The Impact of Multiuser Diversity on Space-Time Block Coding

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Abstract

In this paper, we consider a scenario in which K active data users, each of which is equipped with multiple antenna elements, are served by a multi-antenna element base station (BS). We focus on the downlink channel, where space-time block coding scheme is employed and assume that channel state information (CSI) is reported to the BS by all users and on a per frame basis. Using a scoring function at the BS, time resources are allocated to the user with the best instantaneous effective SNR, facilitating a multiuser diversity mechanism. Using order statistics, we compute histograms and cumulative distribution functions of the effective SNR at the space-time combiner output and assess on the interactions between multiuser diversity and spatial diversity.

I. Introducation

Space-time block coding (STBC) schemes, proposed originally by Alamouti in [1] and Tarokh et al. in [2], were recently adopted by the 3G standardization committees for implementation as one of the transmit diversity modes of 3G wireless networks. These schemes introduce a simple, elegant spatial diversity mechanism that improves the energy efficiency of schemes operating over wireless channels. A vast body of research has been published on the performance and structure of these schemes, focusing on the physical (PHY) layer aspects only. In this paper, we assess on the impact of scheduling algorithms, which may be executed in the access-control (MAC) layer of a wireless network, on the performance of STBC.

One important application to space-time architectures may be high-data-rate downstream wireless Internet access to nomadic users [3]. The nature of such data traffic is asymmetric, requiring a much higher downlink rate from the base station (BS) than that generated in the uplink channel by the user terminal. In addition to traffic asymmetry, data services differ from voice services also in their tolerance to delay. We consider in this paper a scenario where a single BS provides data services to K users, each of which is equipped with multiple antenna elements. Since the data rate that can be supported to each user is proportional to its received SNR, it may be beneficial to use channel state information (CSI) and the unequal latency property of the service to serve multiple users with disparate SNRs. This may improve the overall throughput of the system as compared with the case of equally served users regardless of their channel conditions. The BS applies STBC architecture paired with a scheduling protocol that routes the transmission of packets to users based on their reported CSI in the uplink channel.

Two extreme scheduling algorithms are considered. The first one, denoted "greedy", routes each transmission to the user with the best instantaneous channel conditions. The second, denoted "round robin" (RR), routes the transmission of packets equally across users. Results indicate that the "greedy" scheduler gives rise to multiuser diversity mechanism, where the link with the best instantaneous SNR is selected (out of K independent links) on a per frame basis. The round robin scheduler is equivalent to the single user case since all users are equally served in an orthogonal TDM-like manner.

It is well known that by increasing the spatial diversity order of STBC, performance in terms of reliability and capacity is bounded by AWGN results [4, 5]. Natural questions are therefore: 1) How does multiuser diversity enhance performance further ?, and 2) Is multiuser diversity mechanism equivalent to spatial diversity ? The answer to these questions is provided in this paper.

The multiuser diversity, introduced by the "greedy" scheduler, enhances the averaged SNR and thus improves the performance of the space-time architecture. We study the interactions of multiuser diversity and spatial diversity by computing the statistics of the effective SNR at the space-time combiner output with and without K-fold multiuser diversity. The analysis is based on classical results from order statistics

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[6]. While the spatial diversity mechanism is designed to reduce the probability of deep fades at the output of the receiver combining stage, it also eliminates the peaks (constructive fades) of the Rayleigh fading channel. This phenomenon is shown to limit the performance gain that could have been achieved by the multiuser diversity mechanism only.

The remainder of the paper is organized as follows. In section II we present the system model and the assumptions underlying our analysis. Section III presents a general procedure to derive the statistics of the effective SNR at the output of the receiver combining stage with and without multiuser diversity. In section IV, we investigate these statistics for few specific space-time block coding schemes and demonstrate how multiuser diversity improves frame error rate performance. Finally, section V concludes the paper.

II. System Model

The block diagram of a full rate space-time block code with 2 transmit and 1 receive antenna elements is illustrated in Figure 1. The orthogonality across sequences of symbols, transmitted simultaneously form n_T antenna elements, is exploited at the receiver by the combining rule. The objectives of this combiner are two fold: 1) decouple the received superposition of symbols to create separate decision statistics, and 2) coherently combine the fade coefficients associated with each transmitted symbol to allow for diversity order.

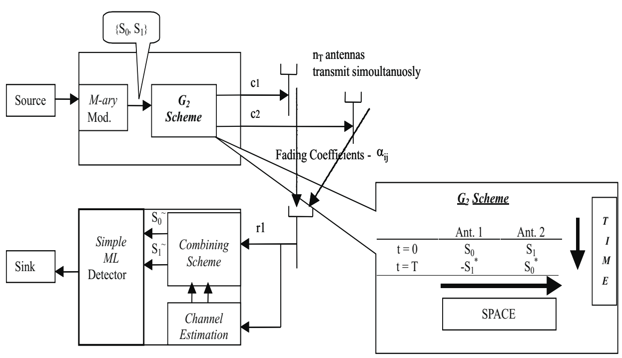

Figure 1: STBC block diagram

The received signal at the kth terminal, receive antenna $j, j \in [1, 2, \cdots, n_R]$, at time t is modeled by,

$$r_k^j(t) = \sum_{i=1}^{n_T} \alpha_{i,j} c_k^i(t) + \eta_k^j(t), \quad (1)$$

where $\eta_k^j(t)$ denotes the AWGN term and is modeled as independent samples of a zero mean complex Gaussian random variable with variance $\frac{n_T}{2SNR}$ per dimension and $c_k^i(t)$ is the symbol transmitted to the kth user from transmit antenna i at time t with average energy normalized to one. The channel is modeled by an $n_T \times n_R$ matrix $\Omega_k = [\alpha_{i,j}^{(k)}]$, whose entry $\alpha_{i,j}^{(k)}$ represents the complex fade coefficient for the path from transmit antenna i to receive antenna j of the kth user. These fade coefficients are assumed to be zero mean complex Gaussian random variables with variance 0.5 per dimension. For simplification purposes, we assume perfect channel tracking at the terminal and perfect feedback of these estimates to the base station. We also assume that the K users experience the same average SNR and that each MIMO link exhibits quasi-static frequency non-selective (flat) fading.

III. Performance Analysis

The effective SNR at the space-time combiner output of the kth user is given by [7, 8]:

$$\gamma = \frac{SNR}{n_T} \sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \left| \alpha_{i,j}^{(k)} \right|^2, \quad (2)$$

This is a chi-square random variable with $n_T n_R$ degrees of freedom. The probability density function of this random variable is given by [9, 4]:

$$f_\gamma(\gamma) = \frac{1}{(n_T n_R - 1)! \left(\frac{SNR}{n_T} \right)^{n_T n_R}} \gamma^{n_T n_R - 1} e^{-\frac{\gamma}{SNR/n_T}}, \quad (3)$$

Let us define new variables μ, n and C as: $\mu = \frac{n_T}{SNR}$, $n = n_T n_R - 1$ and $C = \frac{\mu^{n+1}}{n!}$. This allows us to write a simplified form of (3):

$$f_\gamma(\gamma) = C \gamma^n e^{-\mu\gamma}. \quad (4)$$

In our "greedy" scheduling algorithm, the base station selects (on a per frame basis) the user k with the best instantaneous channel realization. That is:

$$arg\ max_{k \in \{1,2,\cdots,K\}} \sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \left| \alpha_{i,j}^{(k)} \right|^2, \quad (5)$$

This multiuser diversity mechanism creates a new random variable, denoted $\tilde{\gamma}$, which represents the effective SNR at the space-time combiner output with K-fold multiuser diversity. The probability density function of this random variable is computed using order statistics results [6]:

$$g_{\tilde{\gamma}}(y) = K f_\gamma(y) F(y)^{K-1}, \quad (6)$$

where K is the number of users and $F(y)$ denotes the cumulative distribution function (CDF) of the original random variable γ. The CDF of γ is given by:

$$\begin{aligned} F(y) &= \int_{\gamma=0}^{y} f_\gamma(\gamma) d\gamma \quad (7) \\ &= 1 - \mu^{n+1} e^{-\mu y} \sum_{m=0}^{n} \frac{y^m}{m! \mu^{n-m+1}} \end{aligned}$$

Plugging Eqs. (7) and (4) into (6), we obtain a closed form expression for the pdf of the effective SNR at the space-time combiner output with K-fold multiuser diversity.

The mth moment of the effective SNR at the output of the space time combiner with K-fold multiuser diversity is computed by

$$E[y^m] = \int_{y=0}^{\infty} y^m g_{\tilde{\gamma}}(y)dy = K \int_{y=0}^{\infty} y^m f_{\gamma}(y)F(y)^{K-1}dy. \tag{8}$$

In the following section, we apply the results of Eqs. (7) and (8) to assess on the impact of multiuser diversity when employed in conjunction with space-time block coding.

IV. Performance Results

Results in the form of cumulative distribution functions (CDFs) are plotted in Figure 2 using Monte Carlo simulations and the analysis presented herein for STBC with 4Tx-4Rx, paired with $50, 20, 5$ and 1-fold multiuser diversity.

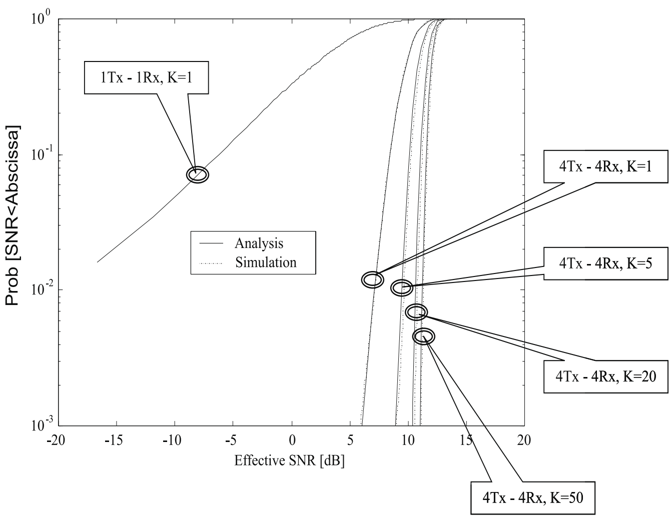

Figure 2: The effect of multiuser diversity on the average effective SNR ; $SNR = 4dB$

As a reference, we include the CDF of a single-input single-output (SISO) link with no multiuser diversity. For a given CDF level of 1%, the effective SNR is enhanced by $2.3, 3.3$ and 4 dB for $K = 5, 20$ and 50 users, respectively. An excellent match is demonstrated between the closed form expression and the Monte Carlo simulation results.

This SNR enhancement yields an improved performance in terms of frame error rate (FER). Figure 3 presents FER results for a rate 3/4 (denoted H_4 in [2]) STBC with QPSK modulation and 4Tx-4Rx antenna array configuration with and without multiuser diversity. The outer error correction code used herein is a constraint length 7, rate 1/2 convolutional code.

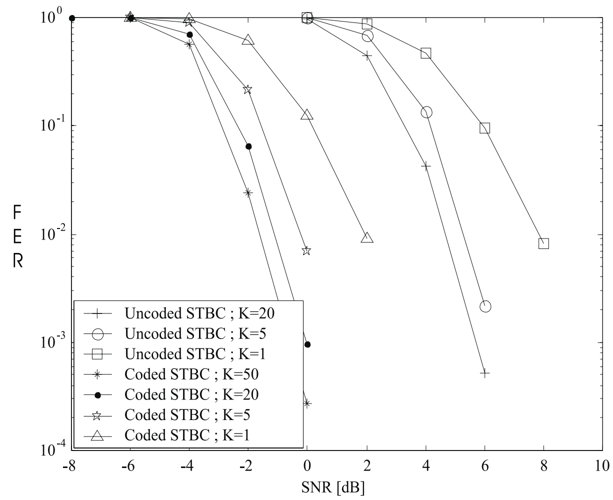

Figure 3: Performance of STBC with K-fold multiuser diversity (4Tx-4Rx, QPSK, H_4, $r = 1/2$)

It can be seen that for both coded and uncoded STBC, multiuser diversity reduces the SNR required to guarantee 1% FER. It is also noticed that in addition to the enhanced mean SNR (shift in the FER curves to the left) there is a slight improvement in the slope of the curves, corresponding to improved diversity order. Using the first and second moments of Eq. (8), Figures 4 and 5 illustrate the effective SNR gain in terms of mean and variance due to multiuser diversity for various array configurations.

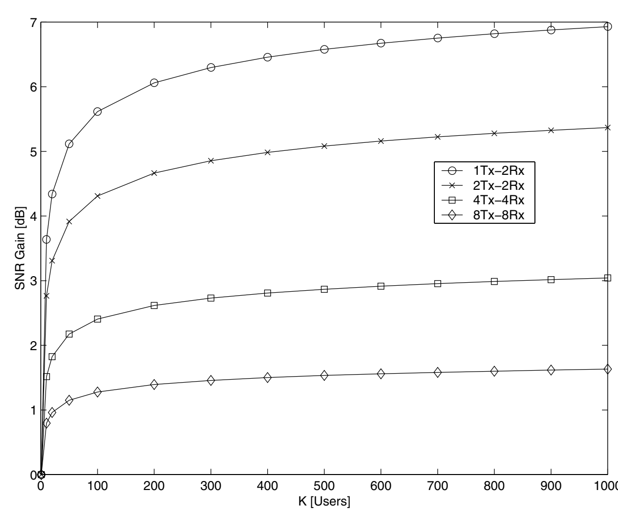

Figure 4: Multiuser diversity over STBC; averaged effective SNR gain

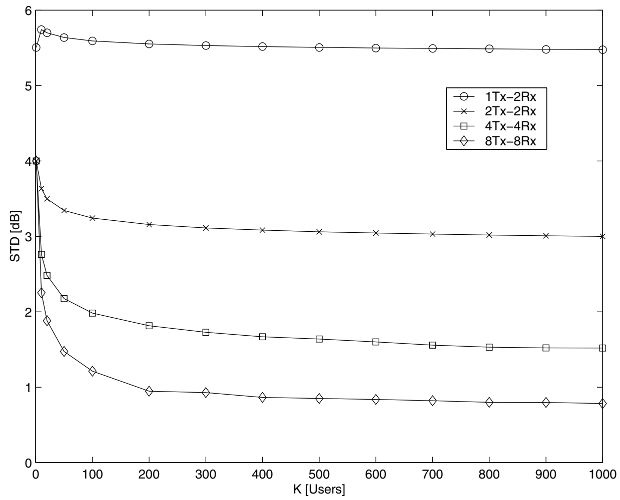

Figure 5: Multiuser diversity over STBC; standard deviation of the effective SNR

It is noticed that the SNR gain reaches a saturation point for all array sizes, where increasing the number of users further (amount of links to choose from) results in diminishing returns in SNR enhancement. More important, it is observed that as the array size increases (higher spatial diversity order), the gain associated with the multiuser diversity decreases. We also notice that the variance of the effective SNR decreases with the number of users thus improving the effective diversity order.

In order to assess on the interactions of multiuser diversity and spatial diversity further, we plot in Figure 6 the CDF of the effective SNR with the "greedy" ($K = 50$) and round robin scheduling algorithms for various spatial diversity orders that arise from transmit diversity only.

It is observed that in the round robin case, as the size of the transmitter array increases, the steepness of the CDF slope increases, indicating the increased spatial diversity order. However, with multiuser diversity, an opposite trend occurs. That is, multiuser diversity with no spatial diversity outperforms schemes that employ both multiuser diversity and spatial diversity. Although this result may seem surprising at first, it can be explained as follows. The "greedy" scheduler takes advantage of the peaks in the Rayleigh fading channel. The spatial diversity order of the STBC does not only reduce the probability of having deep fades but also the probability of having constructive peaks, thus limiting the benefits that could be achieved by multiuser diversity. In other words, while the performance of STBC schemes with multiple transmit antenna elements and a single receive antenna element is bounded by AWGN results, schemes with multiuser diversity can outperform those of a single input single output AWGN channel.

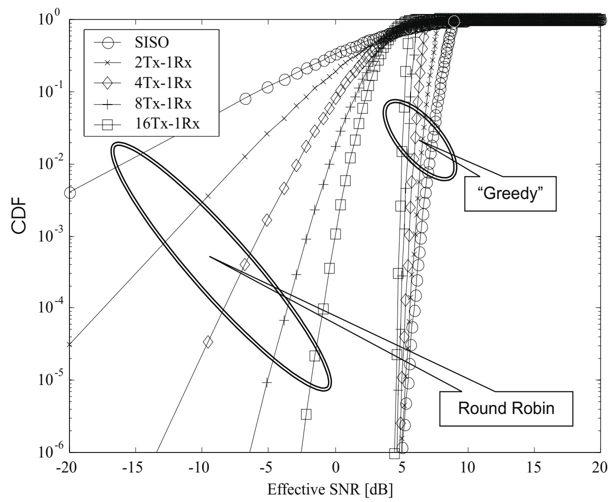

Figure 6: CDF of effective SNR with and without 50-fold multiuser diversity; $n_R = 1$

Figure 7 illustrates the CDF of schemes that employ multiple antenna elements at the receiver. It is observed that the receive diversity provides antenna aperture gain that shifts the curves by 3 and 6 dB, for the 2 and 4 receive antenna elements, respectively.

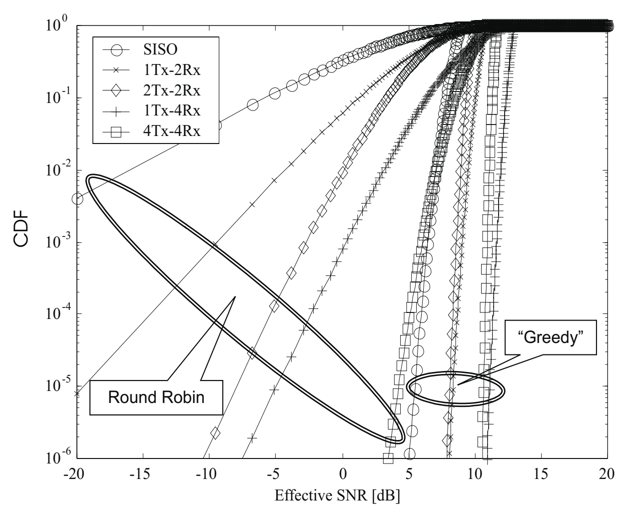

Figure 7: CDF of effective SNR with and without 50-fold multiuser diversity; various array sizes

In the "greedy" mode, it is observed that although the slope of the 4Tx-4Rx configuration may be steeper than the slope of the 1Tx-4Rx configuration, schemes with multiuser diversity would perform better in the case of smaller spatial diversity order.

V. Conclusions

The impact of multiuser diversity on the performance of space-time block codes was investigated. Two scheduling algorithms, known as the "greedy" and round robin approaches, applied different protocols for the transmission of packets to users in the service area. The "greedy" scheduling algorithm was shown to yield multiuser diversity, enhancing the averaged effective SNR. It is shown that the multiuser diversity does not only reduce the variance of the effective SNR but also enhances its mean value. Moreover, spatial diversity is shown to eliminate the peaks of the Rayleigh fading channel, thus limiting the performance gain that could have been achieved by the multiuser diversity mechanism.

Acknowledgments

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References

[1] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal On Selected Ares in Communications*, vol. 16, pp. 1451–1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, pp. 1456 –1467, July 1999.

[3] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushayana, and A. Viterbi, "CDMA/HDR: A bandwidth-efficient high-speed wireless data service for nomadic users," *IEEE Communications Magazine*, pp. 70–77, 2000.

[4] G. Bauch and J. Hagenauer, "Analytical evaluation of space-time transmit diversity with FEC-coding," in *Global Telecommunications Conference - GLOBECOM 2001*, vol. 2, pp. 435–439, 2001.

[5] S. Sandhu and A. Paulraj, "Space-time block codes: a capacity perspective," *IEEE Communications Letters*, vol. 4, pp. 384–386, Dec. 2000.

[6] D. Neal, "Order statistics," *http://www.wku.edu/neal/probability/order.html*, 2002.

[7] R. Gozali and B.D. Woerner, "Theoretic bounds on orthogonal design transmit diversity," in *IEEE International Symposium on Information Theory - ISIT'*, 2002.

[8] R. Gozali and B.D. Woerner, "On the robustness of space-time block codes to spatial correlation," in *IEEE Vehicular Technology Conference - VTC' Spring, Birm., Alabama*, pp. 832–836, 2002.

[9] J.G. Proakis, *Digital Communications.* McGraw-Hill, 1995.