ON THE CONVERGENCE OF MULTISTAGE INTERFERENCE CANCELLATION

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ABSTRACT

In this paper we examine the convergence of multistage interference cancellation for CDMA systems. Specifically, we examine linear and non-linear multistage versions of parallel and successive interference cancellation.

1. INTRODUCTION

Due to the emergence of CDMA as a competitive access scheme for cellular mobile telephony, multiuser detection has received significant attention in the past ten years. While a majority of the research has focused on techniques applicable to systems with short, repeating spreading codes, cellular standards have stuck to systems with long pseudo-random spreading codes. Thus, in this paper we investigate techniques which are applicable to systems with long codes, namely explicit interference cancellation. Interference cancellation comes in two distinct flavors, successive interference cancellation (SIC) [1] and parallel interference cancellation (PIC) [2]. Both can have multiple stages (or iterations). In this paper we examine the convergence of multi-stage cancellation for SIC and PIC when the cancellation is either linear or non-linear.

2. LINEAR INTERFERENCE CANCELLATION

Consider a CDMA system uplink where the received signal at the base station is

$$r(t) = \sum_{k=1}^{K} \sqrt{P_k}\gamma_k(t-\tau_k)b_k(t-\tau_k)s_k(t-\tau_k)+n(t) \quad (1)$$

where P_k, $\gamma_k(t)$, $b_k(t)$, $s_k(t)$ and τ_k are the received power, multiplicative distortion, symbol stream, pseudo-random spreading code and relative delay of the kth mobile respectively and $n(t)$ is AWGN. Further, the symbol (chip) rate of $s_k(t)$ is N times the symbol (information) rate of $b_k(t)$. Interference cancellation systems make decisions based on the decision statistic

$$z_{k,i} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} r_k(t)s_k^*(t-\tau_k)dt \quad (2)$$

where

$$r_k(t) = r(t) - \sum_{j\neq k} \hat{b}_j(t-\tau_j)\hat{\gamma}_j(t-\tau_j)s_j(t-\tau_j) \quad (3)$$

If we define $\mathbf{y}$ as a vector[1] of matched filter outputs before cancellation, then

$$\mathbf{y} = \Gamma R\mathbf{b} + \mathbf{n} \quad (4)$$

where R is the matrix of cross-correlations between the spreading codes, Γ is a diagonal matrix of the channels seen by each signal integrated over a symbol period and $\mathbf{n}$ is a vector of filtered AWGN. In general these vectors and matrices are defined over some block of N_b bits due to asynchronism. However, for simplicity let us assume a synchronous system allowing the vectors and matrices to be defined over a single symbol interval. For a system with long pseudo-random spreading codes, R will change each bit interval.

In a linear multistage interference cancellation system, the decision statistics at each stage are a linear function of the matched filter outputs

$$\mathbf{z}^{(i+1)} = A\mathbf{y} + B\mathbf{z}^{(i)} \quad (5)$$

where if $\mathbf{z}^{(0)} = \mathbf{y}$ then $\mathbf{z}^{(i+1)} = T^{(i+1)}\mathbf{y}$ for some matrix transform $T^{(i+1)}$. Linear interference cancellation can be viewed as an iterative method for solving linear systems of equations [3]. In the above formulation linear

[1] Boldface notation refers to vectors and capital letters refer to matrices.

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parallel interference cancellation (LPIC) is simply Jacobi iterations [4] with $A = I$ and $B = I - R$. Similarly, multistage successive interference cancellation (LSIC) is simply Gauss-Seidel iterations with $A = (R - U)^{-1}$ and $B = -(R - U)^{-1}U$ and U is the upper triangular portion of R. The convergence of Jacobi and Gauss-Seidel iterations is well known [4]. It can be shown that if they converge, both methods will converge to [3]

$$\lim_{i \to \infty} \mathbf{z}^{(i)} = R^{-1}\mathbf{y} \quad (6)$$

Borrowing the understanding of these iterative methods we find that the convergence of LPIC and LSIC is dependent on the spectral radius of B. Specifically, if the spectral radius (or the absolute value of the maximum eigenvalue) of B is greater than one, the technique will not converge. For random code CDMA this can be a very likely event thus giving to the poor performance of linear PIC in even moderately loaded systems. However, since the cross-correlation matrix is symmetric and positive definite, the LSIC method will always converge.

Figures 1 and 2 show simulation results for LPIC and LSIC respectively in AWGN. The E_b/N_o=8dB, $N = 64$ and the system was synchronous (*i.e.*,sampled once per chip) with random phases and random spreading codes. The performance of each IC method is shown versus the number of cancellation stages for system loadings of K=5, 15, 25 and 55 users. Also shown in each plot is the performance of the decorrelating detector for the same loading factors. It can be clearly seen that LPIC converges to the decorrelator for low loading (5 users) but diverges for moderate to high loading. LSIC however converges to the decorrelator in all cases, although the rate of convergence slows as the loading increases. It should be noted that LSIC performs better than the decorrelator prior to convergence, which is not surprising since the decorrelator does not provide the minimum BER for linear multiuser detection schemes. This is discussed more in [3].

One method of improving PIC is partial cancellation[5]. Again we can relate LPIC with partial cancellation to iterative methods by applying the relaxation method to Jacobi iterations. In this case $B = I - \mu R$ where μ is the partial cancellation factor. Convergence is dependent on the eigenvalues of $I - \mu R$. By choosing μ properly we can obtain convergence for heavily loaded systems. Since we wish the spectral radius of B to be less than one, choosing $\mu < \frac{2}{\lambda_{max}}$ where λ_{max} is the maximum eigenvalue of R will guarantee convergence.

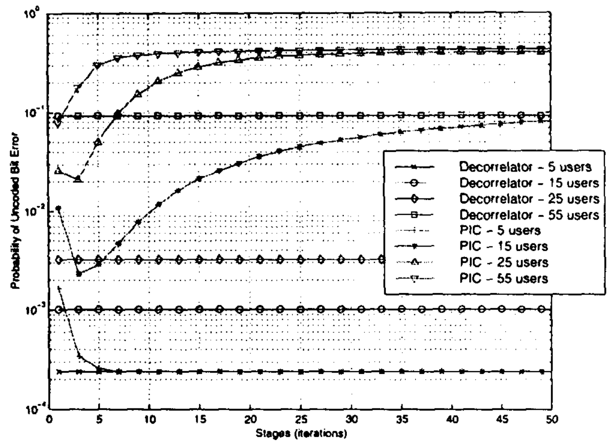

Figure 1: Convergence of Linear Parallel Interference Cancellation (E_b/N_o =8dB, synchronous, AWGN)

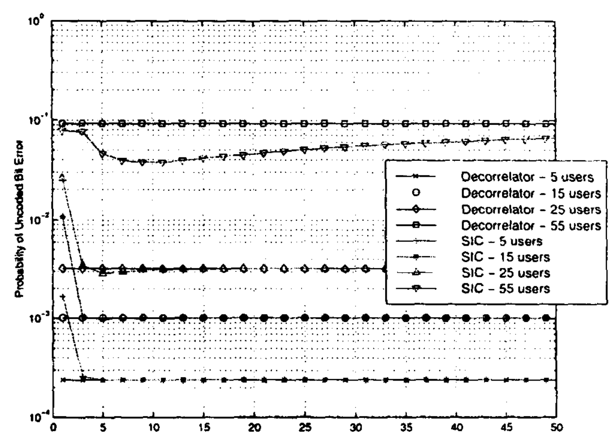

Figure 2: Convergence of Linear Successive Interference Cancellation (E_b/N_o =8dB, synchronous, AWGN)

Figure 3 shows the simulated performance of partial LPIC under the same conditions as Figures 1 and 2 with a partial cancellation factor of $\mu = 0.7$. We now find that LPIC will converge for all loading factors below approximately 50. Note, however, that the convergence is considerably slower than LSIC. If μ were chosen to be smaller, the system would converge at higher loading factors, but convergence overall would be slower.

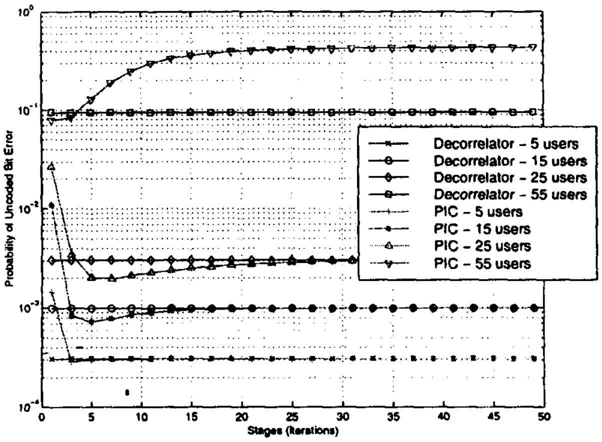

Figure 3: Convergence of Linear Partial Parallel Interference Cancellation (E_b/N_o =8dB, synchronous, AWGN,$\mu = 0.7$)

3. NON-LINEAR INTERFERENCE CANCELLATION

While the convergence of linear interference cancellation algorithms is well understood, the ultimate performance (i.e., that of the decorrelator) is not particularly good in moderate to high loading scenarios. Thus, we are interested in non-linear iterative cancellation methods. In non-linear interference cancellation we can define the decision statistic iteration as

$$\mathbf{z}^{(i+1)} = A\mathbf{y} + B\hat{\Gamma} f\left(\mathbf{z}^{(i)}\right) \quad (7)$$

where $\hat{\Gamma}$ is an estimate of the channel and $f\left(\mathbf{z}^{(i)}\right)$ is some non-linear function of the previous stage decision statistics. We consider three non-linear functions:

$$f_1(\mathbf{x}) = \operatorname{sgn}\left\{\Re\left(\hat{\Gamma}^*\mathbf{x}\right)\right\} \quad (8)$$

$$f_2(\mathbf{x}) = \mu_i \cdot \operatorname{sgn}\left\{\Re\left(\hat{\Gamma}^*\mathbf{x}\right)\right\} \quad (9)$$

$$f_3(\mathbf{x}) = \tanh\left\{\Re\left(\frac{\hat{\Gamma}^*\mathbf{x}}{\sigma_x^2}\right)\right\} \quad (10)$$

where

$$\operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}, \quad (11)$$

$f_3\left(\mathbf{z}^{(i)}\right)$ is the MMSE estimate of $\mathbf{b}$ at stage i [6], σ_x^2 is the variance of the elements of $\mathbf{x}$ which are assumed to have equal variance and $\Re(\mathbf{x})$ is the real part of $\mathbf{x}$. We first consider parallel cancellation in which $A = I$ and $B = I - R$. Concerning convergence, if we assume perfect channel estimation we can rearrange (7) to obtain

$$\mathbf{z}^{(i+1)} = \Gamma\mathbf{b} + \mathbf{n} + (R - I)\Gamma\left(\mathbf{b} - f\left(\mathbf{z}^{(i)}\right)\right) \quad (12)$$

From this equation we can see that if $f\left(\mathbf{z}^{(i)}\right)$ approaches $\mathbf{b}$, the iteration will tend to converge. Nonlinear parallel cancellation was simulated under the same conditions as Figures 1-3. Figure 4 plots results for nonlinear parallel cancellation with $f_1(\mathbf{x})$ and perfect channel estimation while Figure 5 plots results for $f_3(\mathbf{x})$ and perfect channel estimation. Now we find that PIC converges in all loading cases considered. In fact it was found that convergence was even achieved for $K > N$.

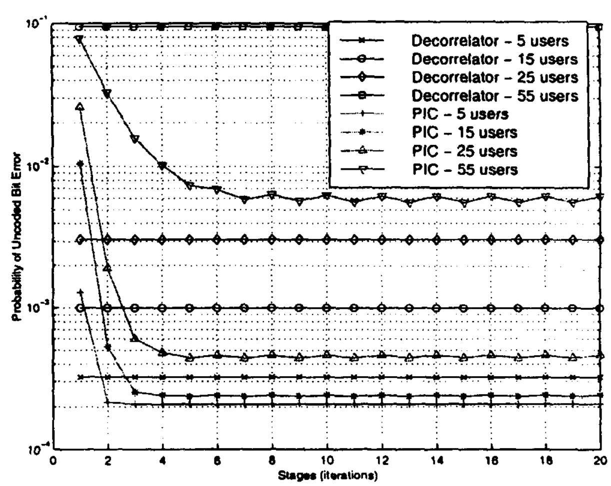

Figure 4: Convergence of Non-Linear Parallel Interference Cancellation with Sign Decision Function and Perfect Channel Estimation (E_b/N_o =8dB, synchronous, AWGN)

If we remove the assumption of perfect channel estimation, we see that non-linear PIC can become unstable. Figure 6 plots the performance of NLPIC with $f_1(\mathbf{x})$ and a 10 symbol average used for channel estimation. The plots indicate that at low loading factors, the iteration will converge, but at high loading factors, the algorithm does not converge. The same was found for $f_3(\mathbf{x})$. When it converges, the sigmoid decision function provides a lower probability of error, however, it is not guaranteed to converge. Partial cancellation has been shown to provide more stable performance for both linear and non-linear cancellation methods [3]. Thus, we consider partial cancellation using $f_2(\mathbf{x})$ with

$$\mu = [0.1, 0.2, 0.3, \ldots, 0.8, 0.9, 0.95, 0.99, 1, 1, 1 \ldots]. \quad (13)$$

Figure 7 plots the performance of the scheme versus cancellation stage in AWGN with a 10 symbol channel estimate. Partial cancellation allows the iteration to converge and provides even better performance than the other non-linear functions. Convergence is however slower than full cancellation.

Finally, we examine non-linear successive interference cancellation. In the case of perfect channel estimation we can rearrange (7) to obtain

$$\mathbf{z}^{(i+1)} = N^{-1}R\Gamma\left(\mathbf{b} - f\left(\mathbf{z}^{(i)}\right)\right) + N^{-1}\mathbf{n}\Gamma f\left(\mathbf{z}^{(i)}\right) \quad (14)$$

where $N = R - U$ and again we see that in the absence of noise, once $f\left(\mathbf{z}^{(i)}\right) = \mathbf{b}$ the system will stay at the correct solution. Figures 8 and 9 present simulation results for successive interference cancellation for $f_1(\mathbf{x})$ with perfect and imperfect channel estimation respectively. Unlike PIC this method converges in all cases examined and provides significantly faster convergence than LSIC and all of the PIC approaches. Channel estimation does not have the same impact on SIC as it has on PIC causing only a slight degradation in performance.

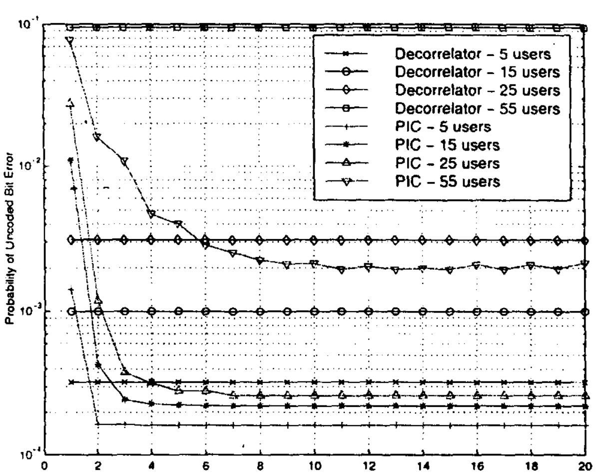

Figure 5: Convergence of Non-Linear Parallel Interference Cancellation with Sigmoid Decision Function and Perfect Channel Estimation (E_b/N_o =8dB, synchronous, AWGN)

4. CONCLUSIONS

In this paper we have examined the convergence of linear and non-linear multistage interference cancellation approaches. Not surprisingly, non-linear interference cancellation was found to converge more rapidly and to lower final bit-error rates. SIC methods were shown to converge in all cases examined for both linear and non-linear. PIC methods were less stable. Convergence was dependent on loading and, for the case of non-linear cancellation, on channel estimation. Partial cancellation, however, was shown to provide stability (*i.e.*,convergence) to PIC although the rate of convergence was slowed.

5. ACKNOWLEDGMENTS

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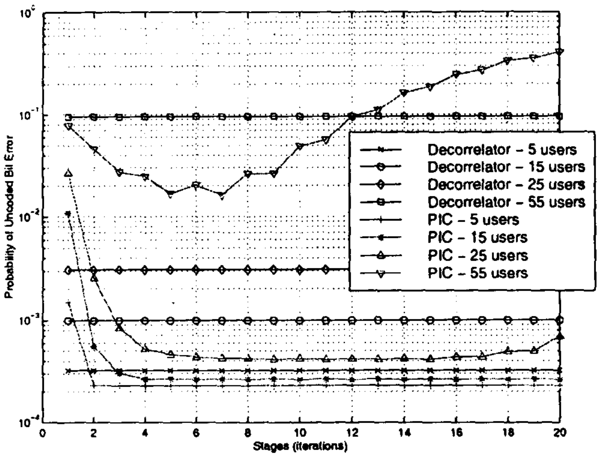

Figure 6: Convergence of Non-Linear Parallel Interference Cancellation with Sign Decision Function and Imperfect Channel Estimation (E_b/N_o =8dB, synchronous, AWGN, 10 symbol avg.)

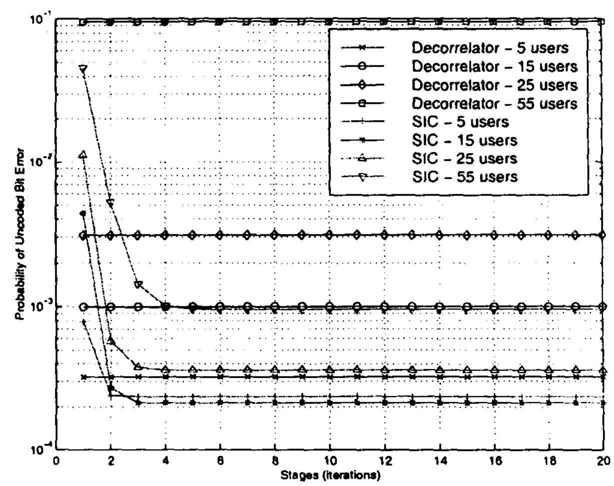

Figure 8: Convergence of Non-Linear Serial Interference Cancellation with Perfect Channel Estimation (E_b/N_o =8dB, synchronous, AWGN)

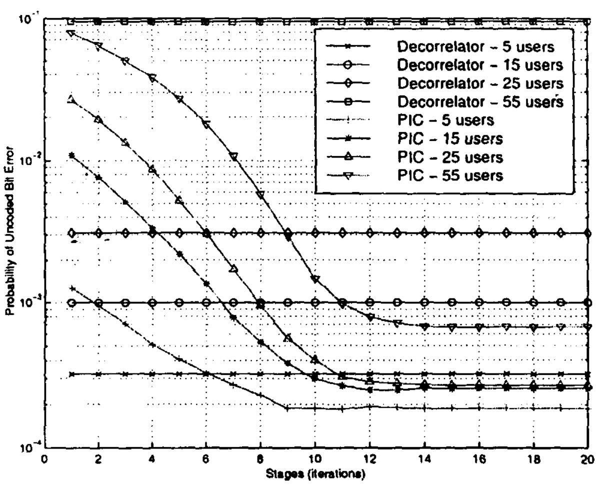

Figure 7: Convergence of Non-Linear Parallel Interference Cancellation with Partial Cancellation and Imperfect Channel Estimation (E_b/N_o =8dB, synchronous, AWGN, 10 symbol avg.)

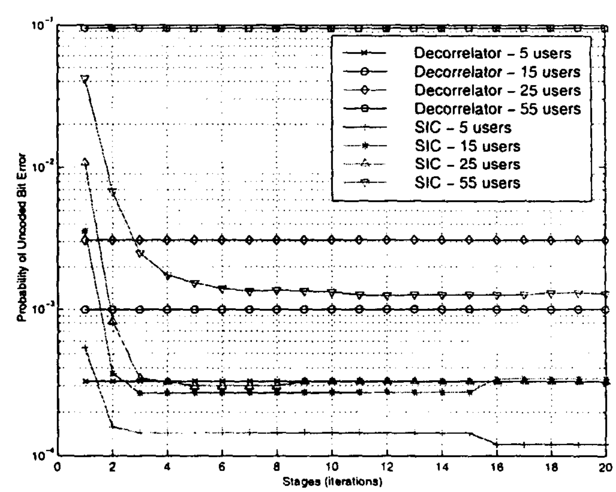

Figure 9: Convergence of Non-Linear Serial Interference Cancellation with Imperfect Channel Estimation (E_b/N_o =8dB, synchronous, AWGN, 10 symbol avg.)