The Impact of Multiuser Diversity on Space-Time Block Coding

Ran Gozali, *Member, IEEE*, R. Michael Buehrer, *Member, IEEE*, and Brian D. Woerner, *Member, IEEE*

***Abstract*—In this letter, analytic performance results are derived for space-time block coding paired with multiuser diversity. We consider a scenario in which K active data users, each of which is potentially equipped with multiple antenna elements, are served by a multi-antenna element base station (BS). We focus on the downlink channel, where a space-time block coding scheme is employed and assume that channel quality information is reported to the BS by all users on a per frame basis. Using a scoring function at the BS, time resources are allocated to the user with the best instantaneous effective signal-to-noise ratio (SNR), facilitating a multiuser diversity mechanism. Using order statistics, we compute histograms and cumulative distribution functions of the effective SNR at the space-time combiner output and assess the interaction between multiuser diversity obtained via scheduling and spatial diversity obtained via the space-time code.**

***Index Terms*—Multi-antenna systems, multiuser diversity, scheduling, space-time codes, spatial diversity.**

I. Introduction

SPACE-TIME block coding (STBC) schemes, proposed originally by Alamouti in [1] and Tarokh *et al.* in [2], were recently adopted by the 3G standardization committees for implementation as one of the transmit diversity modes in 3G wireless networks. These schemes introduce a simple, elegant spatial diversity mechanism that improves the energy efficiency of wireless systems. A vast body of research has been published on the performance and structure of these schemes, focusing on the physical (PHY) layer aspects primarily. In this letter, we address the impact of scheduling algorithms, which may be executed in the medium access-control (MAC) layer of a wireless network, on the performance of STBC.

One potential application of space-time architectures may be high-data-rate downstream wireless Internet access to nomadic users [3]. The nature of such data traffic is asymmetric, requiring a much higher downlink rate [from the base station (BS)] than uplink rate (from the user terminal). In addition to traffic asymmetry, many data services differ from voice services in their tolerance to delay. We consider a scenario where a single BS provides delay tolerant data services to K users, each of which is equipped with multiple antenna elements. Since the data rate that can be supported to each user is proportional to its received SNR, it is beneficial to use channel quality information (CQI) and the delay tolerance of the application to intelligently serve multiple users with disparate signal-to-noise ratios (SNRs) [3]. This improves the overall throughput of the system as compared to traditional "round robin" (RR) scheduling, which routes the transmission of packets equally across users in an orthogonal TDM-like manner [4]. We examine the application of a STBC architecture along with a scheduling protocol that schedules the transmission of packets to users based on the reported CQI. This scheduling protocol, denoted "greedy", routes each transmission to the user with the best instantaneous channel conditions. Previous results have shown that the "greedy" scheduler gives rise to multiuser diversity mechanism, which increases overall throughput [5]. In that work, multiple transmit antennas were used to induce fluctuations in static or Ricean channels through phase sweeping (so called "dumb antennas").

It is well known that by increasing the spatial diversity order of STBC, performance in terms of reliability and capacity is bounded by additive white Gaussian noise (AWGN) channel results [6], [7]. Natural questions are therefore: 1) How does multiuser diversity enhance performance further? and 2) Is multiuser diversity mechanism equivalent to spatial diversity? We answer these questions by deriving analytic expressions for the statistics of the effective SNR at the receiver combining stage with and without multiuser diversity. The analysis is based on order statistics [8] and classical spatial diversity results.

While the spatial diversity mechanism is designed to reduce the probability of deep fades at the output of the receiver combining stage, it also eliminates the peaks (constructive fades) of the Rayleigh fading channel. This phenomenon is shown to limit the performance gain that could have been achieved by the multiuser diversity mechanism [9]. It is important to note that the figure of merit addressed here is the effective SNR (and thus the aggregate throughput of the system) without referring to other quality-of-service (QoS) criteria such as delay and service guarantees that may be provided to users in the system.

II. System Model

The architecture of STBC is based on an orthogonal design at the transmitter, which is exploited at the receiver by a linear combining rule [2]. This combining rule decouples the received superposition of symbols to create separate decision statistics, and coherently combines multiple versions of the transmitted symbol from multiple antennas to allow for diversity improvement.

Manuscript received September 14, 2002. The associate editor coordinating the review of this letter and approving it for publication was Dr. S. Diggavi. This work was supported by the MPRG Industrial Affiliates and the Navy Collaborative Integrated Information Technology Initiative (NAVCIITI) under Grant N00014-00-1-0549.

R. Gozali is with the Communication System Directorate, Rafael Ltd., Haifa 31021, Israel (e-mail: gozali@rafael.co.il).

R. M. Buehrer and B. D. Woerner are with the Mobile and Portable Radio Research Group, Virginia Polytechnic Institute and State University, Blacksburg, VA 24060-0350 USA (e-mail:buehrer@vt.edu; woerner@vt.edu).

Digital Object Identifier 10.1109/LCOMM.2003.812182

The received signal at the kth terminal, receive antenna $j, j \in [1, 2, \ldots, n_R]$, at time t is modeled by

$$r_k^j(t) = \sum_{i=1}^{n_T} \alpha_{i,j}^{(k)} c_k^i(t) + \eta_k^j(t) \tag{1}$$

where $\eta_k^j(t)$ denotes the AWGN term and is modeled as independent samples of a zero mean complex Gaussian random variable with variance $n_T/2\mathrm{SNR}$ per dimension and $c_k^i(t)$ is the symbol transmitted to the kth user from transmit antenna i at time t with average energy normalized to one. The channel is modeled by an $n_T \times n_R$ matrix $\Omega_k = [\alpha_{i,j}^{(k)}]$, whose entry $\alpha_{i,j}^{(k)}$ represents the complex fade coefficient for the path from transmit antenna i to receive antenna j of the kth user. These fade coefficients are assumed to be independent zero mean complex Gaussian random variables with variance 0.5 per dimension. For simplification purposes, we assume perfect channel tracking at the terminal and perfect feedback of these estimates to the base station. We also assume that the K users experience the same average SNR and that each MIMO link exhibits quasi-static[1] frequency nonselective (flat) fading.

III. PERFORMANCE ANALYSIS

The effective SNR at the space-time combiner output of the kth user is given by [10], [11]

$$\gamma = \frac{SNR}{n_T} \sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \left| \alpha_{i,j}^{(k)} \right|^2. \tag{2}$$

This is a chi-square random variable with $2n_T n_R$ degrees of freedom. The probability density function of this random variable is given by [6]

$$f_\gamma(\gamma) = \frac{1}{(n_T n_R - 1)! \left(\frac{SNR}{n_T} \right)^{n_T n_R}} \gamma^{n_T n_R - 1} e^{-\gamma/SNR/n_T}. \tag{3}$$

Let us define new variables μ, n and C as $\mu = n_T/\mathrm{SNR}$, $n = n_T n_R - 1$ and $C = \mu^{n+1}/n!$. This allows us to write a simplified form of (3)

$$f_\gamma(\gamma) = C \gamma^n e^{-\mu\gamma}. \tag{4}$$

In the “greedy” scheduling algorithm, the base station selects (on a per frame basis) the user k with the best instantaneous channel realization. That is

$$\arg \max_{k \in \{1,2,\ldots,K\}} \sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \left| \alpha_{i,j}^{(k)} \right|^2. \tag{5}$$

This multiuser diversity mechanism creates a new random variable, denoted $\tilde{\gamma}$, which represents the effective SNR at the space-time combiner output with K-fold multiuser diversity. The probability density function of this random variable is computed using order statistics [8]

$$g_{\tilde{\gamma}}(y) = K f_\gamma(y) F_\gamma(y)^{K-1}, \tag{6}$$

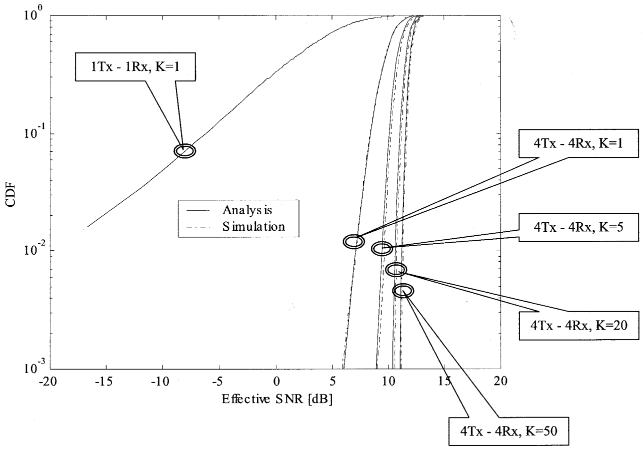

Fig. 1. The effect of multiuser diversity on the average effective SNR ; $\mathrm{SNR} = 4$ dB.

where K is the number of users and $F_\gamma(y)$ denotes the cumulative distribution function (CDF) of the original random variable γ. The CDF of γ is given by

$$F_\gamma(y) = \int_{\lambda=0}^{y} f_\gamma(\lambda) d\lambda = 1 - \mu^{n+1} e^{-\mu y} \sum_{m=0}^{n} \frac{y^m}{m! \mu^{n-m+1}}. \tag{7}$$

Plugging (7) and (4) into (6), we obtain a closed form expression for the pdf of the effective SNR at the space-time combiner output with K-fold multiuser diversity.

The mth moment of the effective SNR at the output of the space time combiner with K-fold multiuser diversity is computed by

$$E[y^m] = \int_{y=0}^{\infty} y^m g_{\tilde{\gamma}}(y) dy = K \int_{y=0}^{\infty} y^m f_\gamma(y) F(y)^{K-1} dy. \tag{8}$$

In the following section, we apply these results to assess on the impact of multiuser diversity when employed in conjunction with STBC.

IV. PERFORMANCE RESULTS

Results in the form of CDFs are plotted in Fig. 1 using Monte Carlo simulations and the analysis presented herein for STBC with 4Tx-4Rx, paired with 50-, 20-, 5-, and 1-fold multiuser diversity. As a reference, we include the CDF of a single-input single-output (SISO) link with no multiuser diversity. For a given CDF level of 1%, the effective SNR is enhanced by 2.3, 3.3, and 4 dB for $K = 5, 20,$ and 50 users, respectively. An excellent match is demonstrated between the closed form expression and the Monte Carlo simulation results.

This SNR enhancement yields an improved performance in terms of frame error rate (FER). Fig. 2 presents FER results for a rate 3/4 (denoted H_4 in [2]) STBC with QPSK modulation and 4Tx-4Rx antenna array configuration with and without multiuser diversity. The outer error correction code used herein is a constraint length 7, rate 1/2 convolutional code.

It can be seen that for both coded and uncoded STBC, multiuser diversity reduces the SNR required to guarantee 1% FER.

---

[1] Quasi-static means that the fading coefficients are constant over a frame length of l symbols and are changed independently between consecutive frames.

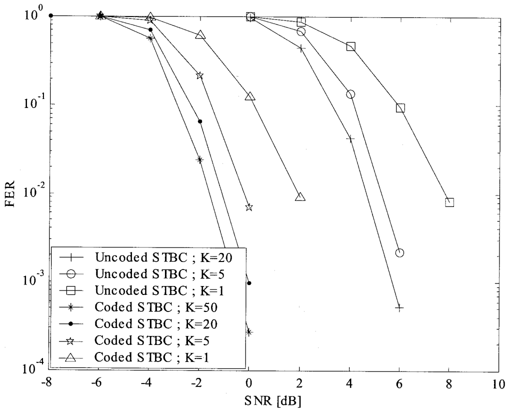

Fig. 2. Performance of STBC with K-fold multiuser diversity (4Tx-4Rx, QPSK, $H_4(r = 3/4)$, conv. coding $r = 1/2$, const. length 7).

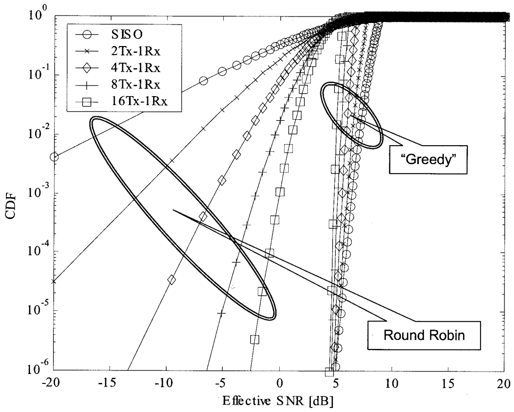

Fig. 3. CDF of effective SNR with and without 50-fold multiuser diversity; $n_R = 1$.

It should also be noted that multiuser diversity enhances the mean SNR (as seen by the shift in the FER curves to the left) as well as improving the diversity order (as seen by the increased slope of the curves).

In order to assess on the interactions of multiuser diversity and spatial diversity further, we plot in Fig. 3 the CDF of the effective SNR with the "greedy" ($K = 50$) and "round robin" scheduling algorithms for various spatial diversity orders that arise from transmit diversity only.

It is observed that in the "round robin" case, as the size of the transmitter array increases, the steepness of the CDF slope increases, indicating the increased spatial diversity order and improved performance. However, with multiuser diversity, this is not the case. Instead, multiuser diversity with no spatial diversity outperforms schemes that employ both multiuser diversity and spatial diversity. Although this result may seem surprising at first, it can be explained as follows. The "greedy" scheduler takes advantage of the peaks in the Rayleigh fading channel. The spatial diversity order of the STBC does not only reduce the probability of having deep fades but also the probability of having constructive peaks, thus limiting the benefits that could be achieved by multiuser diversity. Furthermore, while the performance of STBC schemes with multiple transmit antenna elements and a single receive antenna element is bounded by AWGN results, schemes with multiuser diversity can outperform those of a single input single output AWGN channel.

V. CONCLUSIONS

The impact of multiuser diversity on the performance of space-time block codes has been examined. Two scheduling algorithms, known as the "greedy" and "round robin" approaches, applied different protocols for the transmission of packets to users in the service area. The "greedy" scheduling algorithm yields multiuser diversity, which enhances the average effective SNR and reduces its variance. It was shown that under the comparison of the effective SNR and aggregate throughput, a SISO-based scheduler outperforms a STBC-based scheduler.

REFERENCES

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.

[3] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushayana, and A. Viterbi, "CDMA/HDR: A bandwidth-efficient high-speed wireless data service for nomadic users," *IEEE Commun. Mag.*, pp. 70–77, 2000.

[4] R. Gozali, R. M. Buehrer, and B. D. Woerner, "On the performance of scheduling over space-time architectures," *Proc. IEEE Vehicular Technology Conf.*, pp. 420–424, Sept. 2002.

[5] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1277–1294, June 2002.

[6] G. Bauch and J. Hagenauer, "Analytical evaluation of space-time transmit diversity with FEC-coding," in *Proc. Global Telecommunications Conf. – GLOBECOM 2001*, vol. 2, Nov. 2001, pp. 435–439.

[7] S. Sandhu and A. Paulraj, "Space-time block codes: A capacity perspective," *IEEE Commun. Lett.*, vol. 4, pp. 384–386, Dec. 2000.

[8] H. A. David, *Order Statistics*, 2nd ed. New York: Wiley, 1981.

[9] R. Gozali, R. M. Buehrer, and B. D. Woerner, "The impact of multiuser diversity on space-time block coding," in *Proc. IEEE Vehicular Technology Conf.*, Sept. 2002, pp. 415–419.

[10] R. Gozali and B. D. Woerner, "Theoretic bounds on orthogonal design transmit diversity," in *Proc. IEEE Int. Symp. on Information Theory*, July 2002, p. 304.

[11] ——, "On the robustness of space-time block codes to spatial correlation," in *Proc. IEEE Vehicular Technology Conf.*, May 2002, pp. 832–836.