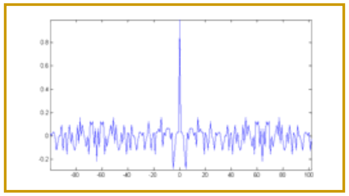
ECE 5660 – Spread Spectrum Communications
Spring 2008



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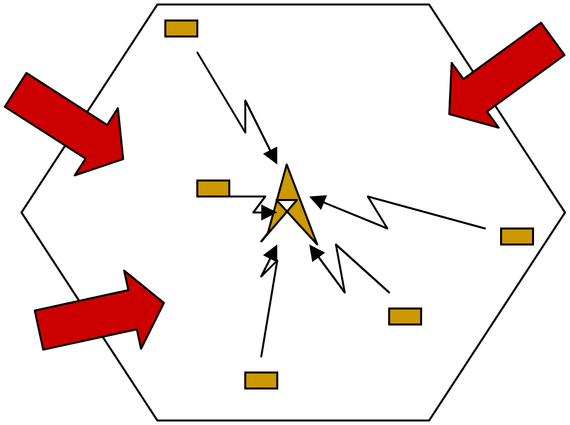
Lecture #19: CDMA – System Capacity

CDMA

- Previously we analyzed the capacity of CDMA system using a simplistic first order model using average interference values
- Today we will take another look at the capacity of CDMA cellular telephony, still making some simplifying assumptions, but with more rigor than last class.
  - The analysis follows the original work of Gilhousen,et. al. from Qualcomm
    - K.S. Gilhousen, et. al, “On the Capacity of a Cellular CDMA System,” *IEEE Transactions on Vehicular Technologies*, vol. 40, no.2, pp. 303-312, May 1991.

Uplink vs. Downlink

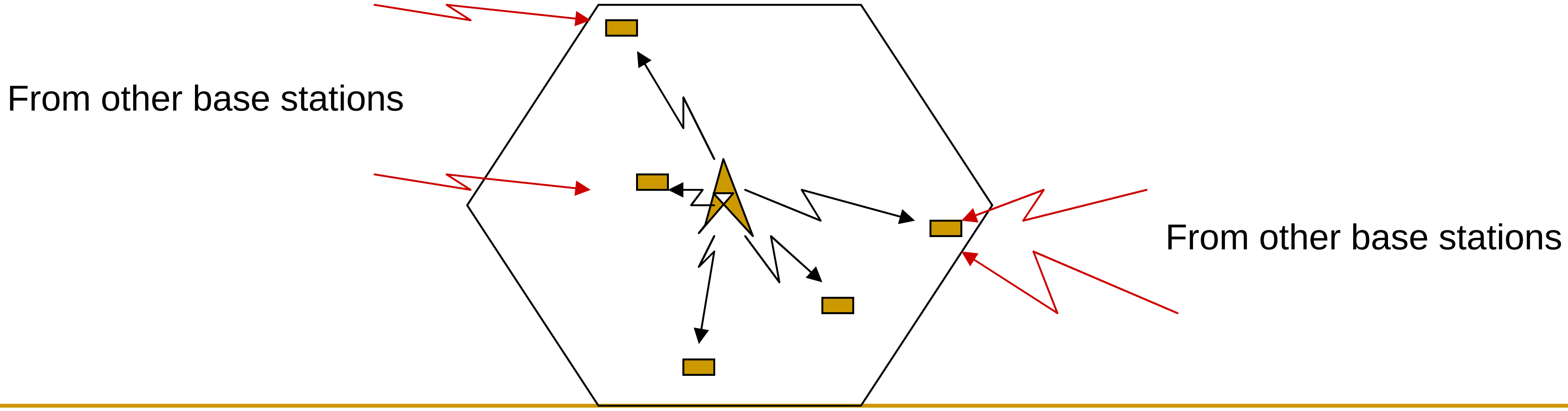
- The uplink and downlink of a CDMA system are fundamentally different.
- The uplink has many uncoordinated users transmitting to a single base station.
  - The path loss from mobile stations can vary greatly and thus the uplink requires strict power control to avoid the near-far problem
  - Mobile station signals arrive asynchronously
  - Interference on the uplink is caused by a large number of low power distributed sources



Out-of-cell interference

Uplink vs. Downlink

- The downlink has a single transmitter sending a signals to many receivers.
  - The path loss of all user signals is the same to a given mobile, thus the need for power control is not as great.
  - All downlink signals are synchronous
  - Interference is caused by a relatively small number of high power sources (i.e., the other base stations)
  - A common pilot is possible and efficient which facilitates coherent demodulation (as well as acquisition and soft handoff)



Uplink vs. Downlink

- As a result, IS-95 (the first cellular CDMA system) had the following characteristics
  - Uplink
    - non-coherent modulation (a pilot was thought to be inefficient)
    - long psuedo-random spreading codes
    - strict (1.25ms) feedback based (closed loop) power control
  - Downlink
    - coherent modulation based on common pilot
    - Walsh codes for separation between users and PN covering code for each base station
    - slow (frame-based) power control
  - The uplink was predicted and widely believed to be the limiting link. This will be seen in our analysis today. However, today most believe that the links are either balanced or downlink limited.

Single Cell Capacity

- Recall that with perfect power control and *K* users operating on the uplink of a CDMA system, the signal-to-interference ratio per user at the base station is

$$SIR = \frac{P}{(K-1)P} = \frac{1}{(K-1)}$$

- Since the noise/interference power is spread over B_T Hz and the data rate is *R*, we can write

$$\frac{E_b}{I_o} = \frac{P/R_b}{(K-1)P/B_T} = \frac{B_T/R_b}{(K-1)}$$

- If we include thermal noise

$$\frac{E_b}{I_o} = \frac{P/R_b}{(K-1)P/B_T + N_o} = \frac{B_T/R_b}{(K-1) + \widetilde{N}/P}$$

- For a required value of E_b/I_o, the number of users that can be supported is then

$$K = 1 + \frac{B_T/R_b}{E_b/I_o} - \frac{\widetilde{N}}{P}$$

Sectorization and Voice Activity

- With sectorization we obtain a reduction in the interference seen. Only a fraction of the cell's *K* users will cause interference to the desired user.
- By monitoring voice activity at the transmitter, we can suppress transmission when the speaker is not active.
- Including a voice activity factor $\nu < 1$ and a sector antenna of *G* the average E_b/I_o is

$$\frac{\overline{E_b}}{I_o} = \frac{B_T / R_b}{\left(K_s - 1\right)\nu / G + \widetilde{N} / P}$$

- The resulting capacity per cell is then

$$K = 1 + \frac{G}{\nu}\left[\frac{B_T / R}{E_b / I_o} - \frac{N}{P}\right]$$

Approximately $3/\nu$ increase in capacity

Impact of Out-of-Cell Interference

- We can include the impact of out-of-cell interference by simply adding a factor *f* to the interference

$$\frac{\overline{E_b}}{I_o} = \frac{B_T / R_b}{(K-1)\nu(1+f)/G + \widetilde{N}/P}$$

$$K = 1 + \frac{G}{\nu(1+f)}\left[\frac{B_T / R_b}{E_b / I_o} - \frac{\widetilde{N}}{P}\right]$$

- Alternatively we can model the out-of-cell interference as a random variable.
- Similarly, instead of using a constant factor for voice activity we can model voice activity as a random variable

Reverse Link Capacity

- E_b/I_o requirement of 7dB, ν = 3/8, W =1.25MHz, R = 8kbps, P/N = 1
  - CDMA single cell capacity of 127
  - Ideal TDMA/FDMA
    - $K = B_T / R_b / Q$
    - With reuse Q = 7 → 1.25MHz/8kbps = 22 users
  - Actual FM/FDMA, 30kHz channels → 6 users
  - Actual TDMA (IS-136) with coding → 3 channels/30kHz →18 users
- Thus, with voice activity and sectorization CDMA has a large advantage over TDMA or FDMA

Second-order Modeling of Interference

- Up to this point, we have assumed that the interference takes on it average value. A more accurate approach is to model the interference as a random variable and determine the *outage probability.*
- Due to voice activity and power control, in-cell interference is modeled as a sum of binary random variables
- Out-of-cell interference is modeled as a Gaussian random variable
- Cell definition
  - ❑ We typically think of a mobile being associated with a particular cell, *i.e.*, the mobile receives its signal from a particular base station
  - ❑ In cellular this association is based on the strongest received signal
    - If we are only concerned with path loss, this translates into associating a mobile with the closest base station
    - Including the effect of shadowing, this is no longer true.
    - Shadowing is typically modeled as log-normal random variable

Second-order Modeling

- The desired user's E_b/I_o is best modeled a random variable defined as

$$\frac{E_b}{I_o} = \frac{B_T / R_b}{\sum_{i=1}^{K_s-1} \psi_i + \frac{I}{P} + \frac{\widetilde{N}}{P}}$$

where the in-cell interference is now represented as a sum of binary random variables (modeling the voice activity) defined as

$$\psi_i = \begin{cases} 1 & \text{with probability } \nu \\ 0 & \text{with probability } 1\text{-}\nu \end{cases}$$

and I/P is approximately a Gaussian random variable with mean and variance that must be calculated

Out-of-Cell Interference

- The received power from a mobile at its cell site (*r* meters away) is proportional to $10^{\xi/10} r^{-\gamma}$ where $2<\gamma<5$ is the path loss exponent and $10^{\xi/10}$ is the log-normal shadowing.

- We assume perfect power control. Consider a user in an adjacent cell a distance r_m from its base station and experiencing a ζ_m dB of shadowing loss to its base station. Further, it is a distance r_o from the base station of interest and experiences a ζ_o dB shadowing loss to the base station of interest.
- Assuming that $\gamma = 4$ the normalized interference in the desired cell is

$$\frac{I(r_o, r_m)}{P} = \left(\frac{10^{\xi_o/10}}{r_o^4}\right)\left(\frac{r_m^4}{10^{\xi_m/10}}\right)$$

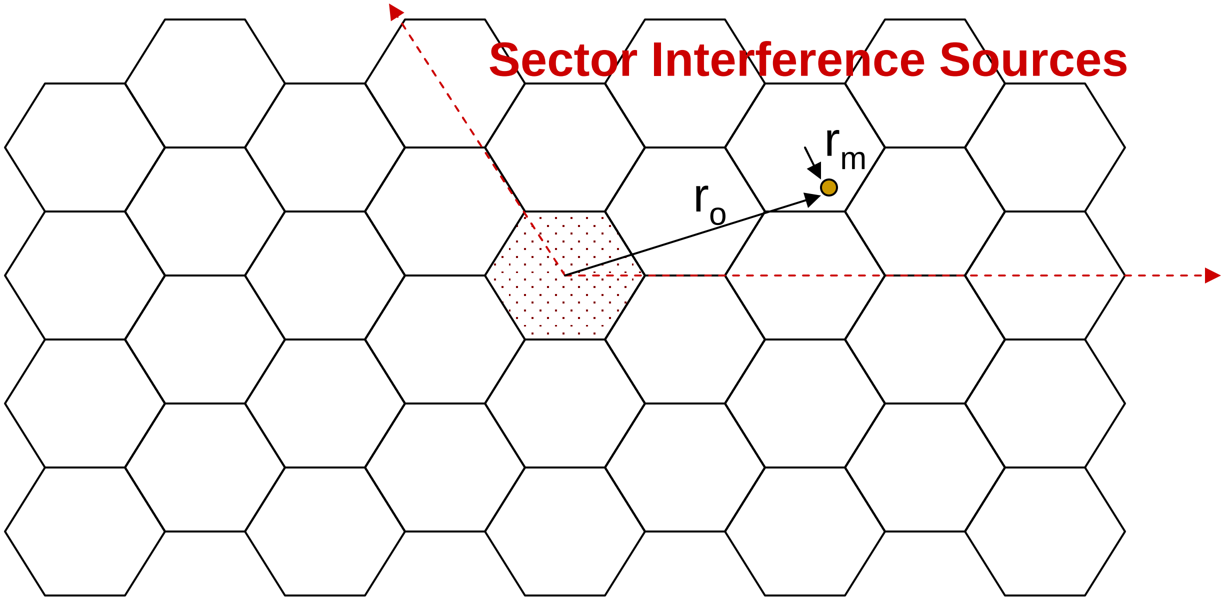
$$= \left(\frac{r_m^4}{r_o^4}\right) 10^{(\xi_o - \xi_m)/10} \leq 1$$

Out-of-Cell Interference – cont.

- Assuming a uniform density of users and normalizing the hexagonal cell radius to unity the density of users is

$$\rho = \frac{2K}{3\sqrt{3}} = \frac{2K_s}{\sqrt{3}}$$

- The total interference is then found by integrating the interference per user over the area of out-of-cell interference



Impact of Out-of-Cell Interference (cont.)

- Thus the total interference is

$$\frac{I}{P}=\iint \psi\left(\frac{r_m}{r_o}\right)^4 10^{(\xi_o-\xi_m)/10}\chi\left(\xi_o-\xi_m,\frac{r_o}{r_m}\right)\rho dA$$

- *where* ψ is voice activity variable which is 1 with probability ν and 0 with probability 1-ν and

$$\chi\left(\xi_o-\xi_m,\frac{r_o}{r_m}\right)=\begin{cases}1 & \text{if } \left(\frac{r_m}{r_o}\right)^4 10^{(\xi_o-\xi_m)/10}\le 1 \\ & \text{or } \xi_o-\xi_m\le 40\log_{10}\left(\frac{r_o}{r_m}\right) \\ 0 & else\end{cases}$$

In other words, we don't include mobiles in the interference calculation that should be assigned as in-cell mobiles

Expected Interference Calculation

$$E\left\{\frac{I}{P}\right\} = E\left\{\iint \psi\left(\frac{r_m}{r_o}\right)^4 10^{(\xi_o-\xi_m)/10}\chi\left(\xi_o-\xi_m,\frac{r_o}{r_m}\right)\rho dA\right\}$$

$$= \iint\left(\frac{r_m}{r_o}\right)^4 E\{\psi\}E\left\{10^{(\xi_o-\xi_m)/10}\chi\left(\xi_o-\xi_m,\frac{r_o}{r_m}\right)\right\}\rho dA$$

$$= \iint\left(\frac{r_m}{r_o}\right)^4 \nu\left(\int_{-\infty}^{40\log(r_o/r_m)} e^{x\ln(10)/10}\frac{e^{-x^2/4\sigma^2}}{\sqrt{4\pi\sigma^2}}dx\right)\rho dA$$

$$= \iint\left(\frac{r_m}{r_o}\right)^4 \nu e^{[\sigma\ln(10)/10]^2}\left\{1-Q\left[\frac{40\log(r_o/r_m)}{\sqrt{2\sigma^2}}-\sqrt{2\sigma^2}\frac{\ln 10}{10}\right]\right\}\rho dA$$

- or

$$E\left\{\frac{I}{P}\right\} = \nu\iint\left(\frac{r_0}{r_m}\right)^4 f\left(\frac{r_0}{r_m}\right)\rho dA$$

- where

$$f\left(\frac{r_0}{r_m}\right) = \exp\left[(\sigma\ln 10/10)^2\right]\left\{1-Q\left[\frac{40}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right)-\sqrt{2\sigma^2}\frac{\ln 10}{10}\right]\right\}$$

Expected Interference Calculation (cont.)

- for σ = 8dB (and ν = 3/8) the numerical integration is

$$E\left\{\frac{I}{P}\right\} = 0.247K_s$$

- The variance of the interference can be shown to be

$$\text{var}\left\{\frac{I}{P}\right\} = \iint \left(\frac{r_0}{r_m}\right)^8 \left[\nu g\left(\frac{r_m}{r_o}\right) - \nu^2 f\left(\frac{r_m}{r_o}\right)\right] \rho dA$$

- where $g\left(\frac{r_m}{r_o}\right) = \exp\left[\left(\sigma \ln 10/5\right)^2\right]\left\{1 - Q\left[\frac{40}{\sqrt{2\sigma^2}}\log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\,\frac{\ln 10}{5}\right]\right\}$

- for σ = 8dB $\text{var}\left\{\frac{I}{P}\right\} = 0.078K_s$

Outage Probability

- We define an outage as the condition where the desired user's E_b/I_o does not meet the requirement of 7dB (approximately what is needed for a 10^{-3} error rate)

$$P_o = \Pr\left\{BER > 10^{-3}\right\} = \Pr\left(\sum_{i=1}^{K_s-1}\psi_i + \frac{I}{P} > \delta\right)$$

where

$$\delta = \frac{B_T/R}{E_b/I_o} - \frac{\widetilde{N}}{P}$$

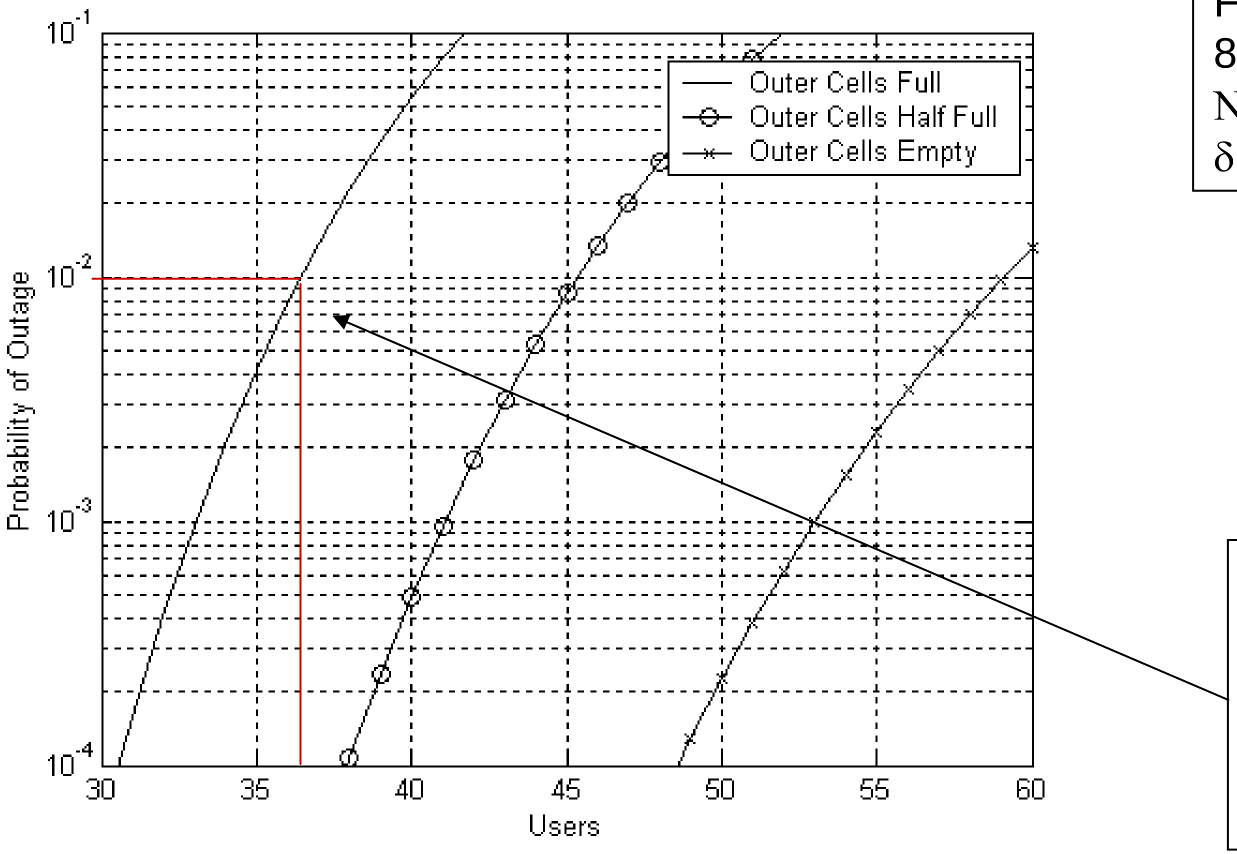
$$\boxed{\frac{E_b}{I_o} = \frac{B_T/R_b}{\sum_{i=1}^{K_s-1}\psi_i + \frac{I}{P} + \frac{\widetilde{N}}{P}}}$$

- This can be calculated as

$$P_o = \sum_{k=0}^{K_s-1}\Pr\left\{\frac{I}{P} > \delta - k \,\middle|\, \sum_i \psi_i = k\right\}\Pr\left\{\sum_i \psi_i = k\right\}$$

$$= \sum_{k=0}^{K_s-1}\binom{K_s-1}{k}\nu^k(1-\nu)^{K_s-1-k}\,Q\left\{\frac{\delta - k - 0.247K_s}{\sqrt{0.078K_s}}\right\}$$

Reverse Link Outage Probability



For *W*=1.25MHz, *R* = 8kbps, E_b/I_o = 7dB, N/P = 1 results in δ = 30

1% outage results in reverse link capacity of 36 users per sector or 108 users per cell

Forward Link Capacity

- Assume that a mobile can see *M* base stations with relative powers

$$P_{T_1} > P_{T_2} > P_{T_3} \cdots > P_{T_M} > 0$$

- Cell site selection is based on the strongest receive signal
- The received E_b/I_o is then

$$\frac{E_b}{I_o} \geq \frac{\beta f_i P_{T_1} / R_b}{\left[\left(\sum_{j=2}^{M} P_{T_j}\right) + N\right] / B_T}$$

- β = fraction of base station power devoted to traffic
    - (1-β) is fraction devoted to pilot
- f_i = fraction of the traffic power devoted to *i* th user

Forward Link Capacity (cont.)

- If we solve for the necessary power fraction for a required E_b/I_o

$$f_i \leq \frac{\left(E_b/I_o\right)_i}{\beta B_T/R_b}\left[1+\left(\frac{\sum_{j=2}^{M} P_{T_j}}{P_{T_1}}\right)_i+\frac{N}{\left(P_{T_1}\right)_i}\right]$$

- where we have the constraint

$$\sum_{i=1}^{K_s} f_i \leq 1$$

- We can define the relative received cell-site powers as

$$\varphi_i \triangleq \left(1+\sum_{j=2}^{K}\frac{P_{T_j}}{P_{T_1}}\right)_i$$

Forward Link Outage Probability

- By summing over φ_i we have the constraint

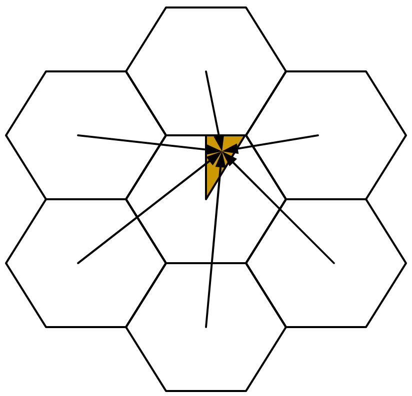
$$\sum_{i=1}^{K_s} \varphi_i \le \frac{\beta B_T / R_b}{E_b / I_o} - \sum_{i=1}^{K_s} \frac{N}{P_{T_i}} \triangleq \delta'$$

- Since we have coherent modulation on the downlink (facilitated by the strong pilot) we require an E_b/I_o of 5dB
- We can support K_s users if we can satisfy the above constraint for E_b/I_o=5dB. The outage probability is

$$P_o = \Pr\left\{BER > 10^{-3}\right\} = \Pr\left\{\sum_{i=1}^{K_s} \varphi_i > \delta'\right\}$$

Distribution of φ_i

- The distribution of φ_i does not lend itself to analysis
- Simulation was used to calculate the distribution of φ_i
- For each point (for a certain granularity) in the triangle below the attenuation to the center cell and each of the surrounding 18 cells was calculated as $10^{\xi_k/10} r_k^{-4} \quad k = 0,1,2\ldots18$
- These values were used to calculate $\varphi_i \triangleq \left(1+\sum_{j=2}^{K}\frac{S_{T_j}}{S_{T_1}}\right)_i$
- This was repeated for each of 65 equally spaced points in the triangle 10000 times



Geometry for simulation (second tier of cells not shown)

Distribution of φ_i (cont.)

- Resulting Histogram of $\varphi_i - 1$

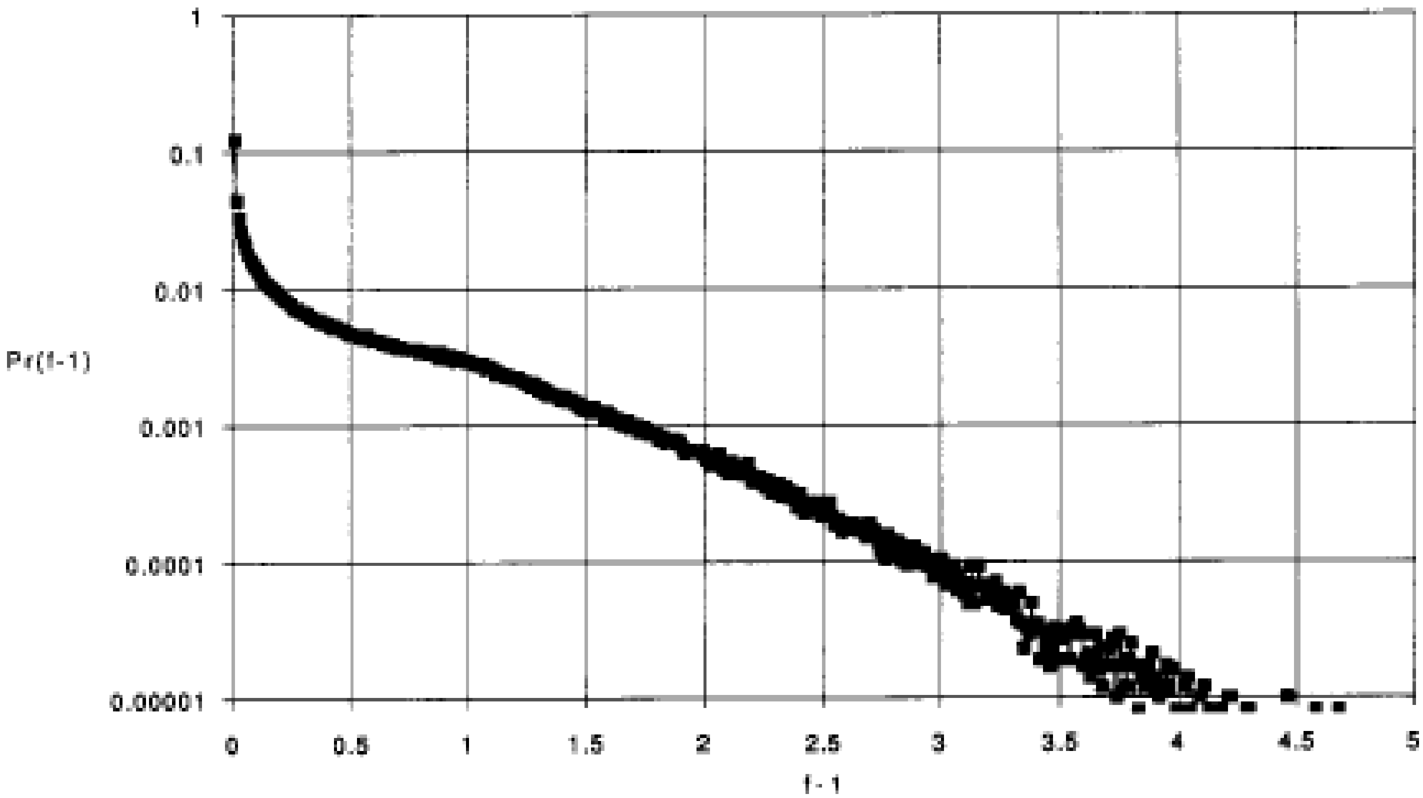


Fig. 4. Histogram of forward power allocation.

K.S. Gilhousen, et. al, “On the Capacity of a Cellular CDMA System,” *IEEE Transactions on Vehicular Technologies*, vol. 40, no.2, pp. 303-312, May 1991.

Outage Probability

- From the histogram we can compute the Chernoff Bound on the Outage Probability as

$$P_o < \min_{\lambda>0} E\left\{\exp\left[\lambda\sum_{i=1}^{K_s}\varphi_i - \lambda\delta'\right]\right\}$$

$$= \min_{\lambda>0}\left[(1-\nu)+\nu\sum_k \Pr_k \exp(\lambda\varphi_k)\right]^{K_s} e^{-\lambda\delta'}$$

- where Pr_k is the probability of φ for bin *k*

Outage Probability

- The resulting outage probability for B_T= 1.25MHz, R = 8kbps, β = 0.8, δ’ = 38

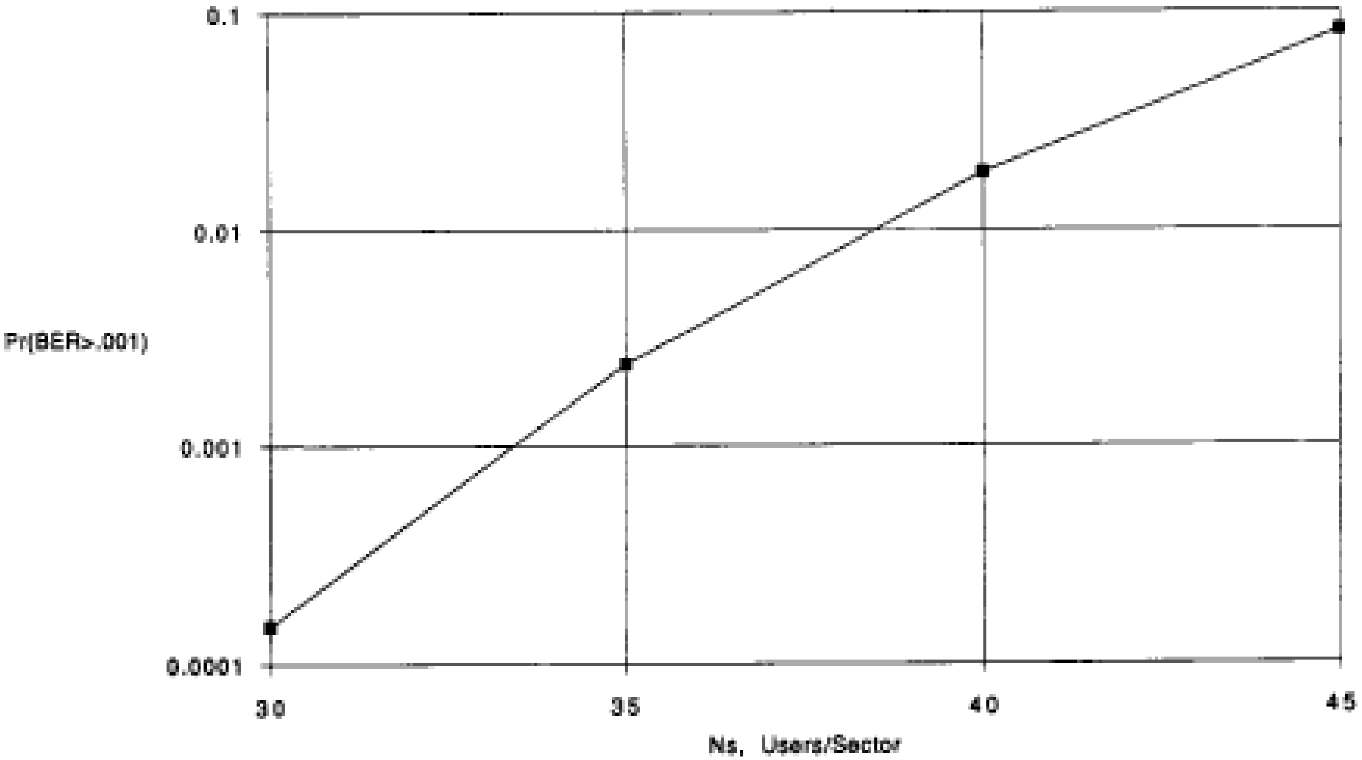


Fig. 5. Forward link capacity/sector. (W = 1.25 MHz, R = 8 kb/s, voice activity = 3/8, pilot power = 20 %).

K.S. Gilhousen, et. al, “On the Capacity of a Cellular CDMA System,” *IEEE Transactions on Vehicular Technologies*, vol. 40, no.2, pp. 303-312, May 1991.

Comparisons

For 1.25MHz channel

Uplink: 36 users per sector (108 per cell) for 10^{-3} BER and 99% outage probability

Downlink: 38 users per sector (114 per cell)

AMPS: 1.25MHz/30kHz/channel = 41

reuse = 7 → 6 channels per cell

TDMA: 1.25MHz/30kHz/3 channels = 123

reuse = 7 → 18 users per cell

Comparisons (cont.)

- Real system capacities show less dramatic gains
  - ~13-15 per sector (~40-45 per cell) for IS-95
  - ~26 per sector (78 per cell) for *cdma2000*
- Idealizations
  - Perfect sector antennas
  - Ignored power needed for soft handoff
  - Perfect power control
  - Ignored intra-cell interference due to non-orthogonal signals and multipath

Complications

- Power Control
  - Power control is the most important function in a CDMA system
    - In TDMA/FDMA RRM is a channel management issue
    - In CDMA RRM is a power/interference management issue
  - We have assumed perfect power control
    - The dynamic range on transmit power can be 80dB
    - Fast Rayleigh fading is very difficult to track
    - Slow log-normal shadowing is easy to track
  - Forward link doesn't require strict power control for single cell system since SINR will remain constant as signal fades
    - Slow power control is necessary to avoid transmitting excess power
- Sectorized antennas
  - Have assumed perfect isolation between sectors which is clearly optimistic

Pole Capacity

- Very often engineers will speak of the "pole capacity" as the maximum (although unattainable) capacity of a CDMA system
- This is calculated from capacity equation as SNR goes to infinity

$$K_s = 1 + \frac{1}{\nu}\left[\frac{B_T / R}{E_b / I_o} - \frac{N}{P}\right]$$

$$K_s^{pole} = \lim_{\frac{P}{N}\to\infty}\left(1 + \frac{1}{\nu}\left[\frac{B_T / R}{E_b / I_o} - \frac{N}{P}\right]\right)$$

$$= 1 + \frac{1}{\nu}\left[\frac{B_T / R}{E_b / I_o}\right]$$

- Example: E_b/I_o requirement of 7dB, ν = 3/8, B_T =1.25MHz, R = 8kbps
  - ❑ $K_s^{pole} = 83$

Pole Capacity (cont.)

- The term “pole” capacity can be understood by examining the SNR required to obtain a fraction of this maximum capacity

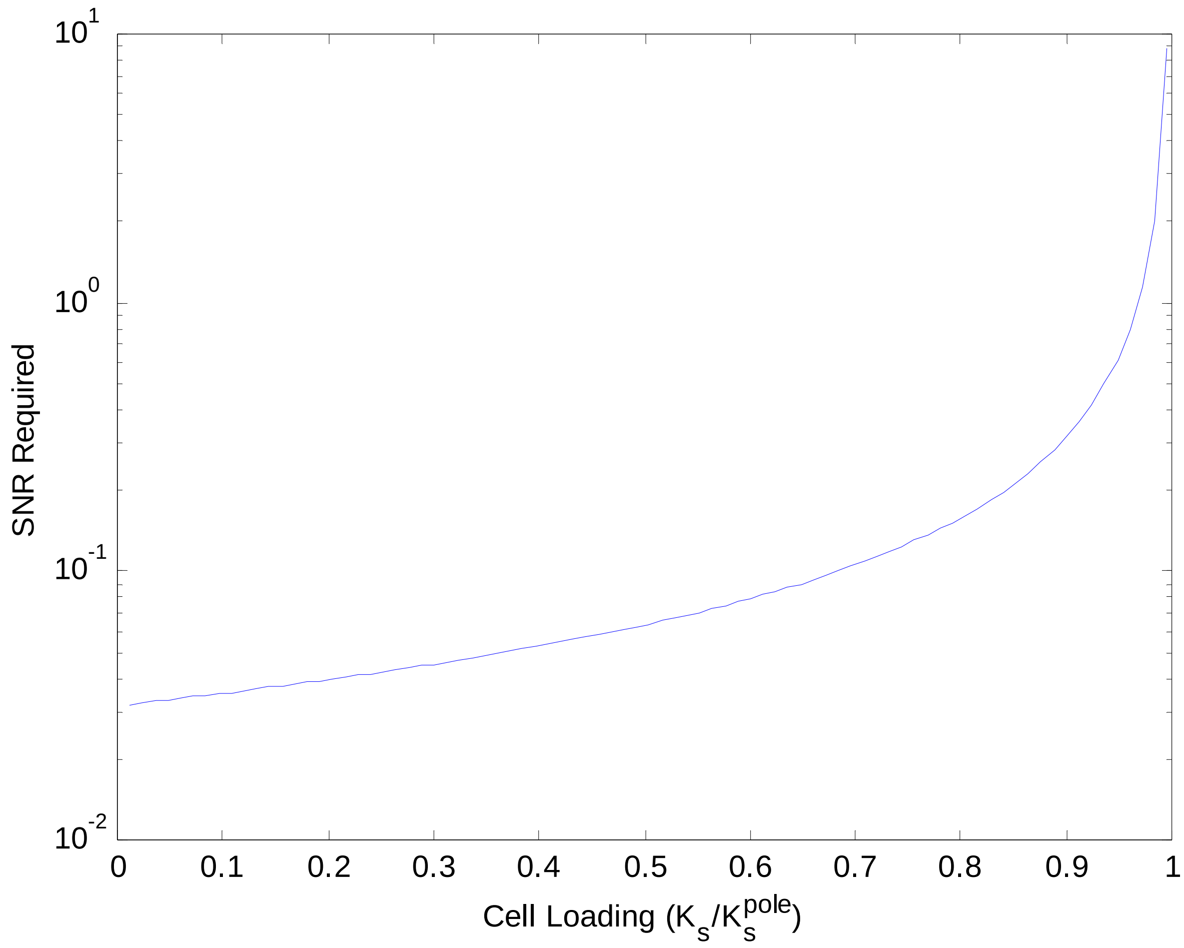
$$\frac{K_s}{K_s^{pole}} = \frac{1+\frac{1}{\nu}\left[\frac{B_T/R}{E_b/I_o} - \frac{N}{P}\right]}{1+\frac{1}{\nu}\left[\frac{B_T/R}{E_b/I_o}\right]}$$

$$\frac{N}{P} = \left(\nu + \frac{B_T/R}{E_b/I_o}\right)\left(1-\frac{K_s}{K_s^{pole}}\right)$$

$$\frac{P}{N} = \frac{1}{\left(\nu + \frac{B_T/R}{E_b/I_o}\right)\left(1-\frac{K_s}{K_s^{pole}}\right)}$$

- Now we can clearly see that the required SNR goes to infinity as we approach the pole capacity

Pole Capacity (cont.)



- E_b/I_o = 7dB
- ν = 3/8
- B_T =1.25MHz
- R = 8kbps
- K_s^{pole} =83

Pole Capacity Impact

- What this tells us is that the required SNR is a non-linear function of the cell loading.
- As the cell loading increases, the required received power goes up very quickly.
- This requirement will have an impact on the range (i.e., size) of the cell
- If the cell loading grows too large, mobiles on the cell boundaries may be unable to generate sufficient power to account for the increased load.
- Thus, the effective cell size can change with load
  - This leads to a phenomenon termed “cell breathing”

Coverage/Capacity Trade-off

- Recall that

$$K = 1 + \frac{B_T / R_b}{E_b / I_o} - \frac{\widetilde{N}}{P}$$

- A simple link budget shows that

$$P_r = \frac{P_t G_t G_r}{L_p}$$

  - where P_t is the transmit power, G_t and G_r are the transmit and receive antenna gains and L_p is the path loss related to the distance *d* and the wavelength λ.

$$L_p = \left(\frac{4\pi d}{\lambda} \right)^2$$

- Thus,

$$K = 1 + \frac{B_T / R_b}{E_b / I_o} - \frac{\widetilde{N}}{P_t G_t G_r} \left(\frac{4\pi d}{\lambda} \right)^2$$

Coverage/Capacity Trade-off (cont.)

$$K = 1 + \frac{B_T / R_b}{E_b / I_o} - \frac{\widetilde{N}}{P_t G_t G_r}\left(\frac{4\pi d}{\lambda}\right)^2$$

- Since the transmit power at the mobile is limited, the number of users is inversely related to the maximum distance
- The maximum distance is typically termed the coverage of the cell.
- Thus, there is a trade-off between capacity and coverage