Space-Time Block Codes for Eight Transmit Antennas

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Abstract

In this paper we present full-rate complex non-orthogonal codes for eight transmit antennas and compare their performance to orthogonal designs that provide the same spectral efficiency. The proposed codes have a structure that allows simplified maximum likelihood decoding at the receiver. The decoding scheme is further simplified for rectangular constellations where the real and imaginary parts can be decoded separately. Finally it is shown that the performance of the non-orthogonal codes can be improved significantly by employing a constellation rotation scheme [7] and this improvement can be attributed to the change in the rank distribution of the code.

1. Introduction

Space-time codes have been recently proposed as a diversity mechanism for wireless fading environments. In [1] Alamouti proposed a simple transmit diversity technique that achieved a diversity order of $2M$ with 2 transmit and M receive antennas. The Alamouti scheme was recognized as a block-coding scheme and this generated interest in developing more powerful codes that can provide higher diversity orders. In [2] space-time block codes (STBC) were derived from the theory of orthogonal designs and several codes of different rates and sizes were presented.

Transmit diversity has emerged as a popular space-time coding approach. This is due to the fact that it is much more economical to place multiple antennas at the base station than in all the mobile devices. However it has been mathematically proven that a full-rate, complex, orthogonal design does not exist for more than two transmit antennas [2]. Several non-orthogonal codes for four transmit antennas have been proposed in [3]-[7].

In [3] a criterion for selecting non-orthogonal space-time block codes is developed. Based upon this criterion a 4 x 4 STBC is selected and it is shown that this code can achieve reasonable performance using an LMMSE decoding approach. In [4] a maximum likelihood (ML) approach for decoding a non-orthogonal code is presented that works on pairs of symbols and it is shown that the performance of the non-orthogonal code is comparable to an orthogonal code that provides the same spectral efficiency. In [5] the ML, MMSE and ZF decoding approaches are compared in terms of the open-loop capacity. It is shown that the non-orthogonal code can achieve more than 95% of the open loop capacity using the ML and MMSE decoding schemes. In [6] the non-orthogonal code is converted to an orthogonal code by exploiting channel state information at the transmitter. Since the resulting code is orthogonal it achieves full diversity order. In [7] a novel concept of constellation rotation is introduced that improves the distance properties of the code and consequently its bit error performance.

In this paper we propose full-rate non-orthogonal codes for eight transmit antennas and a decoding scheme that is much more efficient than the traditional maximum likelihood approach. The performance of these codes is compared to a non-orthogonal code (also called quasi-orthogonal code [4]) for four transmit antennas and a half-rate orthogonal code for eight transmit antennas. It is shown that a simple constellation rotation scheme can substantially improve the performance of the proposed quasi-orthogonal codes and this can be attributed to the change in the rank distribution of the code. Unless otherwise mentioned all codes are assumed to be full-rate and complex.

2. Signal Model

Let us assume that we have N transmit antennas and a single receive antenna. We do not consider the case of multiple receive antennas as that would just increase the diversity order of the system M fold where M is the number of receive antennas. The input to the space-time encoder is a block of symbols $s = (s_1, s_2, \ldots s_L)$ of length L that is transmitted over T time slots resulting in a code rate of L/T. Furthermore each symbol is transmitted from one antenna during each time slot requiring that L be equal to N. The received signal can then be written in the linear model form as

$$r = \sqrt{\frac{\rho}{N}} Hs + w \quad (1)$$

where s is an N x 1 input symbol vector, r is a T x 1 received signal vector, H is a T x N channel matrix, and w is a T x 1 noise vector. The entries of rows of H and w are independent circularly symmetric complex Gaussian random variables with zero mean and unit variance and ρ is the SNR at the receive antenna. The energy per symbol is normalized to one. It is assumed that the channel is frequency non-selective and static during transmission of a single block i.e. over T time slots. The power transmitted from each antenna is scaled by the factor N therefore the total transmit power remains constant. It must be noted that in this form (1) the coding information is embedded in the channel matrix H.

3. Full-Rate Non-Orthogonal Designs

It has been mathematically proven that a full-rate, complex, orthogonal STBC does not exist for more than two transmit antennas [2]. This means that for more than two transmit antennas either the rate or orthogonality of the code has to be sacrificed. Since a full-rate code provides higher spectral efficiency a non-orthogonal code would be preferred over an orthogonal code. However the loss in orthogonality results in a lower diversity order. The diversity order of an STBC is determined by the minimum rank of the matrix $A(c,e)$ over all possible pairs of distinct code words c and e [8]. Here, $A(c,e)$ is the product of the code difference matrix with its Hermitian. The minimum of rth roots of the product of eigen values of $A(c,e)$ over all possible pairs of distinct code words c and e is a measure of the coding gain. We do not try to perform a numerical search to find the code that is best in terms of the above criteria, rather, our approach is to find codes that can be efficiently decoded and amongst those codes find the ones that achieve the best BER performance. We propose the following complex design for 8 transmit antennas.

$$H_8 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 \\ -h_2^* & h_1^* & h_4^* & -h_3^* & h_6^* & -h_5^* & -h_8^* & h_7^* \\ -h_3 & -h_4 & h_1 & h_2 & h_7 & h_8 & -h_5 & -h_6 \\ -h_4^* & h_3^* & -h_2^* & h_1^* & h_8^* & -h_7^* & h_6^* & -h_5^* \\ -h_5 & -h_6 & -h_7 & -h_8 & h_1 & h_2 & h_3 & h_4 \\ -h_6^* & h_5^* & -h_8^* & h_7^* & -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_7 & h_8 & h_5 & -h_6 & -h_3 & h_4 & h_1 & -h_2 \\ -h_8^* & -h_7^* & h_6^* & h_5^* & -h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix}$$
(2)

The above code will be referred to as Quasi-1. It is constructed from the real 8 x 8 design given in [2] and achieves a diversity order of 4. This code has a special structure that allows groups of symbols to be decoded separately. Several other codes that were formed by combining the basic Alamouti block were also found to have similar properties. However these codes have a minimum rank of 2. One possible combination results in the following code.

$$H_8 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 \\ -h_2^* & h_1^* & h_4^* & -h_3^* & -h_6^* & h_5^* & h_8^* & -h_7^* \\ -h_3 & -h_4 & h_1 & h_2 & -h_7 & -h_8 & h_5 & h_6 \\ -h_4^* & h_3^* & -h_2^* & h_1^* & -h_8^* & h_7^* & -h_6^* & h_5^* \\ h_5 & h_6 & h_7 & h_8 & h_1 & h_2 & h_3 & h_4 \\ -h_6^* & h_5^* & h_8^* & -h_7^* & -h_2^* & h_1^* & h_4^* & -h_3^* \\ -h_7 & -h_8 & h_5 & h_6 & -h_3 & -h_4 & h_1 & h_2 \\ -h_8^* & h_7^* & -h_6^* & h_5^* & -h_4^* & h_3^* & -h_2^* & h_1^* \end{bmatrix}$$
(3)

This code will be referred to as Quasi-2. Let's denote the basic 2 x 2 blocks by H_2. These 2 x 2 blocks can be combined to form a 4 x 4 block code

$$H_4 = \begin{bmatrix} H1_2 & H2_2 \\ -H2_2 & H1_2 \end{bmatrix} \quad (4)$$

and these can be further combined to form the above 8 x 8 block code.

$$H_8 = \begin{bmatrix} H1_4 & H2_4 \\ H2_4 & H1_4 \end{bmatrix} \quad (5)$$

These blocks can be combined in several different ways to achieve different 8 x 8 block codes however all of these codes were found to have similar BER performances.

4. Maximum Likelihood Decoding

The maximum likelihood decision rule for the model given in (1) is

$$\min_{s \in \mathcal{A}^8} \left\| \sqrt{\frac{\rho}{N}} Hs - r \right\|^2 \quad (6)$$

where $\mathcal{A}$ is the alphabet. The number of different sequences for a length L block and m constellation points is m^L. For L=8 this turns out to be 65536 for a QPSK constellation and 4.3×10^9 for a 16-QAM constellation. Clearly this is impractical. We propose that matched filtering precede maximum likelihood decoding. Therefore the modified decision rule is

$$\min_{s \in \mathcal{A}^8} \left\| \sqrt{\frac{\rho}{N}} H^H H s - H^H r \right\|^2 \tag{7}$$

or

$$\min_{s \in \mathcal{A}^8} \left\| \sqrt{\frac{\rho}{N}} D s - \tilde{s} \right\|^2 \tag{8}$$

where $\tilde{s}$ represents the matched filter outputs and $D = H^H H$ is an 8 x 8 sparse matrix. The checkerboard plot of D for Quasi-1 is shown in Figure 1.

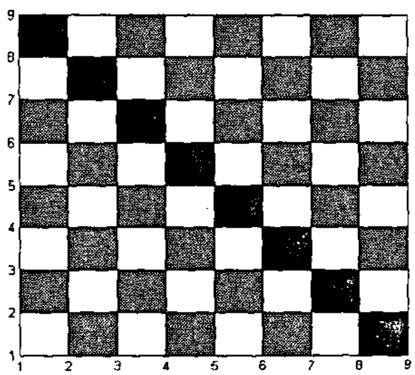

Fig. 1. Checkerboard plot of matrix D. Non-zero off diagonal terms represent interference.

It is seen that there are only 3 non-zero, off diagonal, terms in each row and these terms appear at particular positions in matrix D. This allows the above minimization criterion to be simplified into two separate and independent minimization criteria. It must be noted for an orthogonal code matrix D is diagonal and each symbol can therefore be detected independently.

$$\min_{s_o \in \mathcal{A}^4} \left\| \sqrt{\frac{\rho}{N}} D_o s_o - \tilde{s}_o \right\|^2 \tag{9}$$

$$\min_{s_e \in \mathcal{A}^4} \left\| \sqrt{\frac{\rho}{N}} D_e s_e - \tilde{s}_e \right\|^2 \tag{10}$$

Here s_o and s_e are the 4 x 1 vectors of odd and even symbols, $\tilde{s}_o$ and $\tilde{s}_e$ are 4 x 1 vectors of odd and even matched filter outputs and D_o and D_e are 4 x 4 matrices formed by the non-zero entries of odd and even rows of D respectively. This allows the set of symbols $\{s_1, s_3, s_5, s_7\}$ and $\{s_2, s_4, s_6, s_8\}$ to be decoded separately. The number of possible sequences for each set of equations is $m^{L/2} = 256$ (QPSK) giving a total of 512. Although this is still computationally expensive, the number of computations is reduced by a factor of 128. The combinations per bit is a better measure of the computational complexity, which in this case is 512/16 = 32. The above decoding procedure is applicable to both Quasi-1 and Quasi-2.

The diagonal terms of matrix D are obviously real and the non-diagonal terms are purely imaginary for Quasi-1 and both real and imaginary for Quasi-2. It was found that a code similar to Quasi-2 could be formed by combining the basic Alamouti blocks such that the non-diagonal terms are all real (11). This further reduces the computational complexity as it allows the real and imaginary parts of the transmitted symbols to be decoded separately. This code will be referred to as Quasi-3.

$$H_8 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 \\ -h_2^* & h_1^* & -h_4^* & h_3^* & -h_6^* & h_5^* & -h_8^* & h_7^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* & -h_7^* & -h_8^* & h_5^* & h_6^* \\ h_4 & -h_3 & -h_2 & h_1 & h_8 & -h_7 & -h_6 & h_5 \\ -h_5^* & -h_6^* & -h_7^* & -h_8^* & h_1^* & h_2^* & h_3^* & h_4^* \\ h_6 & -h_5 & h_8 & -h_7 & -h_2 & h_1 & -h_4 & h_3 \\ h_7 & h_8 & -h_5 & -h_6 & -h_3 & -h_4 & h_1 & h_2 \\ -h_8^* & h_7^* & h_6^* & -h_5^* & h_4^* & -h_3^* & -h_2^* & h_1^* \end{bmatrix} \tag{11}$$

The set of symbols that can be decoded separately are $\{s_1, s_4, s_6, s_7\}$ and $\{s_2, s_3, s_5, s_8\}$. The number of possible sequences for each set of equations is reduced to $2(\log_2 m)^{L/2} = 32$ giving a total of 64 and the combinations per bit is reduced to 4 (QPSK). The performance of this code was found to be the same as that of Quasi-2 code therefore it will not be considered in the following sections. It must be noted that this scheme can only work if we have a rectangular constellation where the real and imaginary parts can be decoded separately as is the case with M-QAM. The quasi-orthogonal design given in (2) can also be used to construct a half rate orthogonal code as follows.

$$H_{16} = \begin{bmatrix} H_8 \\ H^*_8 \end{bmatrix} \tag{12}$$

The rate of this code is maximum for an orthogonal code for eight transmit antennas [3] as given by

$$R = \frac{\log_2 N + 1}{2^{\log_2 N}} \quad (13)$$

Since the rate of the code is halved a higher-level modulation has to be used to maintain the same spectral efficiency. The decoding scheme for this code is reduced to simple matched filtering.

5. Constellation Rotation

A novel constellation rotation scheme was presented in [7] that significantly improved the performance of quasi-orthogonal codes introduced in [4]. This motivated us to study the improvement in performance that can be achieved for quasi-orthogonal codes for eight transmit antennas. It was shown in [7] that the performance of a quasi-orthogonal code depends upon the distance properties of the inflated constellation formed by the set of jointly decoded symbols. It was also shown that the minimum distance between any two points in this inflated constellation can be increased by rotating one of the constellations with respect to the other. However a rotation scheme that is based on maximizing this minimum distance does not yield the best results for our codes.

It was found that constellation rotation increased the probability of occurrence of higher ranked code difference matrices and consequently these codes achieve a higher diversity order. The minimum rank is not as important as rank distribution because it only affects the slope of the BER curve at very high SNRs. The rank distribution of the 3 codes for different constellation rotations is shown in Table 1.

Quasi-1	8	6	4	2
Normal	98.7698	1.1470	0.0832	0.0000
Single	99.8905	0.0883	0.0212	0.0000
Uniform	100.0000	0.0000	0.0000	0.0000

Quasi-2	8	6	4	2
Normal	98.0037	1.7891	0.2034	0.0038
Single	99.8137	0.1525	0.0338	0.0000
Uniform	100.0000	0.0000	0.0000	0.0000

Quasi-3	8	6	4	2
Normal	98.0101	1.7703	0.2164	0.0032
Single	99.8105	0.1513	0.0382	0.0000
Uniform	100.0000	0.0000	0.0000	0.0000

Table 1. Rank distribution (%) for different constellation rotation schemes.

Rotating only one of the constellations ($\pi/6$) in each subset significantly changed the rank distribution of the code. For example for Quasi-2 the minimum rank of the code was changed from 2 to 4 and the probability of occurrence of rank 6 and rank 4 matrices was greatly reduced. Furthermore a uniform constellation rotation scheme $(0, \pi/8, 2\pi/8, 3\pi/8)$ for each of the subsets made the quasi-orthogonal codes full rank. However it was found that most of the gain was obtained with a single constellation rotation and there was no further advantage in using the later scheme. Therefore we propose that **for quasi-orthogonal codes those constellation rotation schemes must be sought that improve the rank distribution of the code without sacrificing the coding gain**. This is a very general criterion and can be applied to any quasi-orthogonal code.

6. Simulation Results

The BER performance of Quasi-1 and Quasi-2 codes is compared to that of quasi-orthogonal code for four transmit antennas [4]. It is seen that the Quasi-1 code provides a gain of about 2dB at a BER of 10^{-4} and 4 dB at a BER of 10^{-5} (Figure 2). However the Quasi-2 code provides a constant gain of about 4dB in the high SNR region as it achieves the same diversity order as the code for four transmit antennas (Figure 3). The BER performance of these codes is then compared to that of the complex orthogonal design for eight transmit antennas (Figure 4). Since the rate of this code is halved we used 16-QAM modulation to maintain a spectral efficiency of 2 bits/sec/Hz. It is seen that the Quasi-1 code provides a gain of less than 1 dB whereas Quasi-2 code provides an improvement of about 2 dB in the low to moderate SNR region. However the slope of the orthogonal design is

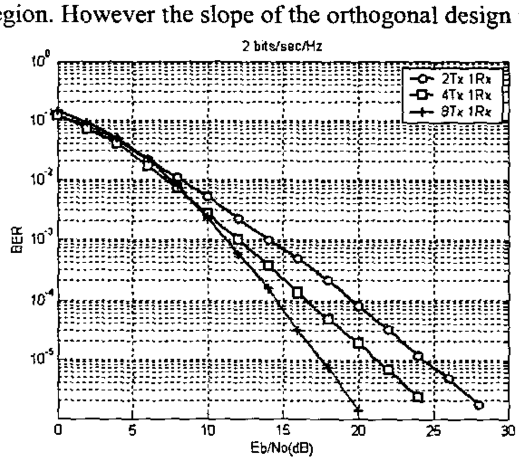

Fig. 2. Comparison of bit error probability of quasi-orthogonal codes (orthogonal for 2 Tx) for different number of transmit antennas. Quasi-1 code for 8 Tx.

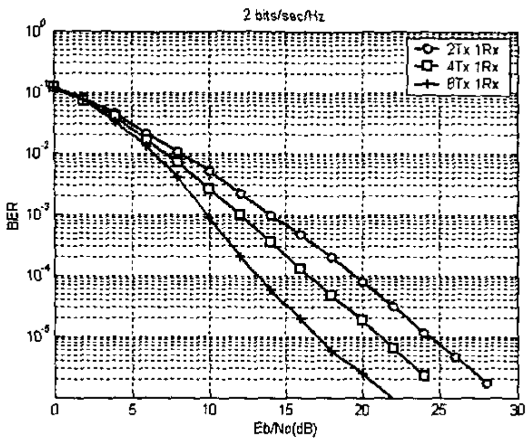

Fig. 3. Comparison of bit error probability of quasi-orthogonal codes (orthogonal for 2 Tx) for different number of transmit antennas. Quasi-2 code for 8 Tx.

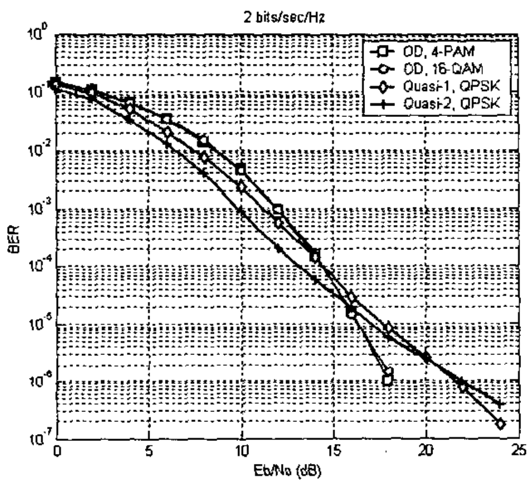

Fig. 4. Comparison of bit error probability of orthogonal and non-orthogonal codes for eight transmit antennas.

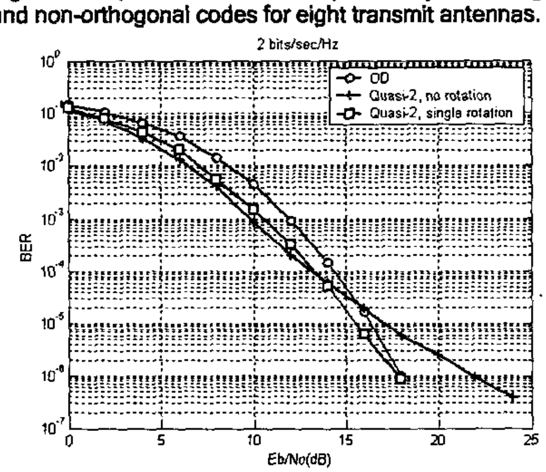

Fig. 5. Bit error probability of Quasi-2 with and without constellation rotation.

much steeper and it performs better at higher SNRs. The crossover point for Quasi-1 is at 14dB whereas the crossover point for Quasi-2 is at 16dB. Since the orthogonal code is half-rate the same performance can also be obtained by using 4-PAM with a full-rate real design. Finally the results of constellation rotation for Quasi-2 code are shown in Figure 5. It is seen that there is a slight degradation in performance at moderate SNRs however the code achieves a higher diversity order and the crossover point is shifted to around 18 dB.

7. Conclusions

Quasi-orthogonal codes out-perform the orthogonal code at low to moderate SNRs. However the orthogonal code performs better at high SNRs. This is because the slope of the BER curve is determined by the diversity order and the orthogonal code achieves full diversity order for eight transmit antennas. The diversity order of a quasi-orthogonal code can be improved by employing a constellation rotation scheme and most of the gains can be obtained by rotating just one of the constellations in each set of jointly decoded symbols. The choice of the modulation/code would depend upon a number of factors including the SNR region the system is operating and the computational complexity that can be afforded.

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