

Game Theoretic Analysis of Joint Link Adaptation and Distributed Power Control in GPRS

Samir Ginde, James Neel, R. Michael Buehrer,
 Mobile and Portable Radio Research Group,
 Virginia Tech,
 Blacksburg, VA 24061
 Email: sginde@vt.edu, janeel@vt.edu, buehrer@vt.edu

Abstract – In this paper, using non-cooperative game theory, we present a joint link adaptation and power control algorithm for GPRS. We propose a utility model and analytically verify the existence of a Nash Equilibrium (NE) in the network. We then propose a solution method and validate our work by simulating a seven-cell distributed system. Non-unique NE are shown to exist. The effects of the key parameters of the game are evaluated. We introduce three Figures of Merit (FOMs) that express the tradeoff between throughput and energy efficiency for the entire network. The FOMs are used to calculate the optimal values of penalty function parameters and to compare different NE. The proposed scheme is finally compared with another multirate power control technique.

Keywords – multirate power control, link adaptation, game theory, General Packet Radio Service (GPRS)

I. INTRODUCTION

Recent efforts, e.g. [1], have motivated the application of game theory to the analysis of wireless networks. This paper is concerned with the application of game theory to the performance of adaptive coding techniques implemented in General Packet Radio Service (GPRS). This is but one of a class of multirate power control problems [2]. GPRS supports link adaptation (LA) in its endeavor to provide greater data rates to GSM users with minimum changes to infrastructure. In GPRS the modulation scheme is Gaussian Minimum Shift Keying (GMSK), but four options for the code rate are specified. These schemes are summarized in Table I.

Simulation curves for throughput versus Signal to Interference and Noise Ratio (SINR) are reproduced from [3] and shown in Fig. 1 for the TU-50 channel with ideal frequency hopping. Note that the throughput axis of this figure is calibrated in units of KBytes/sec.

TABLE I. GPRS CODING SCHEMES [3]

Coding Scheme	Modulation	Code Rate	Data rate/Time slot
CS-I	GMSK	0.49	9.05 kbps
CS-II	GMSK	0.64	13.4 kbps
CS-III	GMSK	0.73	15.6 kbps
CS-IV	GMSK	1	21.4kbps

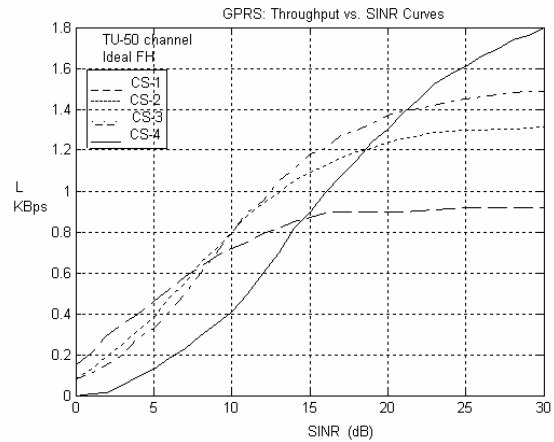


Figure 1. GPRS Throughput vs. SINR Curves [3]

In [4], the sigmoid function (1) of SINR (2) is proposed as an analytical model for throughput

$$L(\gamma) = \frac{A}{1 + e^{-\lambda(\gamma - \delta)}} \quad (1)$$

where

$$\gamma_i = 10 \log_{10} \left(\frac{G_{ii} P_i}{\sum_{j=1, j \neq i}^N G_{ij} P_j + \eta_i} \right) \quad (2)$$

In (1), L is the throughput and A , λ and δ are the parameters of the model. In (2), γ_i is the SINR at receiver i , where G_{ij} is the channel path loss coefficient between transmitter i and receiver j , P_i the power level assigned to link i and η_i , the power of additive noise injected at receiver i . The sets of parameters of (1) that best approximate the simulation curves of Fig. 1 are selected by computer-aided search and summarized in Table II.

TABLE II. PARAMETERS OF SIGMOID MODELS FOR THROUGHPUT VS. SINR CURVE

CS	A kbps	λ	δ dB
CS-1	7.36	0.272	4.75
CS-2	10.52	0.256	8.250
CS-3	11.88	0.256	9.5
CS-4	14.36	0.231	15

This work was sponsored by the Motorola University Partnership in Research Program

The rest of this paper is organized as follows. Section II introduces the game-theoretic concepts applied in this paper. In Section III, the adaptive coding problem in the context of GPRS networks is modeled as a game. Favorable conditions for the existence of Nash Equilibria (NE) are investigated in Section IV, which motivates the solution algorithm presented in Section V. Section VI uses a running example that solves a realistic scenario in a GPRS network by treating it as a game, as well as evaluates the performance of our algorithm.

II. GAME THEORY

In this paper we apply the concept of a game in the *strategic* form. The definitions that follow pertain to strategic form games and will be utilized throughout the paper.

A. Definition 1: Game in the strategic form

A game G in the strategic form consists of [5]

- a finite set of players, denoted by N ,
- for each player $i \in N$, a non-empty set of actions, A_i . The action space of the game is given by $A = \times_{j \in N} A_j$, the Cartesian cross product of the action sets of individual players,
- for each player $i \in N$, a preference relation $R_i(a,b)$ is defined on A . The relation $R_i(a,b)$ is read as the action-tuple a is preferable to another action-tuple b , for player $i \in N$. An action-tuple $(a_i)_{i \in N}$ can be considered to be the vector or profile of actions/strategies chosen by the players, one action per player. The notation a_{-i} is used to indicate the profile of actions taken by all players except player i . Thus it is possible to express an action tuple using the shorthand $a = (a_i, a_{-i})$. The preference relation is usually represented in terms of a utility function $U_i : A \rightarrow \mathfrak{R}$, where the relation $R_i(a,b)$ is equivalent to $U_i(a) > U_i(b)$.

B. Definition 2: Nash Equilibrium

The Nash Equilibrium (NE) is the most commonly used solution concept in game theory [6]. The NE of a strategic game $G = \langle N, A, R \rangle$ is an action-tuple a' which satisfies the following property

$$U_i(a'_i, a'_{-i}) \geq U_i(a_i, a'_{-i}), \forall a_i \in A_i, \forall i \in N \quad (3)$$

Note that a NE does not exist for every game. Before undertaking an attempt at solving the game, it is important to verify the existence of a NE using the Nash Existence Theorem [6].

C. Theorem 1: Nash Existence Theorem

A strategic game $G = \langle N, A, R \rangle$ has at least one NE if $\forall i \in N$ the following conditions hold

- the set A_i of actions is a non-empty, compact and convex subset of a Euclidean space
- the preference relation is continuous and quasi-concave on A_i .

The terms from set theory used in Theorem 1 are concisely defined in [5].

III. GAME THEORETIC FORMULATION

The following formulation of the game is based on Definition 1.

A. Players:

The set of co-channel links, denoted by T , is defined to constitute the set of players in the game. The justification for this definition is intuitive. Co-channel interference (and Gaussian noise) limits the throughput obtained on a GPRS link. Each link attempts to maximize its throughput, by increasing its transmitting power at the cost of increasing interference to other links. This well-understood interaction between links motivates the application of game theory to this and any other power control problem. The total number of co-channel transmitters in the system is finite and given by N .

B. Actions:

Within the scope of this problem, each link has a choice of power levels and code rates. The set of power levels for each link $i \in T$, i.e. $P_i = \{p : p \in [P_{i,\min}, P_{i,\max}]\}$, is chosen to be continuous and closed. The set of code rates is assumed to be discrete. The choices of these two parameters constitute the action pair A_i of each player, as shown in (4).

$$A_i = \{(P_i, r_i) | P_i \in [P_{i,\min}, P_{i,\max}], r_i \in \{r_1, r_2, r_3, r_4\}\} \quad (4)$$

C. Utility Function:

The objective of each link is to maximize its throughput without consuming too much power. Power consumption degrades battery life, and results in interference to other co-channel links. For these reasons, we define the following utility function in (5) that involves throughput and penalizes the excessive use of power.

$$U_i(\mathbf{P}, r_i) = \frac{A(r_i)}{1 + e^{-\lambda(r_i)[r_i - \delta(r_i)]}} - KP_i^q \quad (5)$$

The first term in (5) is the model for the throughput versus SINR from (1). The SINR argument of this function encapsulates the interaction between the players in this game. The second term, the penalty function, is a function of the power assigned to link i , P_i . The index q is an exponent that indicates how much a player is penalized for using too much power. The weight K is chosen to be a constant.

IV. EXISTENCE OF NE

A. Proposition 1

The game of Section III has at least one NE, if code rates of the links are kept constant.

Proof: We apply Theorem 1.

- The action sets P_i are non-empty and convex, by definition. Each P_i is closed since it includes the boundary points $P_{i,\min}$ and $P_{i,\max}$. All points in P_i lie

within the boundary points, thus it is bounded. Therefore the P_i 's are compact.

- The preference relation, defined on the set of power levels is continuous. Since the throughput is monotonically increasing with power, while the penalty function is monotonically decreasing with power, U_i has a single maximum, as shown in Fig. 2 for player 1. Suppose that for player $i \in T$, an arbitrary $P_i = P_i^*$ is chosen (Fig. 2). The set R contains the values of P_i that are more preferable to P_i^* . Due to the unimodal property of U_i , R is a continuous set. The preference relation is also quasi-concave on P_i . Referring to Fig. 2 again, the set R is concave since a line joining any two points in R lies entirely within R . \square

Then by Theorem 1, the game under study has at least one NE, subject to the condition that the coding schemes remain fixed. This is necessary because the set of code rates are discrete and would lead to discontinuous and piecewise-convex action sets. In such a case, by Theorem 1, a NE is not guaranteed to exist. A method for finding the NE is explained in Section V.

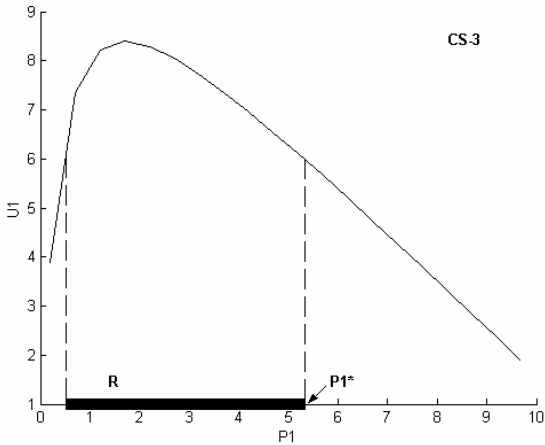


Figure 2. Proof of continuity and quasi concavity of the preference relation

V. NE SOLUTION METHOD

The objective of this game for each player can be stated as: for each player $i \in T$, given the action tuples of the remaining players, i.e. $(A_j)_{j \in T \setminus \{i\}}$ find the action A_i that maximizes its utility function $U_i(\mathbf{P}, \mathbf{r})$. This motivates a distributed solution approach which proceeds as an iterative optimization problem of a scalar objective function, which consists of two recursive steps, and is defined as follows:

Step 1: For each player $i \in T$, given $(P_j)_{j \in T \setminus \{i\}}$, and r_i from Step 2 of the previous iteration, solve

$$P_{i,eqm} = \max_{P_i} U_i(P_i, P_{-i}, r_i), i \in T \quad (6)$$

subject to the constraint $P_{i,min} \leq P_i \leq P_{i,max}$, given a starting guess $(\hat{P}_j)_{j \in T}$. The results of a player i 's optimization routine are used as initial conditions by player $(i+1)$'s routine.

Step 2: The solution of Step 1 is the NE of powers given a fixed choice of coding schemes. Then, given \mathbf{P}_{eqm} find \mathbf{r} which solves the system of equations

$$r_{i,eqm} = \arg \max_{r_i} U_i(r_i, \mathbf{P}_{eqm}), i \in T \quad (7)$$

The solution to this equation yields the set of utility maximizing rates.

Termination: When there is a negligible change in the action tuples $(A_j)_{j \in T}$ obtained at the end of iteration $n-1$ and iteration n respectively, then the solution has converged to a NE. If not, proceed to Step 1 of iteration $n+1$ using $(A_j)_{j \in T}$ obtained at iteration n as the starting guess. The convergence time is determined by the closeness of the initial approximation to the solution.

This iterative procedure continues until all players find that their rates do not change between iterations and the change in their power levels is less than a pre-defined bound, or an upper limit on the number of iterations is reached.

VI. RESULTS

A seven-cell star type network configuration as shown in Fig. 3 forms the basis for our results.

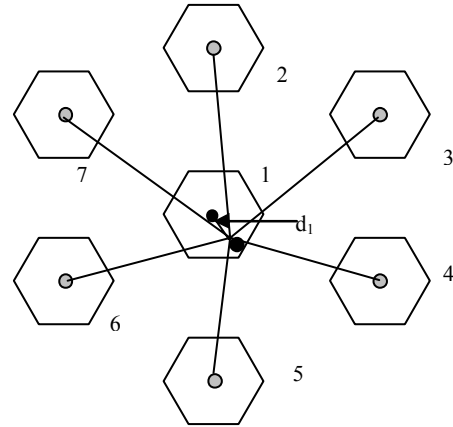


Figure 3. Seven player game, Re-use factor =3. A set of co-channel cells is shown.

The downlink case is considered in all the forthcoming examples. The base-stations of the cells numbered 1 to 7 are co-channel interferers. In computing the path loss coefficients, the distance from each interferer to the receiver of interest is approximated as the distance between the centers of their cells. The frequency re-use factor is set to 3. Therefore the first tier of interfering base stations lie on a circle of radius $3R$, where R is the cell radius. The path loss model is log-distance but log-normal shadowing has not been considered. This permits the establishment of a path loss matrix that remains identical between simulation trials.

Noise is referenced to the maximum received power of the weakest user. Maximum transmitter power is 100 mW and the dynamic range of power is 20 dB. The positioning of the users in the system that stays the same in all examples that follow, is shown in Table III.

TABLE III. SYSTEM TOPOLOGY

Player i	1	2	3	4	5	6	7
d/R	1	0.1	1	0.5	1	1	1

In Table III, the first row indexes the players in accordance with the numbering scheme given in Fig. 3. The second row indicates the distance of a handset from its base station, normalized by the cell radius R .

To evaluate these results more comprehensively we propose a set of three Figures of Merit (FOM) in (8), (9), and (10).

$$FOM1 = \frac{\sum_i L_i}{\sum_i \frac{P_i}{P_{\max}}} \quad (9)$$

$$FOM2 = \sum_i L_i - A_{\max} \sum_i \frac{P_i}{P_{\max}} \quad (10)$$

$$FOM3 = \sum_i L_i \quad (11)$$

FOM1 is the ratio of the system throughput (sum of throughput) to the sum of the fractions of peak power consumed by the links. FOM2 is the difference of system throughput and sum of the fractions of peak power consumed, scaled by peak throughput, A_{\max} . FOM3 is simply the system throughput. The proposed FOMs require that the sum of throughput be maximized. However the first two add the condition that the power consumption be minimized. In all three cases, a higher FOM is preferable.

We consider a Signal to Noise Ratio (SNR) of 100 dB, since our primary interest lies in the interference limited case. We begin with an example based on the model in Fig. 3 with penalty function parameter settings of $K=1$, $q=0.7$, and initial CS choice of 1 for all players. The simulation results for this special case are shown in Table IV.

The results in Table IV are intuitively satisfying. The equilibrium rates reflect the channel quality of the respective links. However, it is clear that the effect of K and q on the utility function must be studied. The initial choice of rates also merits further investigation, since different starting conditions could lead to different equilibria. In Fig. 4, the histogram of FOMs in NE, obtained from an exhaustive search of all the 4^7 different starting rates is plotted for $K=1$, $q=2$.

TABLE IV. EXAMPLE SIMULATION, $K=1$, $Q=0.7$

Player	1	2	3	4	5	6	7
P/P _{max} %	49	7	80	40	72	73	69
CS	1	4	3	3	2	2	2
SINR dB	5.8	28.9	11.9	15.7	10	9.7	10.5

Kbps	4.2	13.8	7.7	9.8	6.4	6.2	6.7
System Throughput = 54.8 kbps							

The striking observations are that there are just four closely spaced, though well-defined NE, of which one clearly dominates. Fig. 5 and Fig. 6 illustrate the effect of q on FOM and power consumption, respectively. FOM1 and FOM2 are small for lower values of q due to excessive power consumption. However, system throughput does not change appreciably with q . Hence, from Fig. 6, $q > 2$ is desirable in the interference limited case. From Fig. 5, FOM1 is observed to be maximized for $q \approx 5$ and FOM2 for $q \approx 2.5$.

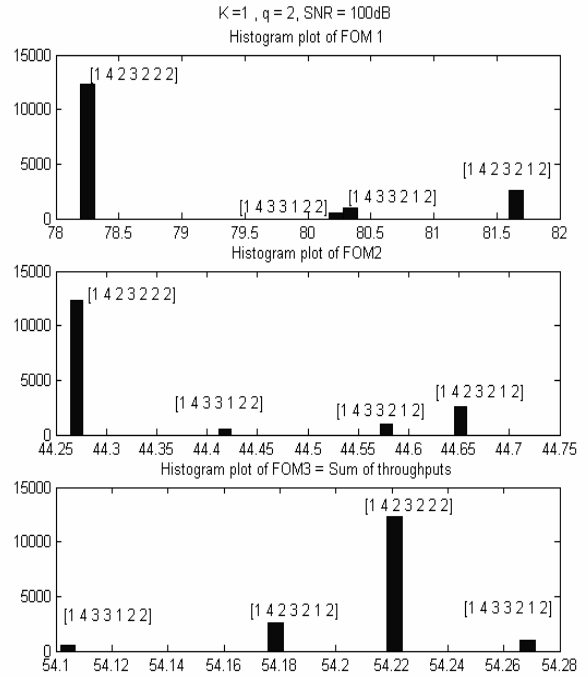


Figure 4. Histogram plot of FOM's in NE

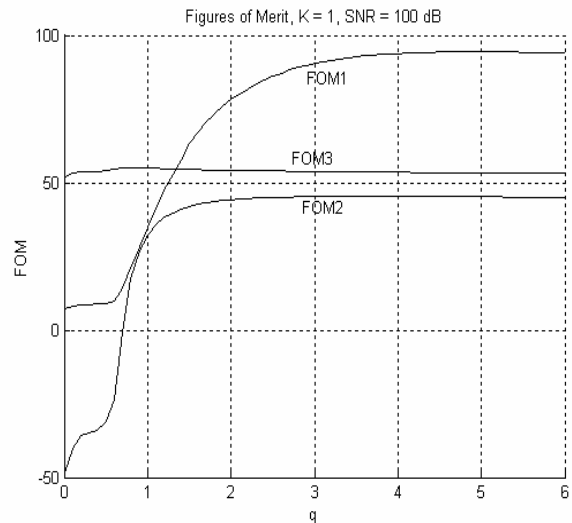


Figure 5. Effect of q on FOM's, SNR = 100 dB

It is interesting to compare this result with well-known algorithms such as the Generalized Selective Power Control (GSPC) [2], under identical conditions. Using GSPC, the equilibrium rate vector is [1 4 2 3 2 2 2] and system throughput is 52.67 kbps. From Fig. 5 it can be observed that system throughput is comparable to this value for $q > 2$.

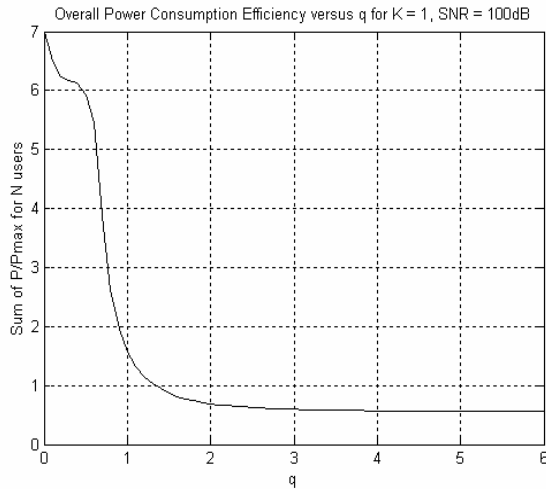


Figure 6. Effect on q on power consumption

At low SNR, there is a greater variability in throughput (Fig. 7). Here it is recommended to select $q > 1$ to optimize the tradeoff between throughput and power consumption.

VII. CONCLUSIONS

In this paper, we applied non-cooperative game theory to solve a multirate power control problem as it applies to GPRS. Game theory facilitated a *general* and *intuitive* method for formulating the problem. Multiple NE were found in this problem depending on the choice of initial rates. However, the sensitivity of these equilibria to different starting rates was found to be low. We introduced a utility model that results in correct decision making by the adaptive modulation algorithm and results in acceptable levels of interference to other co-channel links. This was verified by simulation. We subsequently introduced a set of FOMs that trade off power efficiency for throughput and assist in the selection of the optimal penalty function parameters given the channel conditions. Performance is shown to be comparable, if not better than GSPC, a well-known multirate power control algorithm.

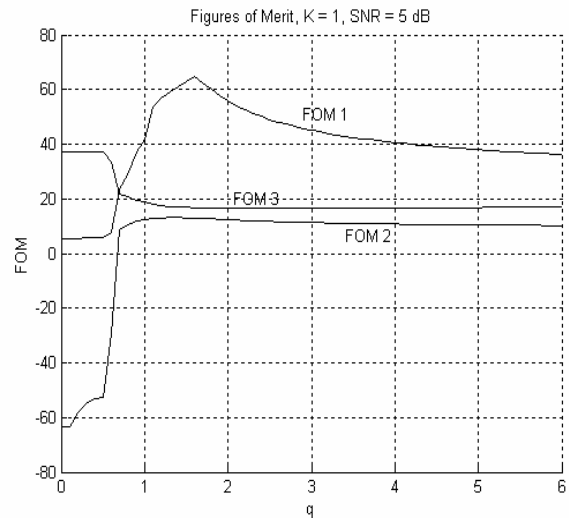


Figure 7. Effect of q on FOM's, SNR = 5 dB

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