Improved CDMA Performance through Bias Reduction for Parallel Interference Cancellation *

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Abstract- **Parallel multistage interference cancellation is a promising approach for practical implementation of multiuser detection. However, as this paper shows, direct implementation of multistage parallel interference cancellation (i.e. complete cancellation of the estimated interference) results in biased estimates and consequently reduced performance. A simple technique that mitigates the effect of the bias and provides significant performance gains over the direct implementation is presented.**

I. INTRODUCTION

Multiuser interference cancellation techniques exploit the structured nature of the multiple access interference and thus offer significant gains in capacity and near-far resistance over the conventional receiver . A number of multiuser receiver architectures have been proposed over the past decade [1, 2, 3, 4, 5, 6]. For a practical implementation, multistage parallel interference cancellation is an attractive approach both in terms of performance and computational complexity [7], as well as its robustness to moderate phase jitter and synchronization errors [8].

In order to minimize the complexity of the receiver, a matched filter bank is commonly used at each stage for estimation. Complete subtraction of the estimated MAI is then performed to reduce the interference affecting each user. It turns out, however that such a straightforward implementation of parallel interference cancellation results in biased decision statistics. In this paper we present an explanation of the source of this bias, and suggest a simple, yet effective technique to mitigate its effects and significantly improve performance for heavily loaded systems. The proposed technique lends itself to practical real-time DSP implementation [9].

Section 2 describes the CDMA system model. Section 3 presents the source of the bias, and in section 4 we describe an effective simple technique that mitigates the effects of the bias and significantly enhances system performance. Simulation results are presented comparing the performance of the direct implementation with that of the proposed technique. Conclusions are presented in section 5.

II. SYSTEM MODEL

The CDMA multiple access model used here is similar to the model described in [8]. The kth user's transmitted signal has the form:

$$s_k(t-\tau_k) = \sqrt{2P_k}\,b_k(t-\tau_k)a_k(t-\tau_k)\cos(\omega_c t+\phi_k), \quad (1)$$

where P_k is the kth user's received signal power, $a_k(t)$ and $b_k(t)$ are the spreading and data waveforms respectively (we assume rectangular pulses for both, a spreading gain of of N chips per bit, and completely random spreading waveforms), τ_k is a time delay that accounts for the asynchronous nature of the system uplink, ϕ_k is the received phase of the kth user relative to some arbitrary reference phase.

The received signal $r(t)$ is given by:

$$r(t) = \sum_{k=1}^{K} s_k(t-\tau_k) + n(t), \quad (2)$$

where K is the number of users, and we assume a single noise source $n(t)$ from a common front end. The decision metric $Z^{(s+1)}_{k,i}$ for the ith bit of the kth user after s stages of cancellation for an S-stage parallel cancellation scheme can be expressed as:

$$Z^{(s+1)}_{k,i} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} \hat{r}^s_k(t)a_k(t-\tau_k)cos(\omega_c t+\phi_k)dt \quad (3)$$

The received signal at stage s is

$$\hat{r}^s_k(t) = r(t) - \sum_{j\neq k} \hat{s}^s_j(t), \quad (4)$$

and the signal $\hat{s}^s_j(t)$ corresponds to the estimated reconstructed signal for user j at stage s and is given by:

$$\hat{s}^s_j(t) = \frac{2}{T}a_j(t)\cos(\omega_c t+\phi_j)\sum_{i=-\infty}^{\infty} Z^{(s)}_{j,i}\,p_T(t-iT). \quad (5)$$

and $p_T(t)$ is a unit pulse function of duration T equal to the bit period.

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III. ANALYSIS OF BIAS IN THE DECISION STATISTICS

In this section we derive the decision statistic for the direct parallel interference cancellation structure, and show the existence of a bias that arises due to MAI and imperfect cancellation. For clarity, first consider a two user ($K = 2$) system. Since only relative delays and phases are important we set $\phi_1 = 0$ and $\tau_1 = 0$. Following the notation of [10], the decision statistic at stage 1 (i.e. before any interference cancellation) for $b_i^{(1)}$, the ith bit of user 1 is:

$$\begin{aligned} Z_{1,i}^{(1)} &= T\sqrt{\frac{P_1}{2}}b_i^{(1)} + \sqrt{\frac{P_2}{2}}\cos(\phi)\Big[b_{i-1}^{(2)}R_{2,1}(\tau) + \\ &\qquad b_i^{(2)}\hat{R}_{2,1}(\tau)\Big] + \eta_i^{(1)}, \end{aligned} \tag{6}$$

where $R_{2,1}(\tau)$ and $\hat{R}_{2,1}(\tau)$ are the continuous-time partial cross-correlation functions of the 1st and the 2nd signature sequences [10], and $\eta_i^{(k)} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} n(t)a_k(t-\tau_k)cos(\omega_c t+\phi_k)dt$. Similarly, for user 2 the decision statistic is given by

$$\begin{aligned} Z_{2,i}^{(1)} &= T\sqrt{\frac{P_2}{2}}b_i^{(2)} + \sqrt{\frac{P_1}{2}}\cos(\phi)\Big[b_i^{(1)}\hat{R}_{2,1}(\tau) + \\ &\qquad b_{i+1}^{(1)}R_{2,1}(\tau)\Big] + \eta_i^{(2)} \end{aligned} \tag{7}$$

At stage 2, the estimated signal for user 2 obtained via equation (5), is subtracted from the received signal $r(t)$ to form a new estimated received signal for user 1 at stage 2. Using equation (3) to compute the decision statistic for user 1 at stage 2 leads to the expression:

$$Z_{1,i}^{(2)} = Z_{1,i}^{(1)} - \frac{\cos(\phi)}{T}\Big[Z_{2,i-1}^{(1)}R_{2,1}(\tau) + Z_{2,i}^{(1)}\hat{R}_{2,1}(\tau)\Big] \tag{8}$$

Substituting into (8) the expression for the decision statistics of users 1 and 2 from equations (6) and (7), and cancelling common terms, one obtains:

$$\begin{aligned} Z_{1,i}^{(2)} &= T\sqrt{\frac{P_1}{2}}b_i^{(1)} - \frac{\cos^2(\phi)}{T}\sqrt{\frac{P_1}{2}}\Big[b_{i-1}^{(1)}\hat{R}_{2,1}(\tau)R_{2,1}(\tau) + \\ &\quad b_i^{(1)}R_{2,1}^2(\tau) + b_i^{(1)}\hat{R}_{2,1}^2(\tau) + b_{i+1}^{(1)}R_{2,1}(\tau)\hat{R}_{2,1}(\tau)\Big] + \\ &\quad \eta_i^{(1)} - \frac{\cos(\phi)}{T}\Big[\eta_{i-1}^{(2)}R_{2,1}(\tau) + \eta_i^{(2)}\hat{R}_{2,1}(\tau)\Big] \end{aligned} \tag{9}$$

Conditioning on $b_i^{(1)}$ and taking the expected value of the decision statistic, we obtain

$$\begin{aligned} E\{Z_{1,i}^{(2)}|b_i^{(1)}\} &= T\sqrt{\frac{P_1}{2}}b_i^{(1)} - \sqrt{\frac{P_1}{2}}E\{\frac{\cos^2(\phi)}{T}\}\times \\ &\qquad b_i^{(1)}E\{(R_{2,1}^2(\tau) + \hat{R}_{2,1}^2(\tau))\}. \end{aligned} \tag{10}$$

Since for random sequences $E\{(R_{2,1}(\tau)\hat{R}_{2,1}(\tau)\} = 0$, $E\{(R_{2,1}^2(\tau)+\hat{R}_{2,1}^2(\tau))\}$ in equation (10) can be expressed as $E\{(R_{2,1}(\tau)+\hat{R}_{2,1}(\tau))^2\}$. We can therefore formulate equation (10) as:

$$\begin{aligned} E\{Z_{1,i}^{(2)}|b_i^{(1)}\} &= T\sqrt{\frac{P_1}{2}}b_i^{(1)} - \sqrt{\frac{P_1}{2}}E\{\frac{\cos^2(\phi)}{T}\}\times \\ &\qquad b_i^{(1)}E\{(R_{2,1}(\tau) + \hat{R}_{2,1}(\tau))^2\}, \end{aligned} \tag{11}$$

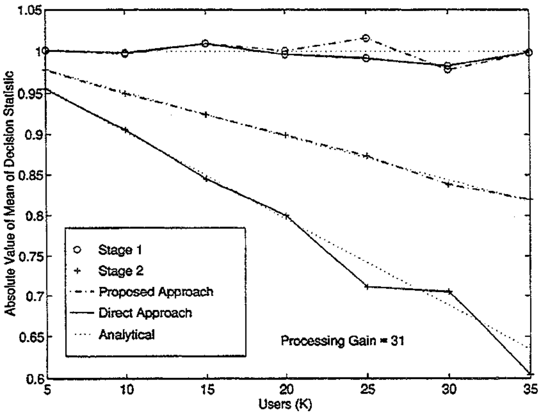

Figure 1: Analytical and simulation results for the mean of the decision statistics for a two stage receiver using direct implementation (i.e. complete cancellation), and the proposed bias mitigation technique (partial cancellation) with $C_K^{(2)} = 0.5$, $N = 31$, $N_o = 0$).

To evaluate this expression, note that $E\{(R_{2,1}(\tau) + \hat{R}_{2,1}(\tau))^2\}$ corresponds to the variance of $\int_0^T a_2(t-\tau)a_1(t)dt$ for a pair of randomly selected sequences since $R_{2,1}(\tau) + \hat{R}_{2,1}(\tau) = \int_0^T a_2(t-\tau)a_1(t)dt$, and $E\{(R_{2,1}(\tau) + \hat{R}_{2,1}(\tau))\} = 0$. It has been shown using the standard Gaussian approximation that the variance of this random variable (normalized for a chip period $T_c = 1$) equals $2N/3$ [11]. Using this result and normalizing for $T_c = 1$, equation (11) becomes

$$E\{Z_{1,i}^{(2)}|b_i^{(1)}\} = N\sqrt{\frac{P_1}{2}}b_i^{(1)}\left[1 - \frac{1}{3N}\right]. \tag{12}$$

Since the estimates of the interfering signals are correlated with the desired user's power and bit value, a bias is produced when they are used to reconstruct and remove the interference. The existence of a bias in the mean of the decision statistics is evident in Equation (12). Extension of this result to a K-user system is straightforward. For K independent users with random signature sequences, following a similar approach one obtains:

$$E\{Z_{1,i}^{(2)}|b_i^{(1)}\} = N\sqrt{\frac{P_1}{2}}b_i^{(1)}\left[1 - \frac{(K-1)}{3N}\right]. \tag{13}$$

From this equation we see that the bias in the mean increases linearly with system loading and is inversely proportional to the processing gain. Similar behavior has also been seen in the synchronous case [12]. However, most analysis, including our previous work [8, 13] assume independence. In Figure 1, simulation and analytical results are shown for the mean of the decision statistic for a direct implementation of a two stage parallel interference canceller as the number of users increases. We see that the analytical results agree with our simulations.

Imperfect interference cancellation not only affects the mean of the decision statistics, but also it affects their variance. Simulation results for the variance

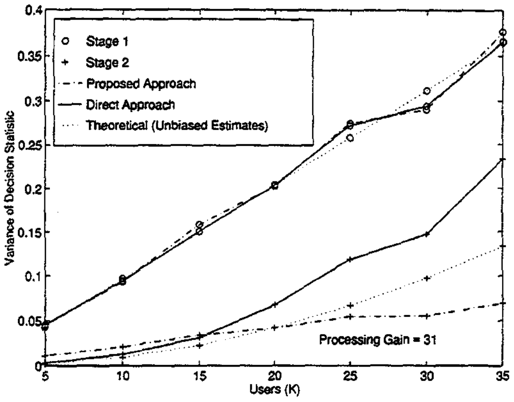

Figure 2: Variance of the decision statistics for direct implementation (i.e. complete cancellation), and the proposed bias mitigation technique (partial cancellation) with $C_K^{(2)} = 0.5$ for a two stage receiver ($N = 31$, $N_o = 0$).

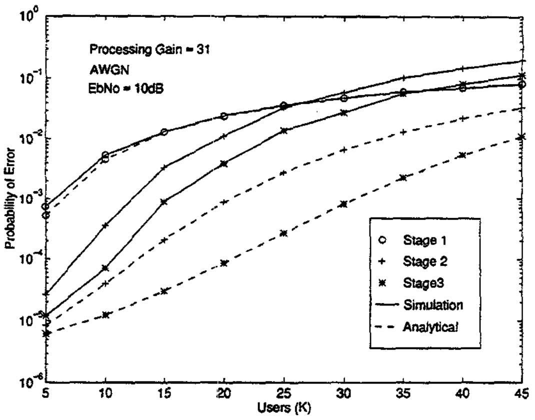

Figure 3: Simulated capacity curves for direct implementation of parallel cancellation (i.e. complete cancellation), and theoretical curves assuming unbiased estimation. ($Eb/No =$ 10 dB and $N = 31$)

of the first and second stages of direct parallel interference cancellation, and analytical results assuming unbiased decision statistics can be seen in Figure 2.

We have also found that the effect of the bias has its strongest effect on the decision statistics in the first stage of interference cancellation. In subsequent stages of cancellation, the influence of the bias diminishes. However, if bias leads to incorrect cancellation at the first stage, the residual effects of these errors may be observed at subsequent stages. Current work is focusing on characterizing the moments of the biased decision statistics and generalizing the results to an arbitrary number of stages of interference cancellation for an asynchronous system.

IV. MITIGATION OF BIAS THROUGH USE OF PARTIAL INTERFERENCE CANCELLATION FACTOR

In this section we propose a simple yet effective way to mitigate the effect of the bias and improve performance of a parallel multistage interference cancellation receiver. Our technique is based on multiplying the channel gain estimates before signal reconstruction by a partial-cancellation factor $0 \leq C_K^{(s)} \leq 1$ that varies with the stage s of cancellation and system loading K. This can also be interpreted as simply modifying the direct cancellation scheme of equation (4) to include the partial-cancellation factor $C_K^{(s)}$ as follows:

$$\hat{r}_k^s(t) = r(t) - C_K^{(s)} \sum_{j \neq k} \hat{s}_j^s(t), \tag{14}$$

Multiplicative factors less than one for interference cancellation have been used in a more complex iterative approach based on maximum likelihood considerations [12, 14], and mentioned in [15] and attributed to the general unreliability of the estimates at early stages.

The contribution of the partial-cancellation factor to mitigating the effect of the bias is twofold: first, it decreases the bias in the mean of the decision statistics, and second, it significantly decreases the variance of the decision statistics after cancellation. The effect of the partial-cancellation factor $C_K^{(s)}$ on the mean of the decision statistics of a two stage of parallel interference canceller is given by

$$E\{Z_{1,i}^{(2)}|b_i^{(1)}\} = N\sqrt{\frac{P_1}{2}}b_i^{(1)}\left[1 - \frac{C_K^{(1)}(K-1)}{3N}\right] \tag{15}$$

The effect of a partial-cancellation factor on the mean and the variance of the decision statistics for a two stage parallel cancellation receiver with $C_K^{(2)} = 0.5$ can be seen in Figures 1 and 2 respectively. An interesting point that can be observed in Figure 2, is that for heavy loads, after interference cancellation, the variance of the new scheme is smaller than the one that would be theoretically obtained with unbiased estimates.

If $Z_{k,i}^{(s)}$ is an unbiased estimate, it has been shown in [13] employing the standard Gaussian approximation, that in an AWGN channel the analytical bit error performance of the multistage parallel cancellation approach at stage s is:

$$P_k^s(E) = Q\left(\left[\frac{1}{2E_b/N_o}\left(\frac{1-\left(\frac{K-1}{3N}\right)^s}{1-\frac{K-1}{3N}}\right) + \frac{1}{(3N)^s} \times \left(\frac{(K-1)^s - (-1)^s}{K}\frac{\sum_j P_j}{P_k} + (-1)^s\right)\right]^{-1/2}\right), \tag{16}$$

where K is the number of users and N is the processing gain. Figure 3 shows a plot of the bit error rate simulation results for the direct implementation of the receiver, together with analytical results assuming unbiased estimation. We can observe the negative effect of the bias by comparing the simulated results with the theoretical ones.

Figure 4 shows the effect of applying a reduced complexity variation of the proposed technique: use a single constant partial-cancellation factor for the first stage of cancellation (in this case we use 0.5), and in the following stages perform complete cancellation. This approach is justified since it can be shown that the bias

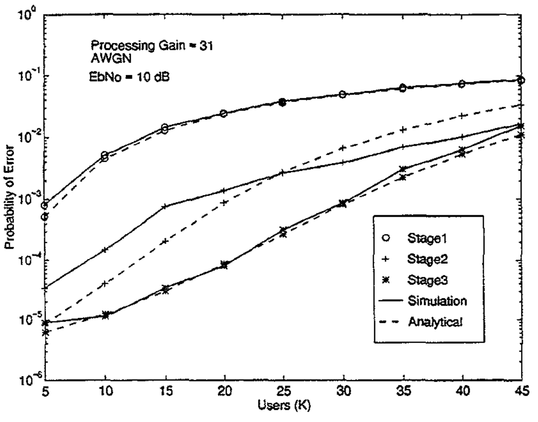

Figure 4: Capacity curves for the proposed approach (partial cancellation) with $C_K^{(2)} = 0.5$ and $C_K^{(3)} = 1$, and, theoretical curves assuming unbiased estimation. (Eb/No = 10dB, and $N = 31$)

varies from stage to stage as $(-K/3N)^s$. Thus in stages $s > 2$, bias reduction is unecessary. Comparison of Figure 4 with Figure 3 shows that the proposed receiver structure results in significant performance improvements over the direct parallel cancellation approach. In fact, the results for the third stage of the proposed receiver are very close to those that would be obtained with unbiased estimation.

Another example of the enhancement in performance offered by the proposed low-complexity technique over the direct implementation can be observed in Figure 5. Both receivers make use of RAKE receivers and maximal ratio combining. The channel corresponds to a 2-ray frequency selective Rayleigh fading channel with parameters taken from measurement data presented in [16], and correspond to one strong main path and a weak second path ($\sigma_1 = 0.93$ and $\sigma_2 = 0.28$). The partial-cancellation factor for the second stage is 0.5 and the third stage performs complete cancellation. We can see from Figure 5 that for these conditions, two stages of the partial-cancellation technique provide better performance than three stages of the direct approach. Also, the simplified partial cancellation technique achieves about one order of magnitude improvement in BER compared to the direct implementation after three stages of interference cancellation.

In many situations the near-far problem can be a limiting factor of the performance of a CDMA receiver. In order to analyze the near-far resistance of the proposed technique, we examined the performance of the receiver for three users in the presence of one interferer whose power varies from 20dB below the power of the desired users to 30 db above. Figure 6 shows that over a wide range of power disparities the proposed scheme shows near-far robustness. It is also interesting to note that even though the near-far resistance of the second stage of the proposed receiever is inferior to that of direct interference cancellation, in stage three the proposed scheme reaps the benefits of bias-mitigation and outperforms the direct approach.

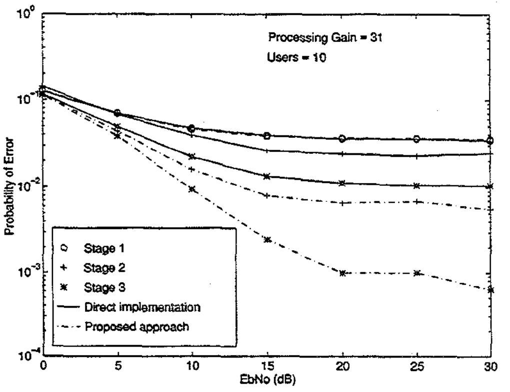

Figure 5: BER vs. E_b/N_o comparison between direct implementation and partial cancellation (with $C_K^{(2)} = 0.5$ and $C_K^{(3)} = 1$) for a 2-ray frequency selective Rayleigh fading channel ($\sigma_1 = 0.93$ and $\sigma_2 = 0.28$, $N = 31$).

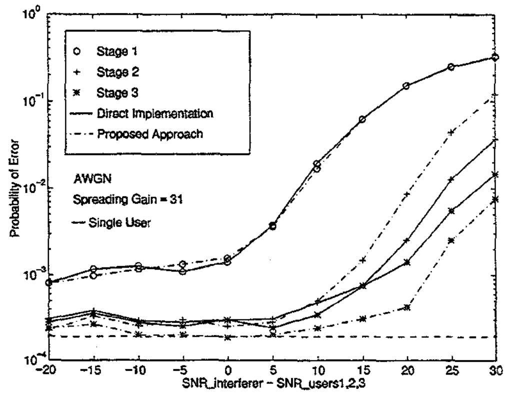

Figure 6: Performance Degradation in Near/Far AWGN Channels (3 users $\overline{E_b/N_o}$=8dB, 1 interferer with varying power. $C_K^{(2)} = 0.75$ and $C_K^{(3)} = 1$, $N = 31$).

Up to this point we have assumed that the receiver structure under study achieves and maintains perfect synchronization during operation. In a more realistic scenario however, this assumption does not hold and the decision statistics are adversely affected by the timing jitter experienced at the receiver. Figure 7 shows the effect of delay estimation errors on the performance of the proposed receiver. The estimation error is assumed to be a Gaussian random variable of zero mean and a given standard deviation measured in fractions of a chip. The simulation results indicate that the proposed receiver structure displays robustness to moderate synchronization and timing errors.

Another attractive feature of the proposed receiver structure is that it lends itself to differentially coherent implementation. In this case, the need for phase estimation and tracking is avoided. Processing is performed on the in-phase and quadrature components of the received signal. Decisions are made on the estimates produced by projecting the current complex-valued decision statistic onto the previous one.

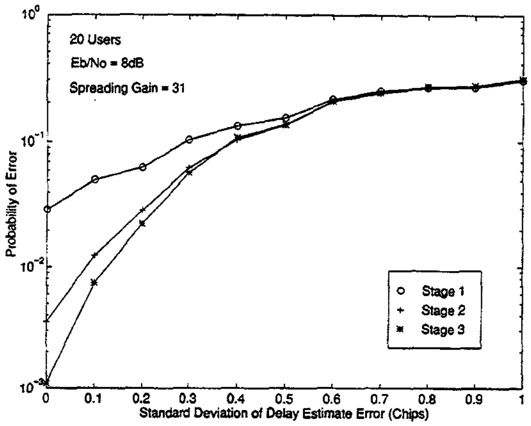

Figure 7: Effects of Delay Estimation Errors on System Performance for AWGN Channels ($C_K^{(2)} = 0.5$, $C_K^{(3)} = 1$ and $N = 31$).

V. CONCLUSIONS

In this paper we have identified a bias in the decision statistics for a direct implementation of the parallel interference cancellation approach. A very simple technique is proposed to mitigate the negative effects produced by the bias. The proposed technique shows near-far robustness and provides significant improvements in CDMA system performance over the direct implementation of parallel interference cancellation.

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