Comments on "Partial Parallel Interference Cancellation for CDMA"

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***Abstract*—In this letter we comment on partial parallel interference cancellation as discussed in the above paper[1] by Divsalar *et al*. The aforementioned work showed that by multiplying symbol estimates by a factor less than unity in early stages of cancellation, the performance of parallel cancellation can be improved relative to full ("brute force") cancellation. In this letter we analyze the improvement of parallel cancellation when using partial cancellation, and provide additional insight into the gains. Specifically, we show that the decision statistic is biased when linear (soft) estimates of the symbol or channel are used for cancellation. Partial cancellation improves performance in this case by reducing the decision statistic bias.**

***Index Terms*— CDMA, multiuser detection, parallel interference cancellations, partial cancellation.**

RECENT work has shown that the performance of parallel interference cancellation in a CDMA system employing BPSK modulation can be improved by performing partial cancellation [1]–[3]. Partial cancellation involves multiplying the symbol estimate of each user by a factor less than unity before attempting cancellation. In this letter we expand upon the results in [1], and show that when linear estimators are used to create symbol and/or channel gain estimates, a bias is introduced in the decision statistic of the next stage. This bias can dominate performance. Partial cancellation reduces this bias and thus improves performance.

In early cancellation stages, the reliability of symbol decisions is worse than at later stages. Since cancelling signals with incorrect symbol estimates will add interference rather than remove it, an intuitive approach is to cancel a fraction of the estimated interference if a symbol estimate is thought to be unreliable, thus reducing the effect of symbol errors. This is termed partial cancellation. The simplest approach is to multiply all symbol estimates by a constant factor less than unity. Since reliability of data and channel estimates will in general improve at later stages of cancellation, the factor at each stage should approach one at the final stage. A more sophisticated approach is to use variable factors based on value of the correlator output. A more thorough discussion of optimizing the partial cancellation factor can be found in [4].

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[1] D. Divsalar, M. K. Simon, and D. Raphelli, *IEEE Trans. Commun.,* vol. 46, pp. 258–268, Feb. 1998.

Results in [1] concentrated on the simple approach using constant factors at each stage. Two symbol estimators were considered—a simple hard limit of the matched filter outputs, and a linear estimate. The linear estimate simply used the correlator output itself (i.e., soft outputs) as the symbol estimate. While perfect channel knowledge was assumed in [1], the second estimate can be viewed as a joint estimate of the channel and symbol assuming perfect carrier/phase recovery. This is a common method of estimating the channel when no pilot or training sequences are available.

Results [1, Figs. 6 and 7] show that partial cancellation provides performance improvements over full cancellation. However, it can be seen that the improvement is dependent on the type of symbol estimate used. When a linear estimate is used [1, Fig. 6], the "brute force" canceller performed very poorly and partial cancellation improved performance dramatically. However, when hard decisions were used for symbol estimates [1, Fig. 7], the performance gains were much more modest.

We ran simulations for the same conditions as in [1] (synchronous BPSK, spreading gain $= 100$) to show BER performance versus loading rather than degradation factor. Figs. 1 and 2 present the simulated BER for an E_b/N_o of 8 dB as the loading increases from 5–95 simultaneous users when using hard and soft symbol estimates, respectively, and assuming perfect carrier recovery and channel estimation. A comparison between Figs. 1 and 2 verifies that partial cancellation is much more crucial to performance when soft symbol estimates are used. In fact, a crossover between full cancellation and the matched filter exists in Fig. 2, unlike Fig. 1. Note that this crossover can also be seen in [1, Fig. 6], while it is absent in [1, Fig. 7].

Fortunately, the use of linear estimates allows straightforward analysis of the decision statistic. Upon examination, we find that in the case of soft decisions, the decision statistic is biased toward the decision boundary after one stage of cancellation. This bias is directly related to the system loading. Additionally, partial cancellation reduces this bias, thus improving performance. This can be explained by recognizing that when correlator outputs are used as symbol (or channel) estimates for cancellation, the estimates of the interference are not uncorrelated with the desired user's bit value. Cancellation thus leads to a reduction in expected value of the desired bit's decision statistic (i.e., a decision statistic which is biased after cancellation). To show this, we consider the output of the matched filter using the notation of [1]

$$Y_1 = \sqrt{\frac{2E_{b_1}}{N_o}}\, a_1 + I_1 + N_1 \tag{1}$$

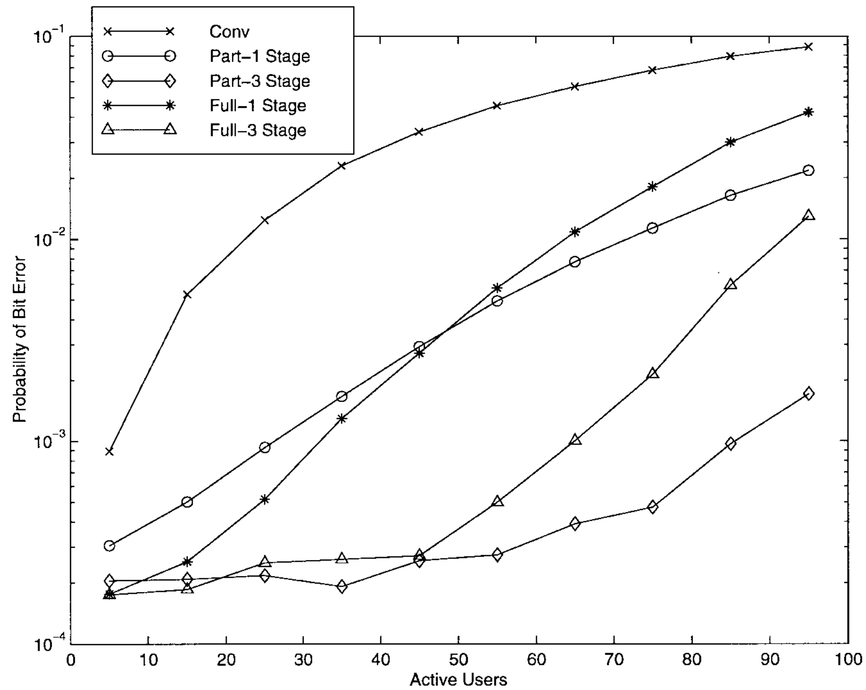

Fig. 1. Probability of bit error for full and partial cancellation versus system loading ($p_1 = 0.6$, $p_2 = 0.8$, $p_3 = 1$) using BPSK in synchronous channel and hard symbol estimates ($E_b/N_o = 8$ dB, spreading gain = 100).

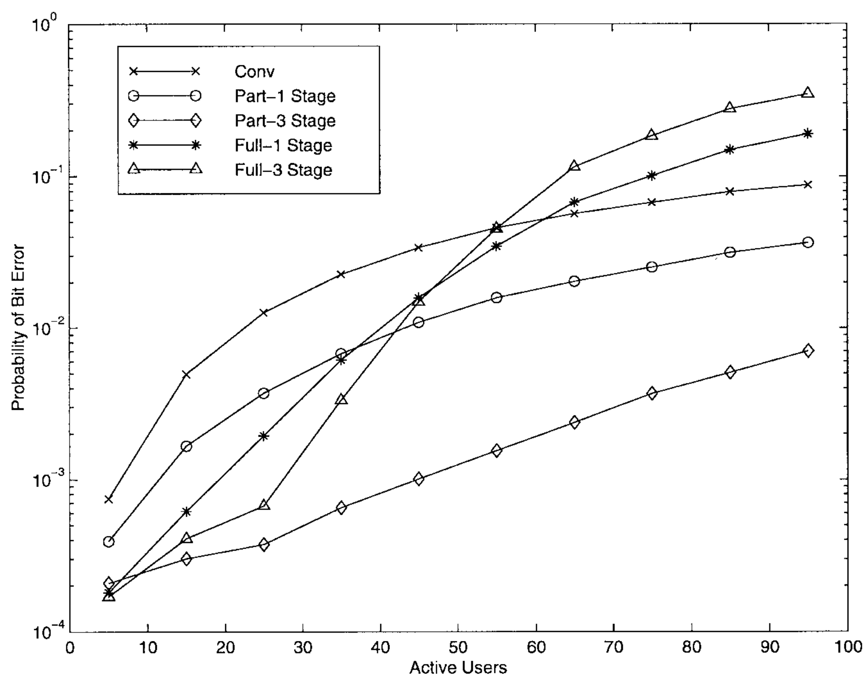

Fig. 2. Probability of bit error for full and partial cancellation versus system loading ($p_1 = 0.3$, $p_2 = 0.8$, $p_3 = 1$) using BPSK in synchronous channel and soft symbol estimates ($E_b/N_o = 8$ dB, spreading gain = 100).

where I_1 and N_1 represent the interference and thermal noise seen in the matched filter output of user 1. Note that Y_1 is not independent of $\{a_k\}$ (the data symbols of the interferers) due to the dependence of I_1 on $\{a_k\}$. Thus when the set $\{Y_k\}$, $2 \leq k \leq K$ is used as estimates of $\{a_k\}$ to cancel interference from Y_1, the expected value of Y_1 after cancellation will be less than $\sqrt{(2E_b/N_o)}a_1$ due to the dependence of each Y_k on a_1. To show this, we take the expected value of $Y_1 - \hat{I}_1$ conditioned on a_1. First recall the definition of $\hat{I}_1$ using $\hat{a}_k = (Y_k/\sqrt{2E_{b_k}/N_o})$:[2]

[2] Please see [1] for definitions of all terms.

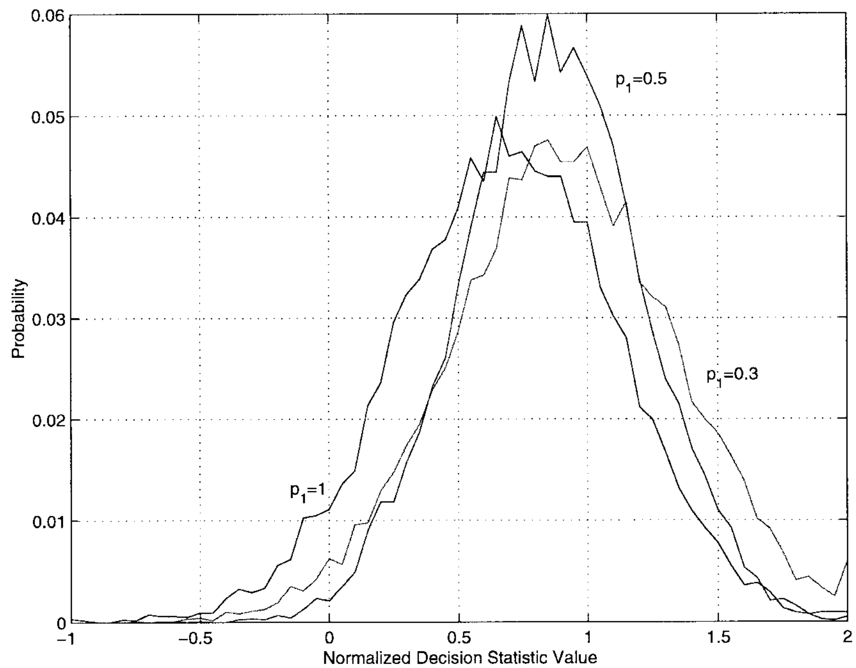

Fig. 3. Probability distribution function of decision statistic after one stage of cancellation using soft symbol estimates for different cancellation factors ($p_1 = 1$, $p_1 = 0.5$, $p_1 = 0.3$) (60 users, spreading gain = 100, synchronous AWGN channel, $E_b/N_o = 8$ dB).

$$\begin{aligned}\hat{I}_1 &= \sum_{k=2}^{K} Y_k \cos(\phi_k - \phi_1)\gamma_{1,k} \\ &= \sum_{k=2}^{K} \sqrt{\frac{2E_{b_k}}{N_o}}\, a_k \cos(\phi_k - \phi_1)\gamma_{1,k} + \sum_{k=2}^{K} \\ &\quad \cdot \left[\sum_{i\neq k} \sqrt{\frac{2E_{b_i}}{N_o}}\, a_i \gamma_{k,i} \cos(\phi_i - \phi_k)\right] \cos(\phi_k - \phi_1)\gamma_{1k} \\ &\quad + \sum_{k=2}^{K} N_i \cos(\phi_k - \phi_1)\gamma_{1k}. \end{aligned} \tag{2}$$

Thus the expected value of $Y_1 - \hat{I}_1$ conditioned on a_1 is

$$\begin{aligned} &E[Y_1 - \hat{I}_1 | a_1] \\ &= E\Bigg[\sqrt{\frac{2E_{b_k}}{N_o}}\, a_1 + \sum_{k=2}^{K} \sqrt{\frac{2E_{b_k}}{N_o}}\, a_k \cos(\phi_k - \phi_1)\gamma_{1,k} \\ &\qquad + N_1 - \sum_{k=2}^{K} \sqrt{\frac{2E_{b_k}}{N_o}}\, a_k \cos(\phi_k - \phi_1)\gamma_{1,k} \\ &\qquad - \sum_{k=2}^{K} \sum_{i\neq k} \sqrt{\frac{2E_{b_i}}{N_o}}\, a_i \cos(\phi_i - \phi_k) \\ &\qquad \cdot \cos(\phi_k - \phi_1)\gamma_{k,i}\gamma_{1,k} \\ &\qquad - \sum_{k=2}^{K} N_i \cos(\phi_k - \phi_1)\gamma_{k,i}\gamma_{1,k} \Bigg| a_1\Bigg] \\ &= E\Bigg[\sqrt{\frac{2E_{b_k}}{N_o}}\, a_1 - \sum_{k=2}^{K} \sum_{i\neq k} \sqrt{\frac{2E_{b_i}}{N_o}}\, a_i \cos(\phi_i - \phi_k) \\ &\qquad \cdot \cos(\phi_k - \phi_1)\gamma_{k,i}\gamma_{1,k} + \tilde{N}_1 \Bigg| a_1\Bigg] \\ &= E\Bigg[\sqrt{\frac{2E_{b_k}}{N_o}}\, a_1 \\ &\qquad - \sum_{k=2}^{K} \Bigg(\sqrt{\frac{2E_{b_k}}{N_o}} a_1 \gamma_{k1}\gamma_{1k} \cos^2(\phi_k - \phi_1) \\ &\qquad - \sum_{\substack{i\neq k \\ i\neq 1}} \sqrt{\frac{2E_{b_i}}{N_o}}\, a_i \cos(\phi_i - \phi_k) \\ &\qquad \cdot \cos(\phi_k - \phi_1)\gamma_{k,i}\gamma_{1,k} + \tilde{N}_1\Bigg)\Bigg| a_1\Bigg] \\ &= \sqrt{\frac{2E_{b_k}}{N_o}}\, a_1 - \sqrt{\frac{2E_{b_k}}{N_o}}\, a_1 \sum_{k=2}^{K} E[\gamma_{1k}^2 \cos^2(\phi_k - \phi_1)] \\ &= \sqrt{\frac{2E_{b_k}}{N_o}}\, a_1 \left(1 - \frac{K-1}{2N}\right) \end{aligned} \tag{3}$$

where, for synchronous transmission, $\gamma_{k,1} = \gamma_{1,k}$ and $\mathrm{var}(\gamma_{1,k}) = (1/N)$ [5], and we have defined $\tilde{N}_1 = N_1 - \sum_{k=2}^{K} N_i \cos(\phi_k - \phi_1)\gamma_{k,i}\gamma_{1,k}$. This reduction in the mean of the decision statistic can be devastating to performance as seen in Fig. 2. Fig. 3 plots the normalized histogram of the decision statistic after one stage of cancellation when using soft symbol estimates with partial cancellation factors of $p_1 = 1$ (full cancellation), $p_1 = 0.5$, and $p_1 = 0.3$, and normalizing by $\sqrt{(2E_{b_k}/N_o)}a_1$. This histogram was taken from a simulation of 60 simultaneous, synchronous BPSK DS-CDMA signals operating in AWGN with $(E_b/N_o) = 8$ dB and a spreading gain of 100. Full cancellation results in a large bias in the statistic $(\overline{Y_1 - \hat{I}_1} \approx 0.7)$. If we include a

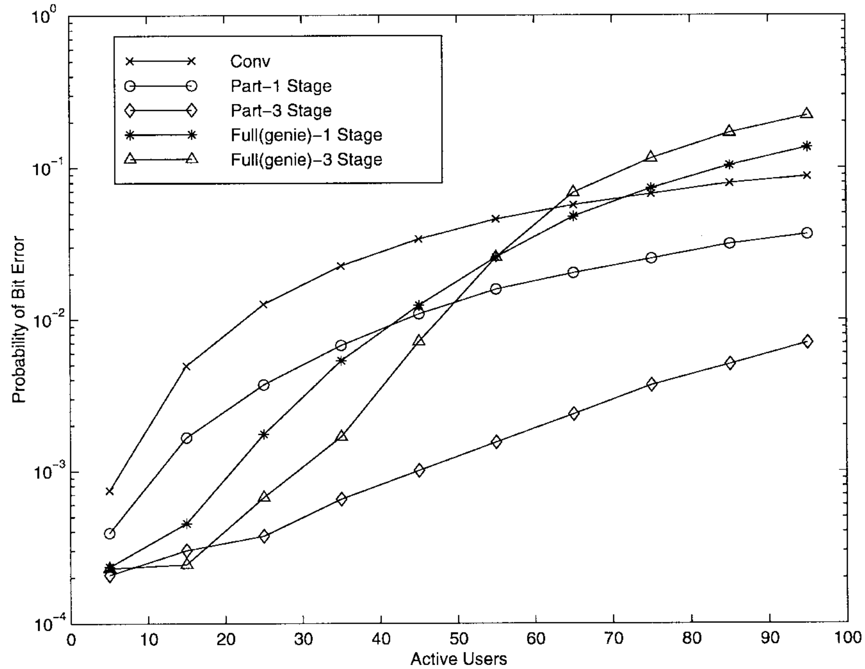

Fig. 4. Probability of bit error for full cancellation (using Genie symbol estimates for cancellation) and partial cancellation (soft symbols estimates and $p_1 = 0.3$, $p_2 = 0.8$, $p_3 = 1$) versus system loading using BPSK in synchronous channel and soft symbol estimates ($E_b/N_o = 8$ dB, spreading gain = 100).

cancellation factor p_1 in the preceding derivation, we find that the bias is reduced to $\sqrt{(2E_{b_k}/N_o)}a_1(1-p_1(K-1)/(2N))$. Using a partial cancellation of 0.3 nearly eliminates the bias $(\overline{Y_1 - \hat{I}_1} \approx 0.9)$, but does not significantly reduce the variance since little interference is removed. Using a factor of 0.5 both improves the variance and reduces the bias $(\overline{Y_1 - \hat{I}_1} \approx 0.85)$, providing the best performance in this situation. This improved performance is then passed to later stages, as demonstrated in the results of Fig. 2. The specific partial cancellation value which optimizes performance is dependent on loading as discussed in [1] as well as [4].

To further demonstrate the fact that this bias dominates performance when soft estimates are used, we also ran simulations using a Genie canceller which used perfect bit knowledge in cancellation (unfortunately, the Genie was unwilling to pass the information on to the detector). In other words, we removed the effect of symbol errors on cancellation but maintained the soft weighting. This can be viewed as perfect bit estimation with correlator outputs used for channel gain estimation. Fig. 4 plots the simulated BER performance versus system loading (5–95 simultaneous users) for such a receiver in a synchronous AWGN channel with spreading gain 100 and $E_b/N_o = 8$ dB. The partial cancellation receiver used the cancellation factors $p_1 = 0.3$, $p_2 = 0.8$, and $p_3 = 1$.[3] We can see that even with perfect symbol estimates for cancellation, the bias induced by the soft correlator output severely degrades performance. In fact, very little performance improvement is seen in Fig. 4 versus Fig. 2.

As a final note, (3) indicates that when using a single symbol soft estimate in a synchronous channel without partial cancellation, interference cancellation is limited to $2N+1$ users.[4] For $K > 2N+1$, the expected value of the decision statistic is actually inverted from the data, thus causing a BER of 0.5. This limit has also been observed in simulation.

In this letter we have analyzed the improvement of partial parallel cancellation over full cancellation in CDMA systems. Specifically we have shown that when soft correlator outputs are used for channel or symbol estimates for cancellation (which is often unavoidable), a bias is introduced in the decision statistic which will dominate performance at moderate to high loading. Partial interference cancellation is beneficial because it reduces this bias. For further discussion of this topic, please see [2] and [3].

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[3] Partial cancellation factors were taken from [1] for comparison purposes.

[4] In an asynchronous channel, the limit would be $3N+1$ users. This is due to the fact that the normalized cross-correlation between random spreading codes in a synchronous system with random phases is $1/2N$, while it is $1/3N$ in an asynchronous system [5], [6].