Spatial Fading Correlation Function of Circular Antenna Arrays With Laplacian Energy Distribution

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***Abstract*—In this letter, we derive an analytical formula for the fading correlation function of a uniform circular array (UCA) when the angular energy follows a Laplacian distribution. The fading correlation for a UCA is a function of antenna spacing, array geometry, as well as the distribution of the received energy in terms of the angle-of-arrival (AOA). Computer simulations are carried out to verify the analytical results.**

***Index Terms*—Angle spread, fading correlation, uniform circular array (UCA), uniform liner array (ULA).**

I. Introduction

ANTENNA arrays have been widely used in mobile radio communications to improve signal quality, thereby increasing system coverage, capacity, and link quality [1]. Among antenna array systems, the uniform linear array (ULA) is the most common form employed in cellular and PCS systems. The fading correlation function for the ULA is well known for uniform and Gaussian angle-of-arrival (AOA) distributions [2]–[4]. However, unlike the ULA, the performance of the uniform circular array (UCA) has not been extensively studied for mobile radio communications. Recently, there has been increased interest in using uniform circular arrays [5]. In order to evaluate the system performance of a UCA, a model for spatial correlation is needed. Thus, in this letter we derive the general spatial fading correlation for the UCA. This letter is organized as follows. In Section II, we present a description of the spatial vector channel model assumed for a circular array and derive an analytical formula for the fading correlation function for the UCA. In Section III, we present the analytical results of the spatial fading correlation function as a function of the antenna spacing and the mean AOA. Computer simulations are carried out to verify the analytical results. Concluding remarks are given in Section IV.

II. Spatial Vector Channel Model

Fig. 1 shows the array geometry assumed in this work. Note that although a four-element uniform circular antenna array is shown and assumed for specific results, the analytical formula derived is applicable to any number of elements. In order to derive the fading correlation function, it is necessary to develop the vector channel model for the uniform circular array. The vector channel model is based on the mathematical spatial channel model developed in [3], [2]. In [2] the spatial channel model assumes that the central AOA is determined by the physical position of a dominant reflector with respect to the received antenna array. An illustration of the wave propagation for the spatial channel model is shown in Fig. 1. Although the work in [3], [2] assumes a uniform AOA distribution at the received antenna array, measurements show that the AOA distribution in general has a shape which more closely resembles a Gaussian or Laplacian distribution [6]. In this letter we assume a Laplacian distribution which is given by

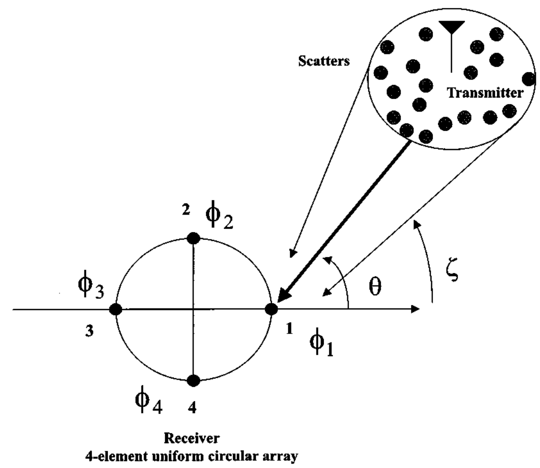

Fig. 1. Four-element uniform circular array with Laplacian AOA distribution in mobile radio communications.

$$f(\zeta) = C_l e^{-a|\theta-\zeta|}, \qquad -\pi+\theta \le \zeta \le \pi+\theta \tag{1}$$

where C_l is a normalizing constant chosen to make $f(\zeta)$ a density function and can be shown to be

$$C_l = \frac{a}{2(1-e^{-a\pi})} \tag{2}$$

and a is a decay factor which is related to the angle spread.[1] Specifically, as a increases, the angle spread decreases. Following the notation and definitions used in [7], the array manifold vector $\mathbf{v}(\zeta)$ for a uniform circular array can be written as

$$\mathbf{v}(\zeta) = \begin{bmatrix} v_1(\zeta) \\ v_2(\zeta) \\ \vdots \\ v_M(\zeta) \end{bmatrix} = \begin{bmatrix} e^{-j2\pi(R/\lambda)\sin(\varsigma)\cos(\zeta-\phi_1)} \\ e^{-j2\pi(R/\lambda)\sin(\varsigma)\cos(\zeta-\phi_2)} \\ \vdots \\ e^{-j2\pi(R/\lambda)\sin(\varsigma)\cos(\zeta-\phi_M)} \end{bmatrix} \tag{3}$$

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[1]We define angle spread as the standard deviation of the distribution.

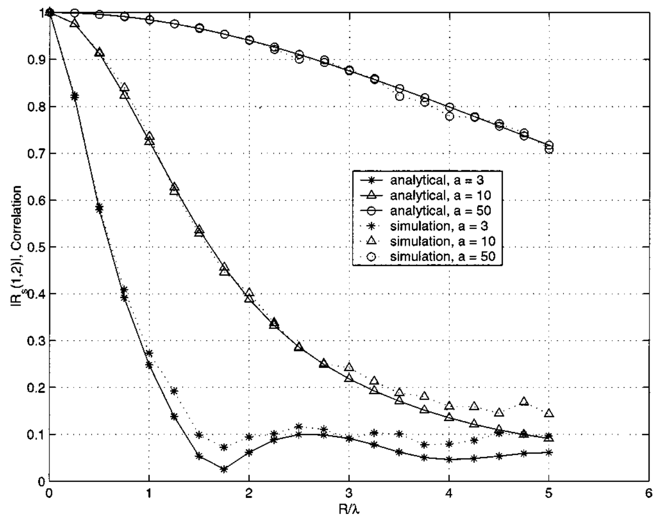

Fig. 2. The fading correlation between antennas 1 and 2.

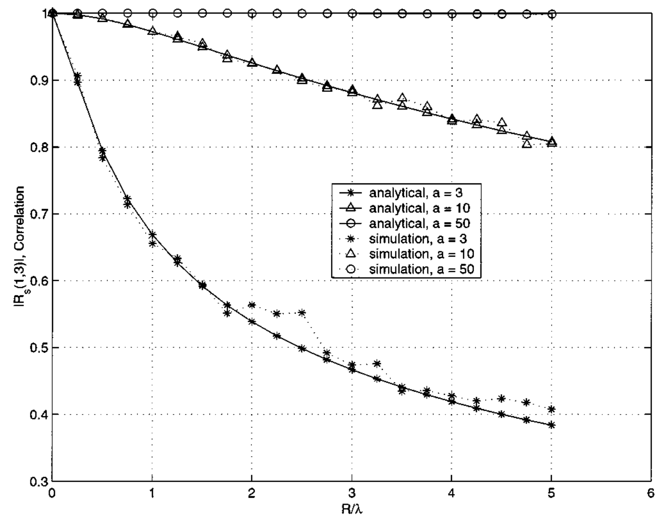

Fig. 3. The fading correlation between antennas 1 and 3.

where R is the circular radius of the antenna array, ς is the elevation angle, and λ is the wavelength of the center frequency of interest. For simplicity, only azimuth angles are considered in the propagation geometry ($\varsigma = 90°$), but the results can be generalized to three dimensions. ϕ_m is the angle of mth element in azimuthal plane as shown in Fig. 1. The array spatial correlation between the mth and nth antenna element is defined as

$$\mathbf{R}_s(m, n) = E\{v_m(\zeta)v_n(\zeta)^*\} = \int_\zeta v_m(\zeta)v_n(\zeta)^* f(\zeta)\, d\zeta \quad (4)$$

where the superscript $*$ denotes the complex conjugate. In the Appendix, it is shown that the real and imaginary part of $\mathbf{R}_s(m, n)$ for a Laplacian AOA distribution can be expressed as

$$\mathrm{Re}\{\mathbf{R}_s(m, n)\} = 2\left\{ J_0(Z_c) + 2\sum_{k=1}^{\infty} \frac{a^2(1-e^{-a\pi})}{a^2+4k^2} \cdot J_{2k}(Z_c)\cos[2k(\theta+\alpha)] \right\} \quad (5)$$

$$\mathrm{Im}\{\mathbf{R}_s(m, n)\} = 4C_l \sum_{k=0}^{\infty} \frac{a(1+e^{-a\pi})}{a^2+(2k+1)^2} J_{2k+1}(Z_c) \cdot \sin[(2k+1)(\theta+\alpha)] \quad (6)$$

where Z_c is related to the antenna spacing and α is the relative angle between the mth and nth antenna element, as defined in the Appendix. $J_n(x)$ is an nth-order Bessel function of the first kind.

III. Results

To demonstrate the usefulness of this formula, we consider a four-element UCA. Due to the symmetry of the UCA we only need to evaluate the fading correlations $|\mathbf{R}_s(1, 2)|$ and $|\mathbf{R}_s(1, 3)|$. We plot the fading correlations $|\mathbf{R}_s(1, 2)|$ and $|\mathbf{R}_s(1, 3)|$ as function of the array radius R and the angle spread parameter a. Figs. 2 and 3 present $|\mathbf{R}_s(1, 2)|$ and $|\mathbf{R}_s(1, 3)|$ when θ (central AOA) $= 0°$ versus R for various values of a based on (5) and (6). As expected, the fading correlation decreases as R increases. Further, as a increases (i.e., angle spread decreases) the fading correlation increases. It is also seen that $|\mathbf{R}_s(1, 2)|$ gives a lower correlation than $|\mathbf{R}_s(1, 3)|$ for the same parameters R and a which is expected. For the purpose of verification, computer simulations were carried out for the situations in Figs. 2 and 3. We use Monte Carlo simulations to evaluate the correlation based on (4). We generate 800 000 instances of the channel realization for a given data point. It is found that the simulation results exhibit good agreement with the analytical results.

IV. Conclusions

In this letter we derived an analytical formula for the fading correlation for the UCA as a function of antenna spacing, the central AOA, and an angle spread parameter based on a Laplacian distribution of angular energy. The results show that as expected, the fading correlation decreases as the array radius R increases and as the angle spread parameter a decreases (or as angle spread increases). Computer simulations were carried out to verify the analytical results.

Appendix

Using the definition of (4), the array spatial correlation between the mth and nth antenna element for uniform circular arrays can be expressed as

$$\mathbf{R}_s(m, n) = \int_{-\pi+\theta}^{\pi+\theta} F(\zeta) C_l e^{-a|\zeta-\theta|}\, d\zeta \quad (7)$$

where

$$F(\zeta) = e^{-j2\pi(R/\lambda)\{[\cos\phi_m - \cos\phi_n]\cos(\zeta) + [\sin\phi_m - \sin\phi_n]\sin(\zeta)\}}. \quad (8)$$

Let

$$K_1 = 2\pi \frac{R}{\lambda}[\cos(\phi_m) - \cos(\phi_n)] \quad (9)$$

$$K_2 = 2\pi \frac{R}{\lambda}[\sin(\phi_m) - \sin(\phi_n)]. \quad (10)$$

Using the definitions of (9) and (10), we can rewrite (7) as

$$\mathbf{R}_s(m,\, n) = C_l \int_{-\pi+\theta}^{\pi+\theta} e^{-j(K_1\cos(\zeta)+K_2\sin(\zeta))} e^{-a|\zeta-\theta|} d\zeta. \tag{11}$$

Let

$$\sin(\alpha) = \frac{K_1}{\sqrt{K_1^2+K_2^2}} \tag{12}$$

$$\cos(\alpha) = \frac{K_2}{\sqrt{K_1^2+K_2^2}} \tag{13}$$

$$Z_c = \sqrt{K_1^2+K_2^2}. \tag{14}$$

Furthermore using the definitions of (12)–(14), we can rewrite (11) as

$$\mathbf{R}_s(m,n) = C_l \int_{-\pi+\theta}^{\pi+\theta} e^{-jZ_c\{\sin(\alpha)\cos(\zeta)+\cos(\alpha)\sin(\zeta)\}} e^{-a|\zeta-\theta|} d\zeta. \tag{15}$$

The term $e^{-jZ_c\{\sin(\alpha)\cos(\zeta)+\cos(\alpha)\sin(\zeta)\}}$ can be written as

$$e^{-jZ_c\{\sin(\alpha)\cos(\zeta)+\cos(\alpha)\sin(\zeta)\}} = e^{-jZ_c\{\sin(\alpha+\zeta)\}}. \tag{16}$$

Thus, we can rewrite (15) as

$$\mathbf{R}_s(m,\, n) = C_l \int_{-\pi+\theta}^{\pi+\theta} e^{-jZ_c\{\sin(\alpha+\zeta)\}} e^{-a|\zeta-\theta|}\, d\zeta. \tag{17}$$

Letting $x = \zeta - \theta$, (17) can be expressed as

$$\mathbf{R}_s(m,\, n) = C_l \left\{ \int_0^{\pi} e^{-jZ_c\{\sin(\alpha+\theta+x)\}} e^{-ax}\, dx + \int_0^{\pi} e^{-jZ_c\{\sin(\alpha+\theta-x)\}} e^{-ax}\, dx \right\}. \tag{18}$$

The real and imaginary parts of (18) can be evaluated as follows:

$$\mathrm{Re}\{\mathbf{R}_s(m,\, n)\} = C_l \left\{ \int_0^{\pi} \cos[Z_c \sin(\alpha+\theta-x)] e^{-ax}\, dx + \int_0^{\pi} \cos[Z_c \sin(\alpha+\theta+x)] e^{-ax}\, dx \right\} \tag{19}$$

$$\mathrm{Im}\{\mathbf{R}_s(m,\, n)\} = C_l \left\{ \int_0^{\pi} \sin[Z_c \sin(\alpha+\theta-x)] e^{-ax}\, dx + \int_0^{\pi} \sin[Z_c \sin(\alpha+\theta-x)] e^{-ax}\, dx \right\}. \tag{20}$$

By making use of the well-known series, $\cos[Z_c\sin(\varphi)]$ and $\sin[Z_c\sin(\varphi)]$ can be further represented as, respectively

$$\cos[Z_c\sin(\varphi)] = J_0(Z_c) + 2\sum_{k=1}^{\infty} J_{2k}(Z_c)\cos(2k\varphi) \tag{21}$$

$$\sin[Z_c\sin(\varphi)] = 2\sum_{k=0}^{\infty} J_{2k+1}(Z_c)\sin((2k+1)\varphi). \tag{22}$$

By integrating (19) and (20) with the substitution of (21) and (22), respectively, we can obtain (5) and (6).

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