EE 5654 - Digital Communications Spring 2005

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Lecture #5

Pulse Shaping, Bandwidth Efficiency and Demodulation

Motivation

- Recall the signaling method for several *M*-ary modulation techniques:
  - *M*-ASK

$$s_m(t) = \mathrm{Re}\left\{A_m g(t) e^{j2\pi f_c t}\right\} \qquad m = 0,1,\ldots M-1$$

$$= A_m g(t)\cos(2\pi f_c t) \qquad A_m = 2m+1$$

$$\widetilde{s_m}(t) = A_m g(t)$$

  - *g(t)* can be any general pulse shape, although we typically assume square pulses

Motivation (cont.)

- *M*-PSK

$$s_m(t) = \mathrm{Re}\left\{ g(t) e^{j2\pi m/M} e^{j2\pi f_c t} \right\} \qquad m = 0,1,\ldots M-1$$

$$= g(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$$

$$\widetilde{s_m}(t) = g(t) e^{j2\pi m/M}$$

Motivation (cont.)

- QAM

$$s_m(t) = \mathrm{Re}\left\{ g(t)\left[a_m + jb_m\right] e^{j2\pi f_c t} \right\} \qquad m = 0,1,\ldots M-1$$

$$= g(t)\, a_m \cos(2\pi f_c t) + g(t)\, b_m \sin(2\pi f_c t)$$

$$\widetilde{s_m}(t) = g(t)\left[a_m + jb_m\right]$$

$$a_m \in \left\{ -\sqrt{M}+1, -\sqrt{M}+3 \ldots \sqrt{M}-1, \right\}$$

$$b_m \in \left\{ -\sqrt{M}+1, -\sqrt{M}+3 \ldots \sqrt{M}-1, \right\}$$

Power Spectral Density of Digital Signals

- The complex baseband of each of these modulation techniques can be represented as a special case of Pulse Amplitude Modulation:

$$s(t) = \sum_n b_n p(t - nT)$$

- Where b_n is the data sequence and $p(t)$ is the pulse shape used.
- The pulse is a deterministic quantity, but the data sequence is random. Thus, we rely on the power spectral density for spectral information

Spectrum of Digital Signals

- The power spectral density of a digital waveform depends on the pulse spectrum *P(f)* and the autocorrelation of the data $R_b(k)$ [including the data variance σ_b^2 and mean m_b]. If data are independent:

$$P_x(f) = \frac{|P(f)|^2}{T} \sum_{k=-\infty}^{\infty} R_b(k) e^{j2\pi kfT}$$
$$= \underbrace{\frac{\sigma_b^2}{T} |P(f)|^2}_{continuous} + \underbrace{\frac{m_b^2}{T} \sum_{k=-\infty}^{\infty} \left| P\left(\frac{n}{T}\right) \right|^2 \delta\left(f - \frac{n}{T}\right)}_{discrete}$$

- For BPSK modulation $\sigma_b^2=1$ and $m_b=0$:

$$P_{bpsk}(f) = \frac{1}{T} |P(f)|^2$$

Definitions of Bandwidth for Baseband Signals

- Absolute Bandwidth

$$W(f) = 0, \text{ for } |f| > B$$

- *X* dB Bandwidth

$$10\log_{10}\left[\left|\frac{\max\{W(f)\}}{W(f)}\right|^2\right] > X \text{ dB}, |f| > B$$

- *Y* % Power Bandwidth

$$\frac{\int_{-B}^{B} |W(f)|^2 df}{\int_{-\infty}^{\infty} |W(f)|^2 df} \geq \frac{Y}{100}$$

- First Null Bandwidth

Note that usually BW is defined over *positive* frequencies

What determines bandwidth?

- From our previous discussion the pulse shape dominates the spectral characteristics
- Bandwidth determined by
  - Pulse duration (i.e., the symbol rate)
    - Bit rate
    - Modulation scheme
  - Pulse shape

Pulse Shaping - Why Does it Matter?

- One way of reducing bandwidth requirements is through efficient quantization
- Sample rate: f_s samples/second.
- Bit rate out of L-level quantizer:

$$f_s \log_2 L = f_s \cdot n \text{ bits/second}$$

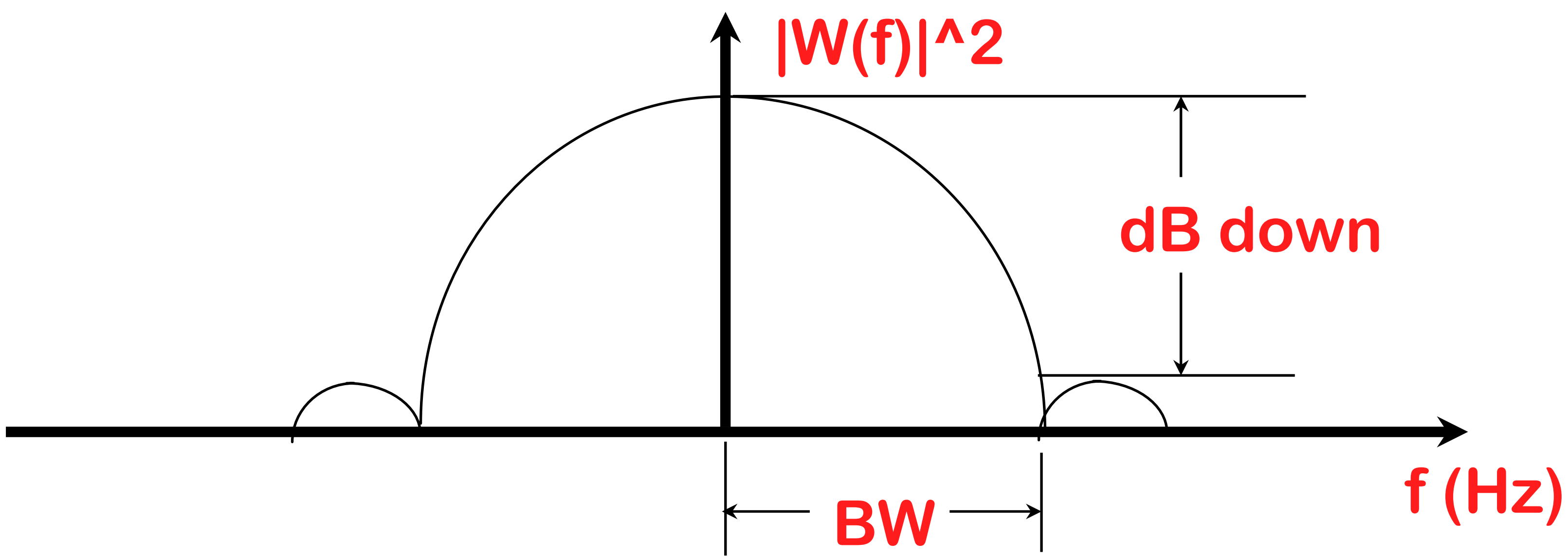
- For the modulation schemes considered Bandwidth is given by:

$$\text{BW} = \left(C_{PS} \cdot \frac{f_s \cdot n}{\log_2 (M)} \right) \text{Hz}$$

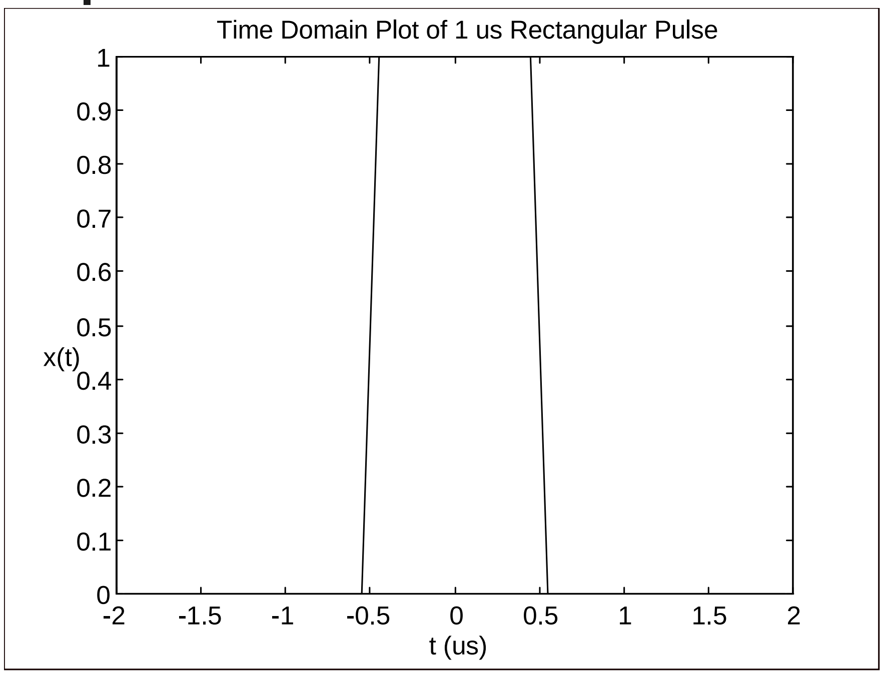
- The constant C_{PS} depends on the pulse shape

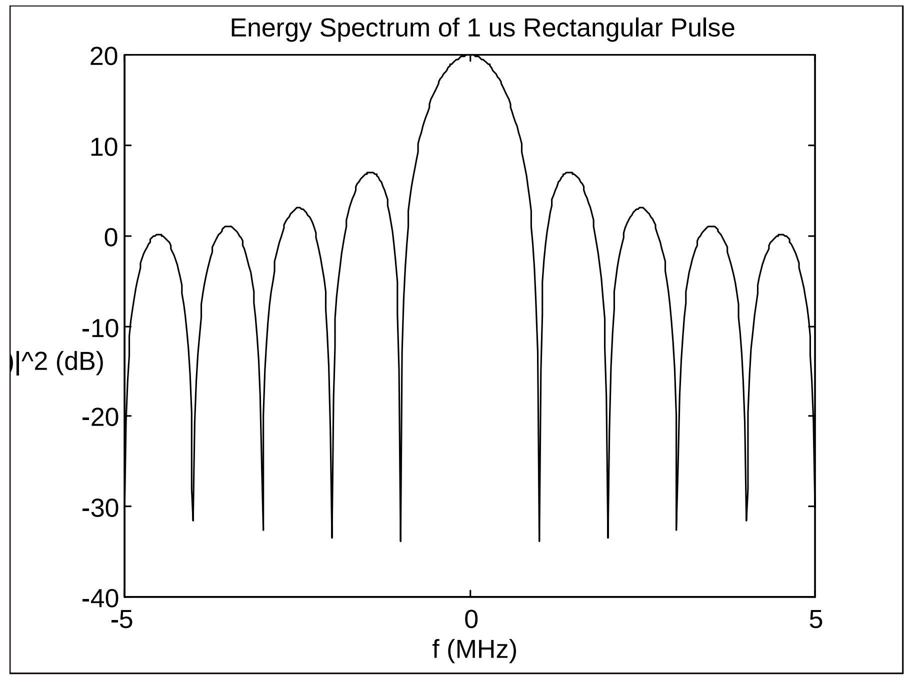
Design Criteria for Pulse Shapes

- Two important characteristics
  - First null bandwidth
  - Size of sidelobes
- Would like to “round off corners” of pulses



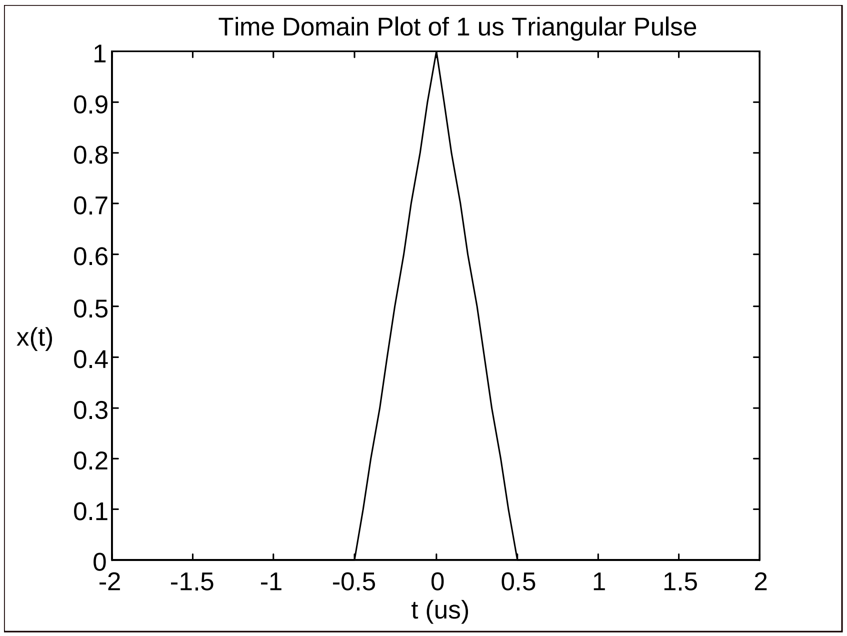
Rectangular Pulse

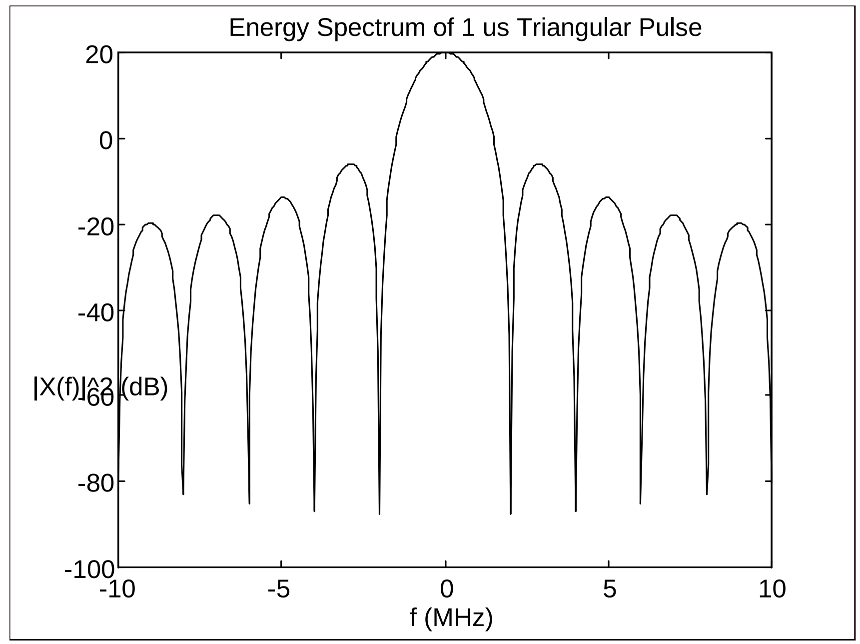




- **First Null BW: 1/T = 1 MHz**
- **First Sidelobe: 13.6 dB down**

Triangular Pulse

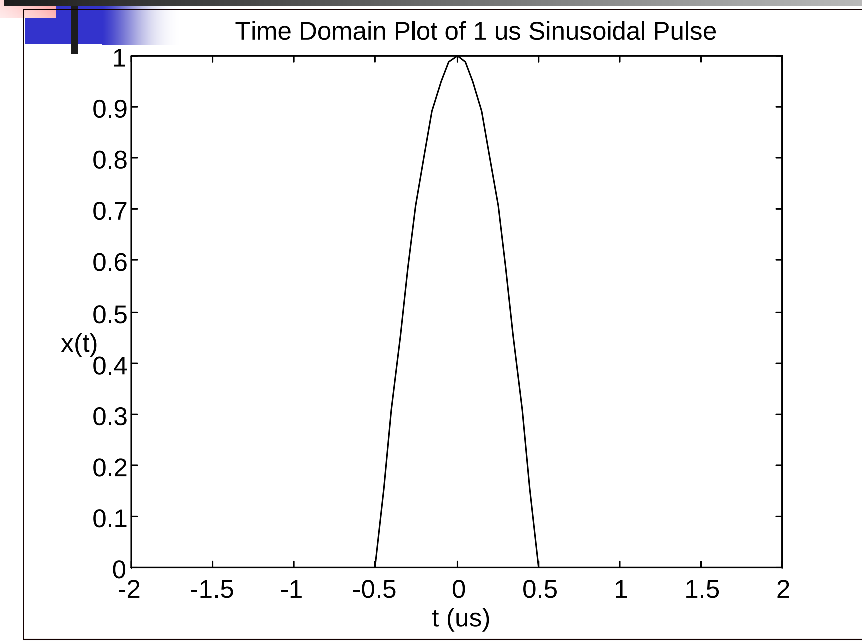


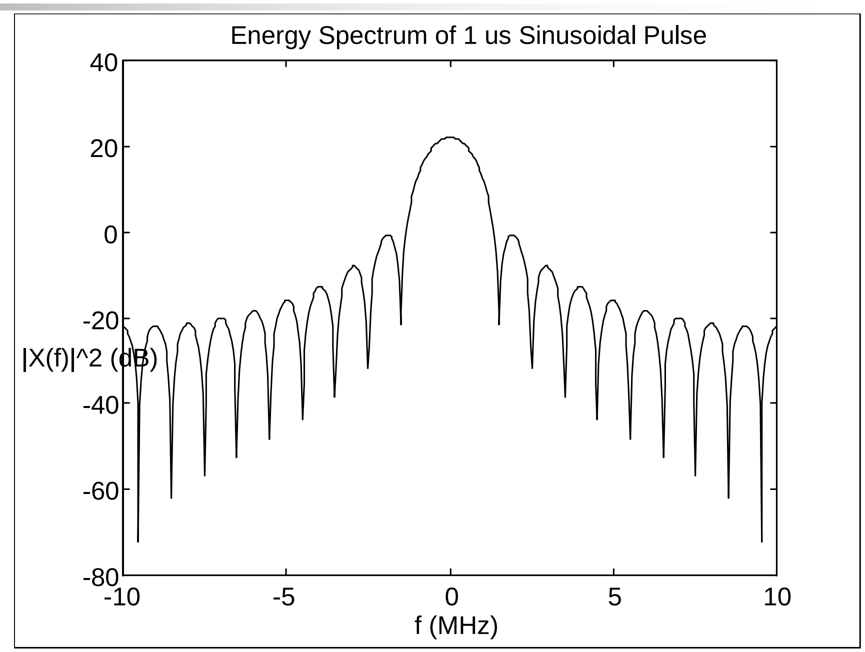


- **First Null BW: 2/T = 2 MHz**
- **First Sidelobe: 26 dB down**

Reduced side-lobes but larger main lobe as compared to square pulse

Sinusoidal Pulse Shape

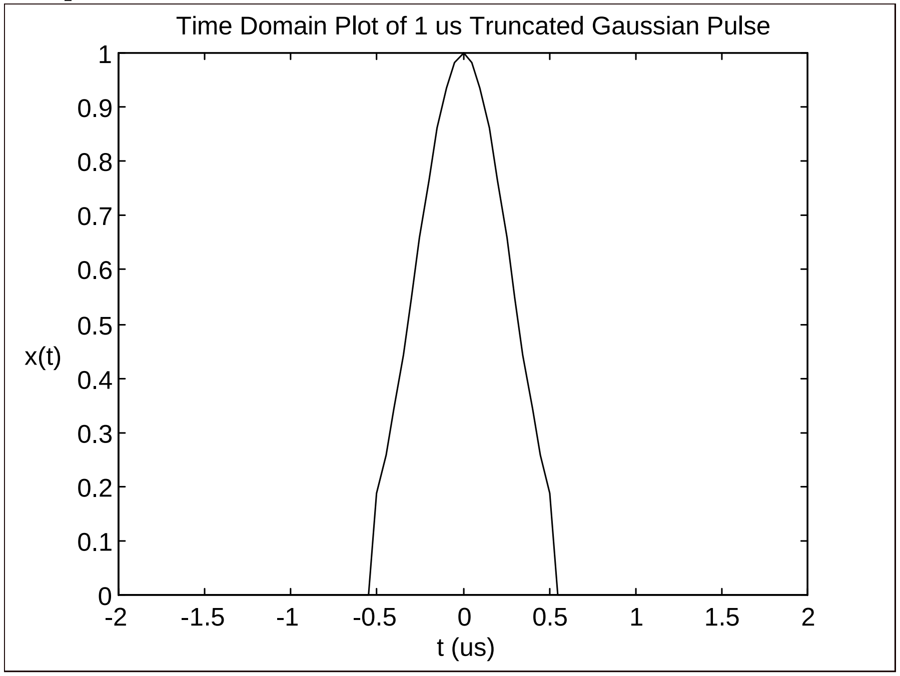


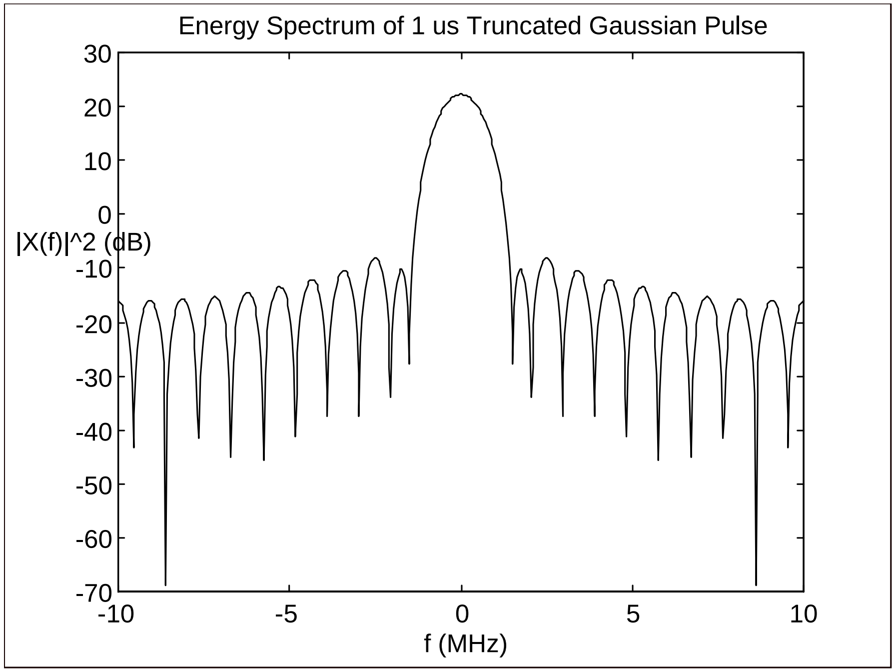


Reduced side-lobes but larger main lobe as compared to square pulse. Larger sidelobes and smaller main lobe as compared to triangular pulse

- **First Null BW: 1.5/T = 1.5 Mhz**
- **First Sidelobe: 22 dB down**

Truncated Gaussian Pulse Shape

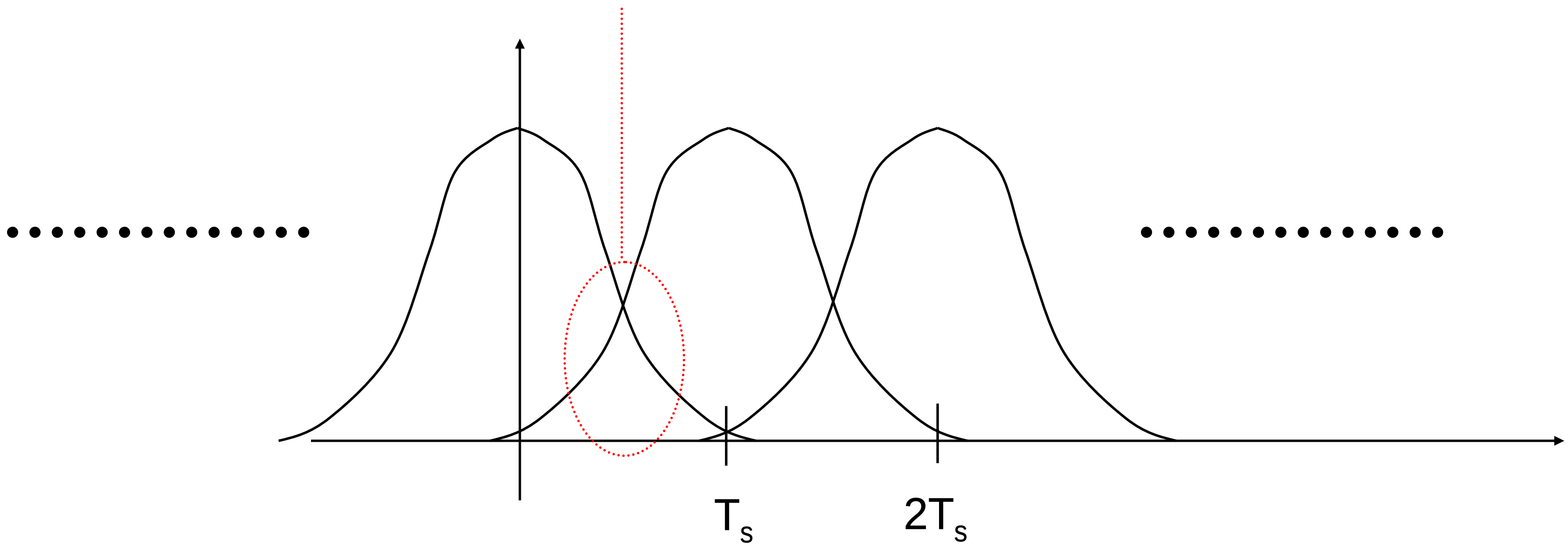




- **First Null BW: 1.5 / T = 1.5 MHz**
- **Largest Sidelobe: 31 dB down**

Inter-symbol Interference

- Though we may refine our pulse shape further, it is clear that we are close to the limit.
- One way to achieve a more narrow spectrum is to use longer duration pulses.
- However, if pulses overlap they may produce inter-symbol interference or ISI



Nyquist's First Criteria for Zero ISI

- Overlapping pulses will not cause inter-symbol interference if they have zero amplitude at the time we sample the signal.
- Mathematically:

$$p(kT_s) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

  - where k is an integer and T_S is one symbol duration

- One function which can do this is the *sinc* function

Using the *sinc* function as a pulse shape

- The pulse shape is:

$$p(t) = \frac{\sin\left(\pi \frac{t}{T}\right)}{\pi \frac{t}{T}}$$

- The baseband transmit signal is

$$s(t) = \sum_n b_n p(t - nT)$$

$$= \sum_n b_n \frac{\sin\left(\pi \frac{t - nT}{T}\right)}{\pi \frac{t - nT}{T}}$$

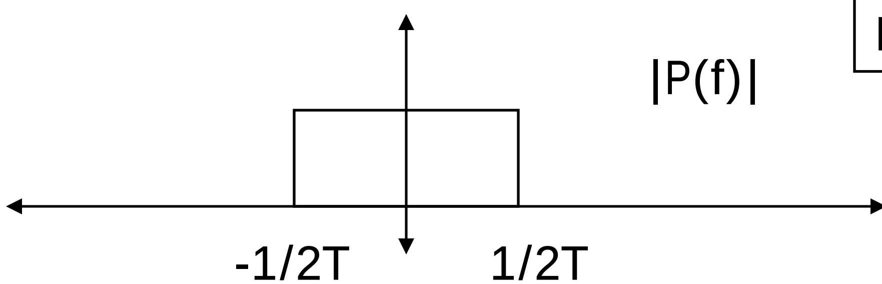
- And the resulting PSD is (for PSK modulation)

$$P_{bpsk}(f) = \frac{1}{T}\left|P(f)\right|^2 = \frac{1}{T}\left|\mathcal{F}\left\{\text{sinc}(t/T)\right\}\right|^2 = \frac{1}{T}\left|\Pi\left(\frac{f}{T}\right)\right|^2$$

Using the *sinc* function as a pulse shape

- Satisfies Nyquist criterion (no ISI)
- Great BW properties

Baseband BW = $R_s/2$
Bandpass BW = R_s



- However, it requires a non-causal signal since the pulse must start before *t=0*. We can accommodate some non-causality through delays, however, we must truncate the pulse somewhere to avoid infinite delay.

Raised Cosine Pulse Family - Satisfies the Nyquist Criteria

- Frequency Domain:

$$H_e(f) = \begin{cases} 1, & |f| < f_1 \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi(|f| - f_1)}{2 f_\Delta}\right)\right], & f_1 \le |f| \le B \\ 0, & |f| > B \end{cases}$$

  - *B* is the absolute bandwidth of the filter

$$f_\Delta = B - f_0, f_1 = f_0 - f_\Delta, r = f_\Delta / f_0$$

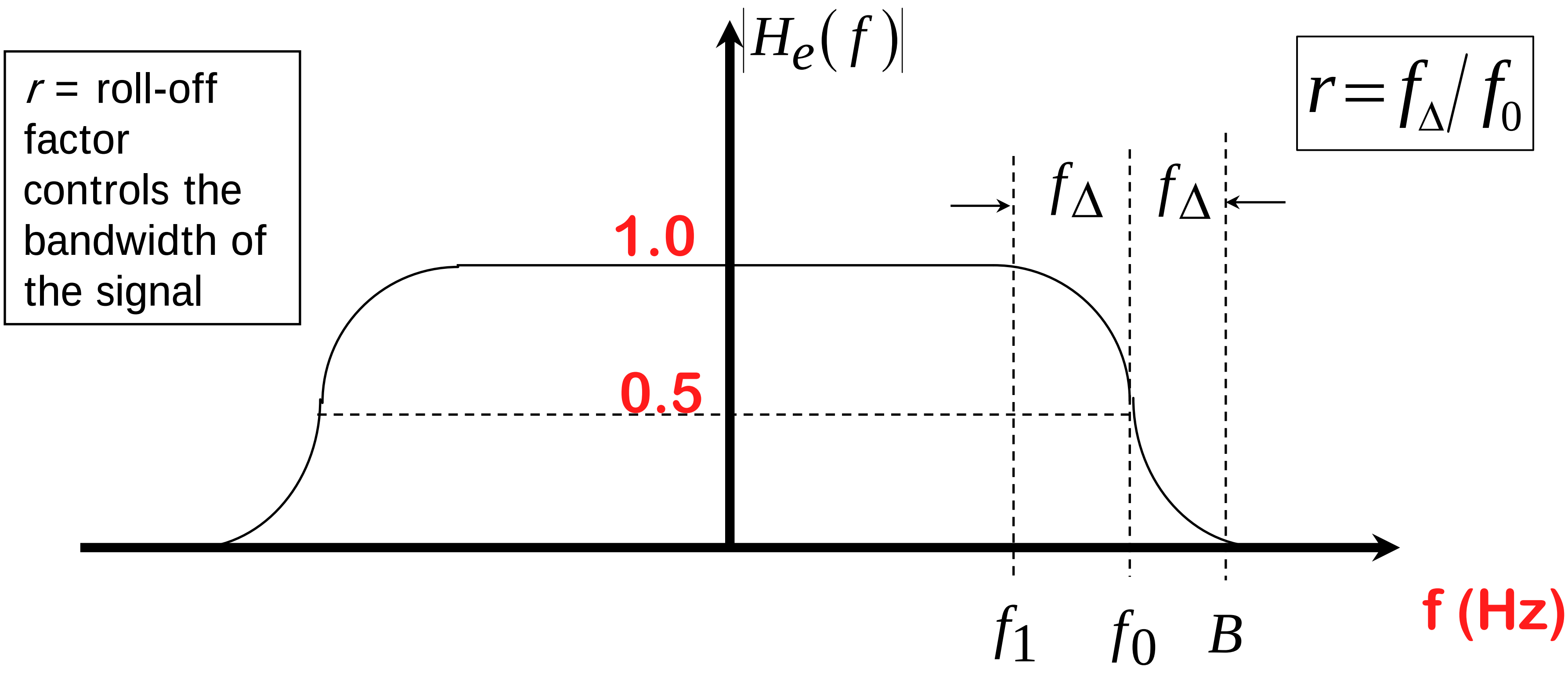
r = roll-off factor

- Time Domain:

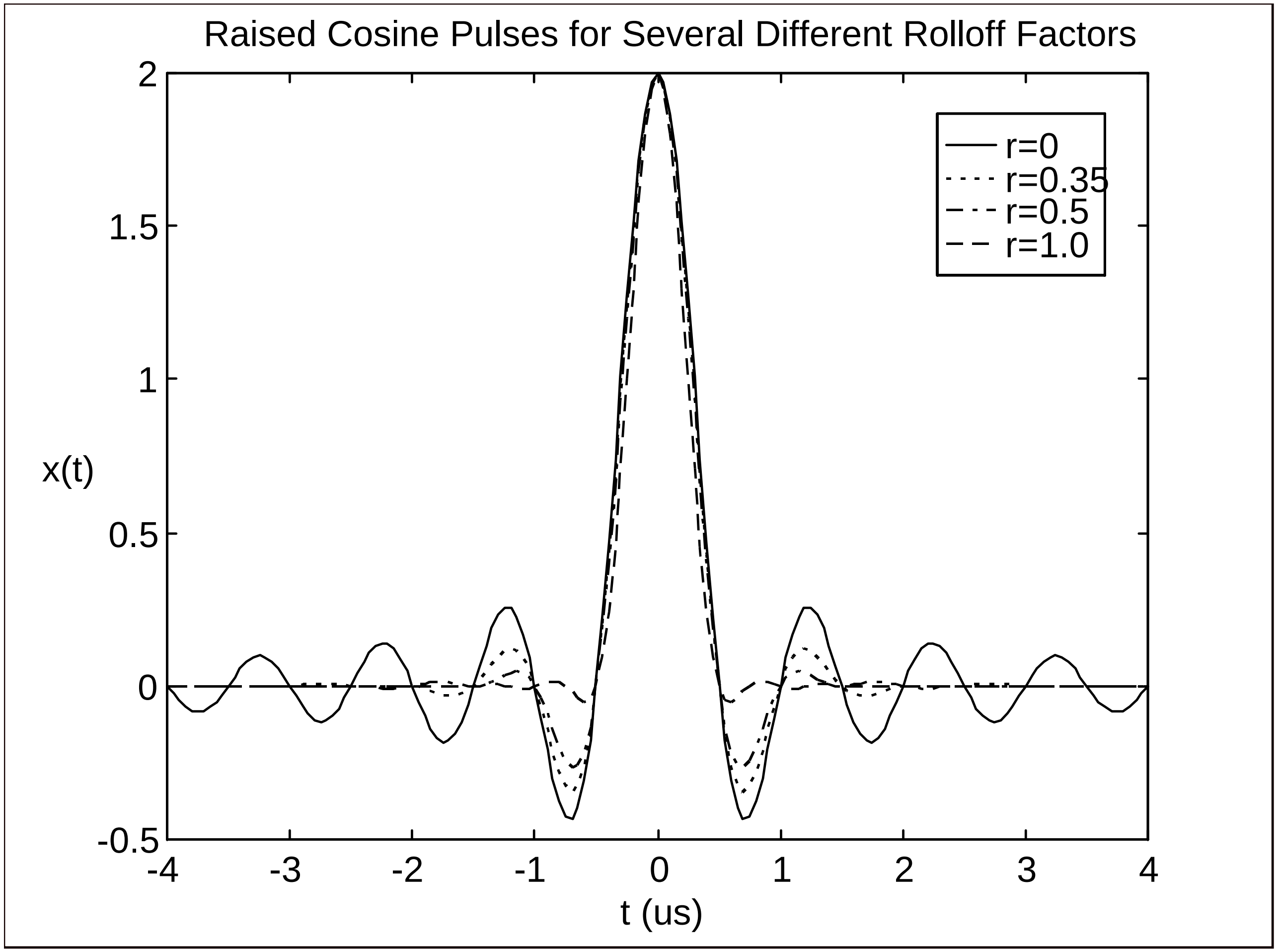
$$h_e(t) = F^{-1}\{H_e(f)\} = 2 f_0 \left(\frac{\sin(2\pi f_0 t)}{2\pi f_0 t}\right) \cdot \left[\frac{\cos(2\pi f_\Delta t)}{1 - (4 f_\Delta t)^2}\right]$$

Spectrum of Raised Cosine Pulse

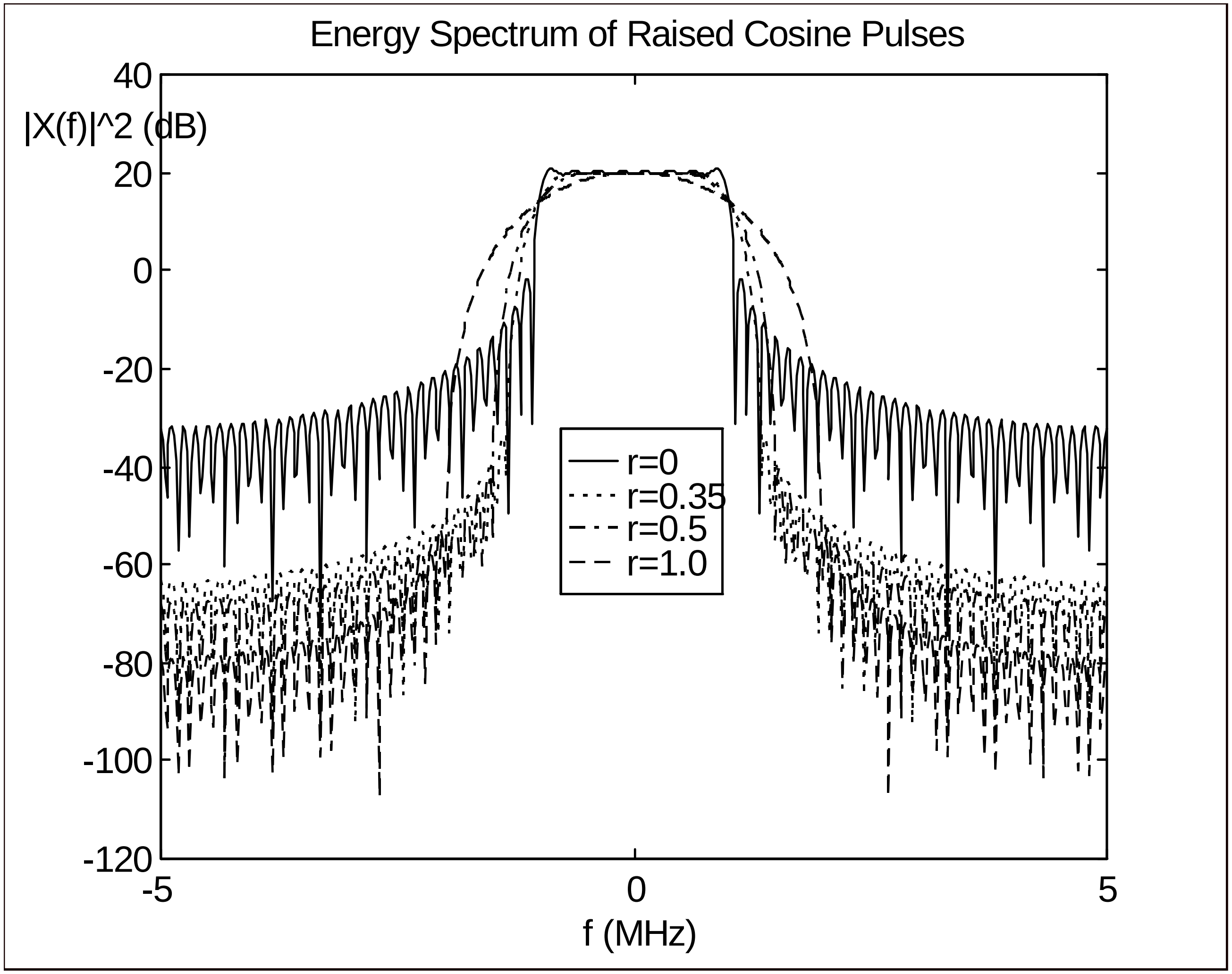
- r=0 corresponds to *sinc* function



Raised Cosine Pulse - Time Domain



Raised Cosine Pulse - Frequency Domain

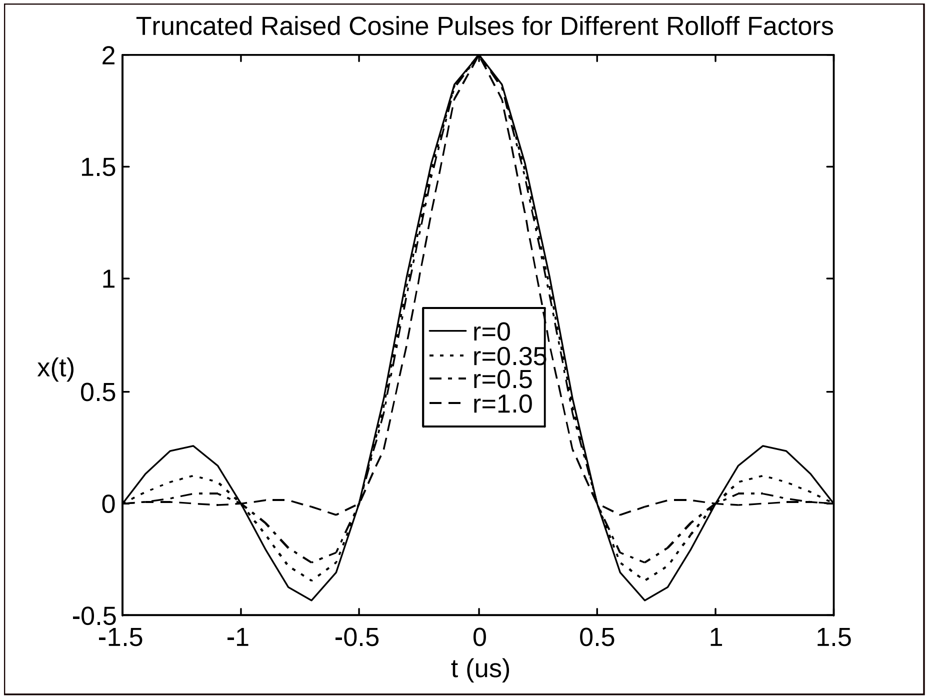


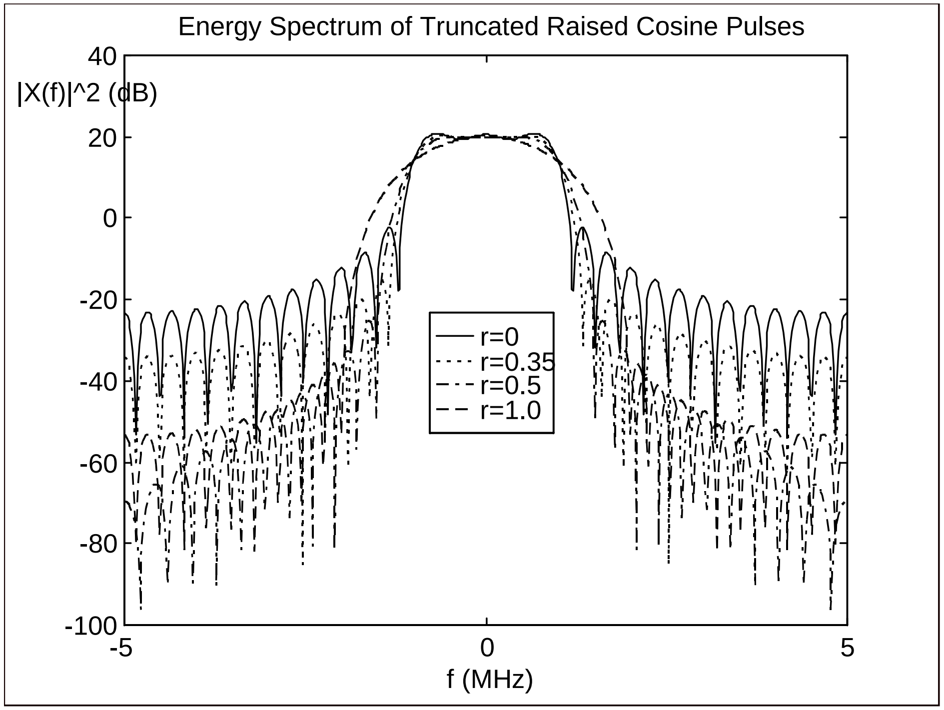
Implementation of Raised Cosine Pulse

- Can be digitally implemented with an FIR filter
- Analog filters such as Butterworth filters may approximate the tight shape of this spectrum
- Practical pulses must be truncated in time
  - Truncation leads to sidelobes - even in RC pulses
- Sometimes a “square-root” raised cosine spectrum is used when identical filters are implemented at transmitter and receiver
  - We will discuss this more when we talk about “matched filtering”

Truncated Raised Cosine Pulses

- Truncating raised cosine pulse to finite duration results in some sidelobes





Bandwidth Requirements for Bandpass Modulation

- Optimum Pulse Shaping:

$$BW = R_s = \frac{R_b}{\log_2 M}$$

- Rectangular Pulse Shaping (a good rule of thumb):

$$BW = 2R_s = \frac{2R_b}{\log_2 M}$$

- Raised Cosine Pulse Shaping:

$$BW = R_s \cdot (1+r) = \frac{R_b}{\log_2 M} \cdot (1+r)$$

- If quadrature channel is not used (e.g. BPSK), then BW is twice as large.

Bandwidth Efficiency:

- **Definition:** $\eta_B = R_b / \mathrm{BW}$ (bits/sec)/Hz
- Measures how efficiently a modulation type uses bandwidth
- Typical Values (assuming optimum pulse shaping):
  - BPSK: 1 bits/sec/Hz
  - QPSK: 2 bits/sec/Hz
  - 8-ary PSK: 3 bits/sec/Hz
  - 16 QAM: 4 bits/sec/Hz
  - 2-ary FSK: 0.5 bits/sec/Hz
  - 8-ary FSK: 3/8 bits/sec/Hz

Signal Space Representation

- The transmitted signal can be represented as:

$$s_m(t) = \sum_{k=1}^{K} s_{m,k} f_k(t) \quad ,$$

where $$s_{m,k} = \int_0^T s_m(t) f_k(t) dt \quad .$$

- The noise can be respresented as: $$n(t) = n'(t) + \sum_{k=1}^{K} n_k f_k(t)$$

where $$n_k = \int_0^T n(t) f_k(t) dt$$

and $$n'(t) = n(t) - \sum_{k=1}^{K} n_k f_k(t)$$

Signal Space Representation (continued)

- The received signal can be represented as:

$$r(t) = \sum_{k=1}^{K} s_{m,k} f_k(t) + \sum_{k=1}^{K} n_k f_k(t) + n'(t)$$

$$= \sum_{k=1}^{K} r_k f_k(t) + n'(t)$$

where $r_k = s_{m,k} + n_k$

Signal Space Representation

- How does pulse shaping impact signal space representation?
- We include the pulse in the basis function
  - Constellation doesn't change!

Example: *M*-PSK

- Assuming square pulses:

$$s_i(t) = \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right)\Bigg|_0^T, \quad i = 0, \ldots, M-1$$

- With general pulse shaping

$$s_i(t) = g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M} i\right)\Bigg|_0^T, \quad i = 0, \ldots, M-1$$

Example: M–PSK (cont.)

$$s_i(t) = \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right)\Bigg|_0^T, \; i = 0, \ldots, M-1$$

$$f_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)\Big|_0^T \qquad f_2(t) = -\sqrt{\frac{2}{T}} \sin(2\pi f_c t)\Big|_0^T$$

$$c_{1i} = \sqrt{\frac{T}{2}} \cos\left(\frac{2\pi}{M} i\right) \qquad c_{2i} = \sqrt{\frac{T}{2}} \sin\left(\frac{2\pi}{M} i\right)$$

$$s_i(t) = c_{1i} f_1(t) + c_{2i} f_2(t)$$

Example: *M* –PSK (cont.)

$$s_i(t) = g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M} i\right)\Bigg|_0^T, \quad i = 0, \ldots, M-1$$

$$f_1(t) = \sqrt{\frac{2}{T}\frac{1}{E_g}} g(t)\cos(2\pi f_c t)\Bigg|_0^T \qquad f_2(t) = \sqrt{\frac{2}{T}\frac{1}{E_g}} g(t)\sin(2\pi f_c t)\Bigg|_0^T$$

$$c_{1i} = \sqrt{\frac{T}{2}E_g} \cos\left(\frac{2\pi}{M} i\right) \qquad c_{2i} = \sqrt{\frac{T}{2}E_g} \sin\left(\frac{2\pi}{M} i\right)$$

$$s_i(t) = c_{1i} f_1(t) + c_{2i} f_2(t)$$

Average Symbol Energy

- Without Pulse shaping

$$E_s = \frac{T}{2}\cos^2\left(\frac{2\pi}{M}i\right) + \frac{T}{2}\sin^2\left(\frac{2\pi}{M}i\right)$$

$$= \frac{T}{2}$$

$$\mathbf{s}_i = \left[\sqrt{E_s}\cos\left(\frac{2\pi}{M}i\right) \quad \sqrt{E_s}\sin\left(\frac{2\pi}{M}i\right)\right]$$

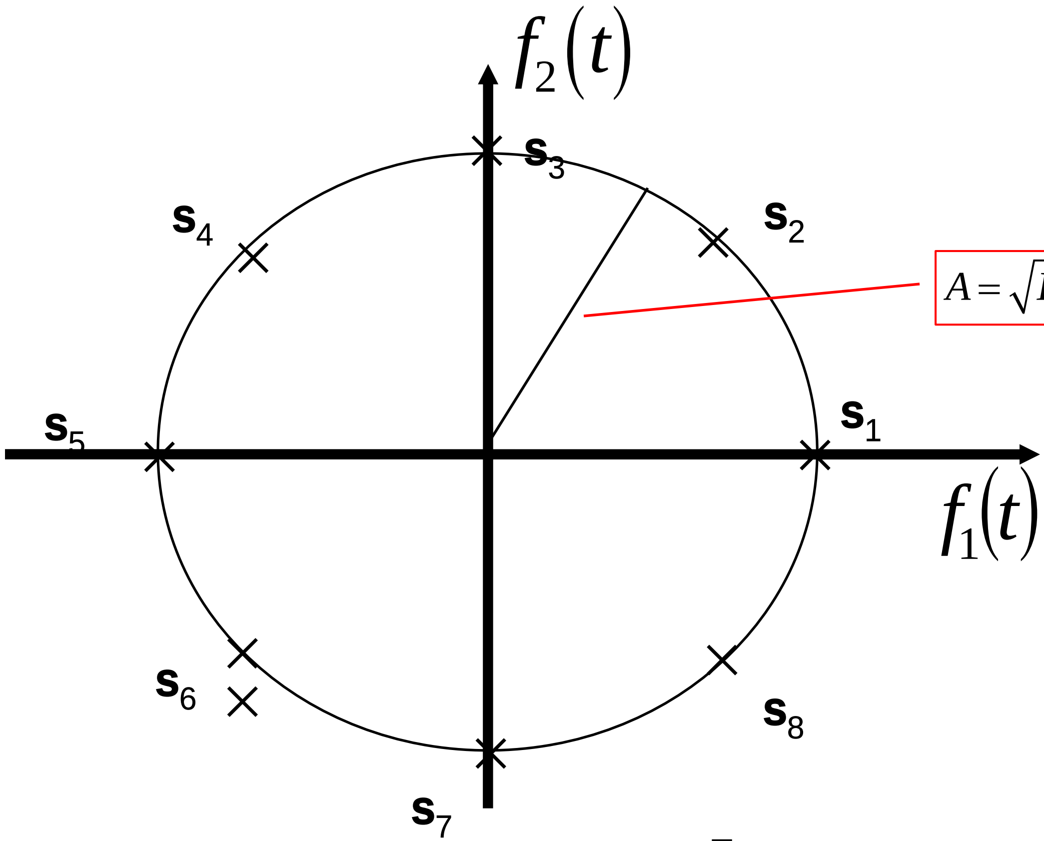
- With pulse shaping

$$E_s = \frac{T}{2}E_g\cos^2\left(\frac{2\pi}{M}i\right) + \frac{T}{2}E_g\sin^2\left(\frac{2\pi}{M}i\right)$$

$$= \frac{T}{2}E_g$$

$$\mathbf{s}_i = \left[\sqrt{E_s}\cos\left(\frac{2\pi}{M}i\right) \quad \sqrt{E_s}\sin\left(\frac{2\pi}{M}i\right)\right]$$

Ex: 8-ary PSK (either case)



$$\mathbf{s}_i = \left[\sqrt{E_s}\cos\left(\frac{2\pi}{M}i\right) \quad \sqrt{E_s}\sin\left(\frac{2\pi}{M}i\right) \right]$$