on the target BER for the system. If the error rate is fairly high, (greater than 10^{-4}) uncoded QPSK is actually the best scheme. However if the target error rate is relatively low (less than 10^{-5}) coded 8-PSK is the best scheme

Solution (5)

- We know that QAM provides better performance than PSK, so we would like to also consider 8-QAM, 16-QAM and 64-QAM as shown in the table below. For a coded QAM system with code rate r the BER performance of 8-QAM can be shown to be

$$P_b = \frac{4}{3} Q\left(\sqrt{\frac{9E_b}{7N_o} r}\right)$$

- while the BER performance of 16-QAM can be shown to be

$$P_b = \frac{3}{4} Q\left(\sqrt{\frac{4E_b}{5N_o} r}\right) + \frac{1}{2} Q\left(3\sqrt{\frac{4E_b}{5N_o} r}\right)$$

- Finally, the BER performance of 64-QAM can be shown to be

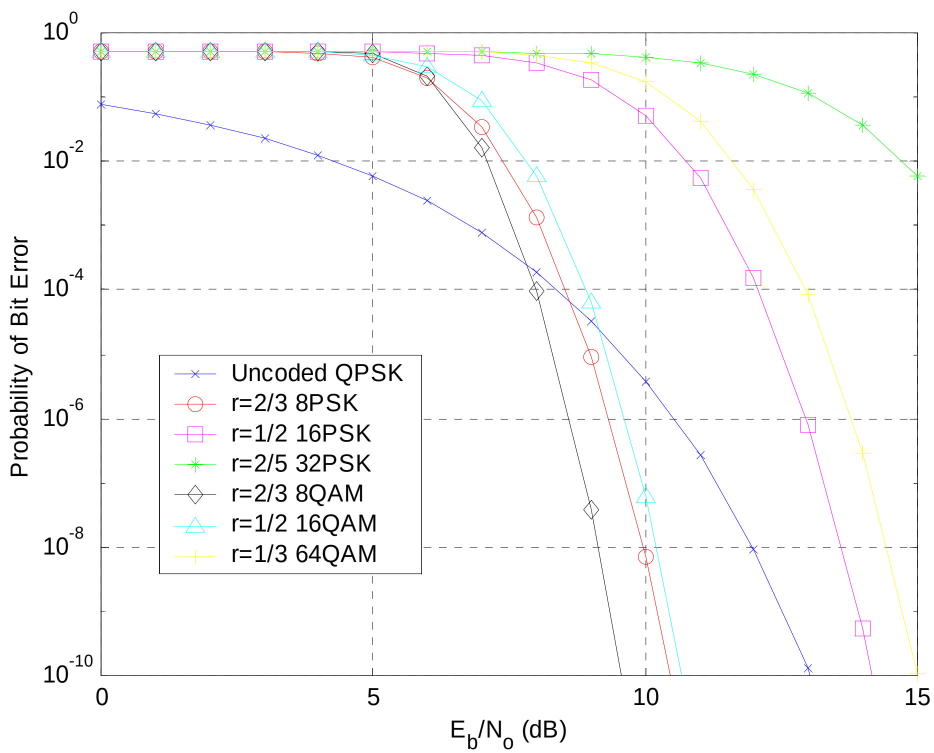
$$P_b = \frac{7}{12} Q\left(\sqrt{\frac{6E_b}{21N_o} r}\right) + \frac{1}{12} Q\left(3\sqrt{\frac{6E_b}{21N_o} r}\right) - \ldots$$

$$\frac{1}{12} Q\left(5\sqrt{\frac{6E_b}{21N_o} r}\right) + \frac{1}{6} Q\left(9\sqrt{\frac{6E_b}{21N_o} r}\right) + \ldots$$

$$\frac{1}{12} Q\left(11\sqrt{\frac{6E_b}{21N_o} r}\right) - \frac{1}{12} Q\left(13\sqrt{\frac{6E_b}{21N_o} r}\right)$$

Solution (6)

- with QAM:



Modulation	Coding	*k*	*r*	*kr*
QPSK	None	2	1	2
8-QAM	BCH (255,171)	3	~2/3	2
16-QAM	BCH (255,131)	4	~1/2	2
64-QAM	BCH (255,87)	6	~1/3	2

Solution (7)

- The resulting performance of QAM schemes is shown in the figure on the previous slide. Now we see a slightly different story. QAM provides approximately 1.5dB improvement over PSK for $M=8$ and about 4dB improvement for $M=16$. The best scheme appears to be 8-QAM with 2/3 rate BCH coding, unless high error rates are to be tolerated in which case uncoded QPSK is better.

Example 4

- You are in charge of transceiver design for a satellite communications project. There is 100MHz of bandwidth available for the system, a total transmit power of 200W, a path loss of 50dB, and a noise power spectral density at the detector of -130dB/Hz. How many 200kbps channels are possible with raised cosine filters with r = 0.5 if a BER of 10^{-4} must be maintained?

- Available modulation schemes: QPSK, 16-QAM, 64-QAM
- Available coding schemes: rate ½ K=7, conv. code (hard decision); rate 1/3 K=7 conv. code (hard decision)
  - r = ½ $\rightarrow d_{free} = 10$, $c_{d\_free} = 36$
  - r = 1/3 $\rightarrow d_{free} = 15$, $c_{d\_free} = 1$

Solution (1)

- Recall that for hard decision decoding, the pairwise error event probability is

$$P_e \le \sum_{d=d_{\text{free}}}^{\infty} a_d \cdot \left(\sqrt{4p(1-p)}\right)^d$$

$$\approx a_{d_{free}} \cdot \left(\sqrt{4p(1-p)}\right)^{d_{free}}$$

- and the error event probability is

$$P_e \le \sum_{d_H=d_{\text{free}}}^{\infty} a_d \cdot P_2(d) \approx a_{d_{\text{free}}} \cdot P_2(d_{\text{free}})$$

- The bit error probability can be approximated as

$$P_b \approx c_{d_{\text{free}}} \cdot P_2(d_{\text{free}})$$

Solution (2)

- The number of channels can be determined as
  - $C = B / (1+r)*R_s$
  - $= 100MHz / (1.5)*200/(k*r)$

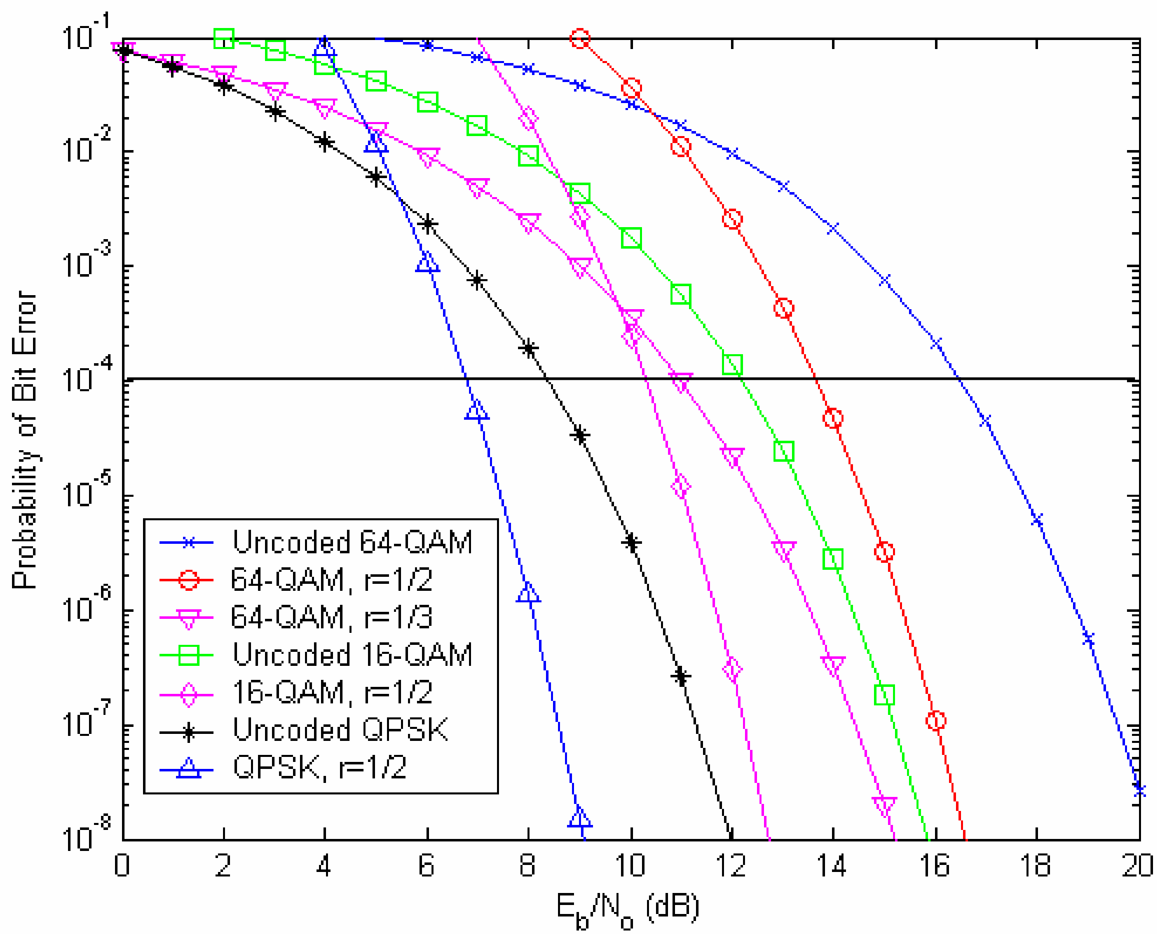
	QPSK	16-QAM	64-QAM
r = 1	666	1333	2000
r=1/2	333	666	1000
r=1/3	222	444	666

E_b/N_o

- E_b = Pr / R_b = 10*log10(100/C) – 10*log10(200000) – 50dB

C	P_c	E_b/N_o
2000	-10dBW	17dB
1000	-7dBW	20dB
1333	-8.2dBW	18.8dB
666	-5.2dBW	21.8dB
444	-3.5dBW	23.5dB

Solution



Uncoded 64-QAM can obtain 10^{-4} at 17dB. Thus, we can support 2000 channels