Chapter 2: Modeling and Problem Formalization

"If we can really understand the problem, the answer will come out of it, because the answer is not separate from the problem." - Jiddu Krishnamurti

Before proceeding to develop solution techniques for cognitive radios interactions, we must first define what we wish to solve. This chapter presents a general model of the interactions of cognitive radios applicable to both procedural and ontological radios and refines the aspects of their behavior that we wish to be able to analyze.

2.1 A General Model of Cognitive Radio Interactions

Consider the interactive cognitive radio problem previously illustrated in Figure 1.18 and repeated in Figure 2.1. In this interaction problem, each radio reacts to observations of the outside world by choosing some adaptation (or waveform) that the radio believes will help bring it closer to its goal, whatever that goal may be. At any given point in time, the observation a cognitive radio makes will be a function of the *passive operating environment* of the network (the channel conditions and interference environment that would be observed if no cognitive radios were operating in the environment) and the decision processes of the cognitive radios – decision processes that may be implemented via procedures or via a reasoning engine. Regardless of the implementation of the decision process, by definition, the cognitive radios are guided in their adaptations by some goal.

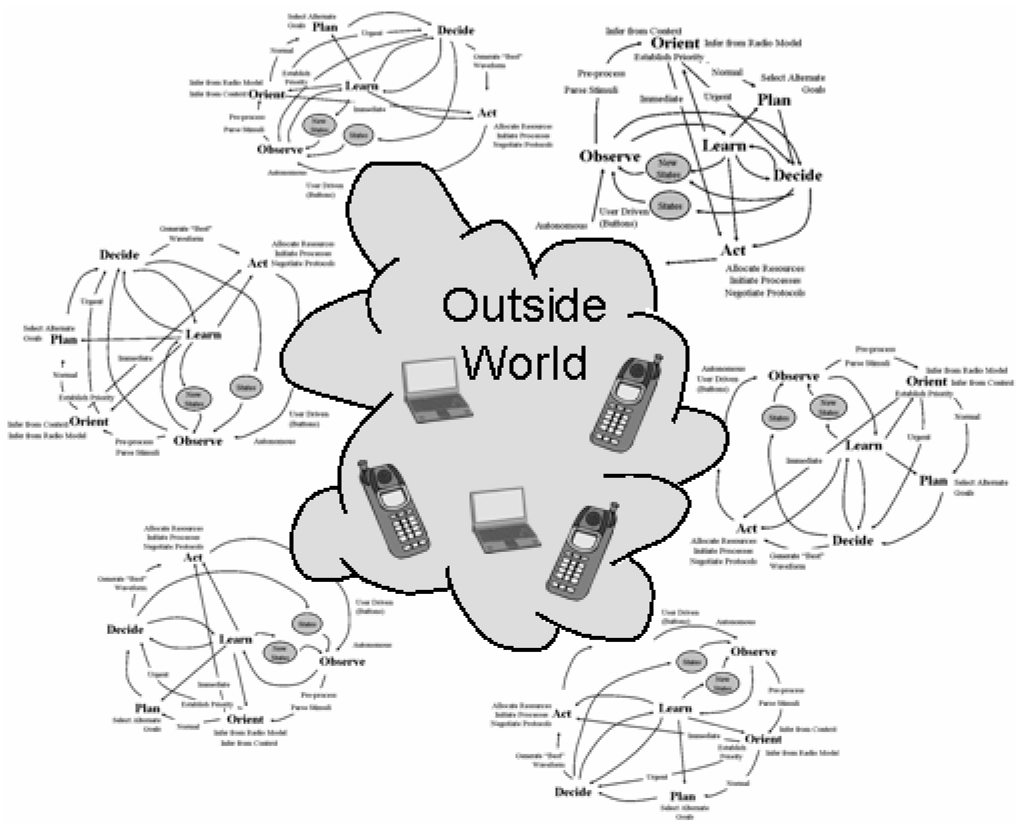

Figure 2.1: The interactive cognitive radio problem. [Neel06]

While application specific variables and models are introduced as needed later in this document, the following presents a collection of symbols and conventions that captures the general features of cognitive radio interaction and can be fashioned into a usable model of cognitive radio interactions.

- N – The (finite) set of cognitive radios in the network where n is the number of elements in N, $|N|$.

- i, j – Particular devices in N.

- A_j – The set of actions available to radio j. While these sets are quite limited for many radios, they include all available adaptations to the radio. As the adaptations can include a number of independent types of adaptations, e.g., power levels, modulations, channel and source coding schemes, encryption algorithms, MAC algorithms, center frequencies, bandwidths, and routing algorithms, A_j will generally be a multidimensional set.

To simplify matters, we assume we are analyzing adaptations only over a short time interval so the A_j will not be a function of time, i.e., the radios are not learning new actions while they are adapting. This is consistent with the earlier discussion that cognitive radios' learning processes are expected to be performed during sleep or prayer processes [Mitola_00]. However, if we consider time scales that spanned these sleep or prayer processes, A_j could be expected to grow as radio j learns new waveforms.

- A – The *action space*, i.e., set of all possible combinations of actions by the radios in the network. Throughout, we assume that A is formed by the Cartesian product of each radio's action sets, i.e., $A = A_1 \times A_2 \times \cdots \times A_n$. For some algorithms, it is convenient to think of A as a vector space with orthogonal bases A_1 through A_n.
- a – A particular combination of actions where each radio in N has implemented a particular action (waveform), i.e., a point in A or an *action vector*. Radio j's contribution to a is written as a_j, and the choice of actions by all cognitive radios other than j is written as a_{-j}.
- O – The set of all possible *observed outcomes* of the outside world as determined by the choice of actions available to each cognitive radio and the passive operating environment.
- o_j – An observation made by or supplied to radio j. For instance, an SINR measurement.
- o – A vector of observed outcomes where all radios have observed an outcome. For instance, o may represent a vector of SINR measurements with each measurement associated with a particular cognitive radio. Frequently, we refer to this as an *outcome.*
- d_j – The *decision rule* which describes how radio j updates its decisions based on observations.

Strictly, d_j is a function that relates actions to outcomes, i.e., $d_j : o_j \to A_j$. However, while the observed outcome may only be statistically related to the action vector, we will assume for the purposes of analysis that the relationship between actions and observed outcomes is known and treat each decision rule as a function that relates action vectors to actions, i.e., $d_j : A \rightarrow A_j$.

For procedural cognitive radios, the decision rule may be explicitly given; for ontological cognitive radios, we may have to make broad generalizations such as the implemented decision rule selects a *locally optimal* action or the radio behaves *selfishly*. A more formal treatment of decision rules for ontological radios is presented in Chapter 4. However, throughout this report, we assume that each radio's decision rule is guided by its goal or *utility function*.

- $u_j(a)$ – The *utility function* which describes how much value radio j assigns to action vector a. Throughout this report we assume these values or utilities are described using real numbers, i.e., $u_j : A \rightarrow \mathbb{R}$. In general, the utility function expresses some goal that the radio is working towards whether explicitly (ontological cognitive radio) or implicitly (procedural cognitive radio).

Again, a practical implementation of a cognitive radio's goal would associate numbers with the radio's observed outcomes, o_j, and not the action vector as other radios' actions will not generally be directly observable. However, for purposes of analysis we assume that the analyst knows the relation between actions and observed outcomes so that the analyst can express the utility function in terms of the action vector. Therefore, for analysis purposes, these utility functions capture the actions of the cognitive radios and the passive operating environment.

Although elided in the introduction to this section, the exact times at which radios make their decisions can significantly influence the behavior of a network. In military circles, there is much effort placed on getting inside the enemy's decision loop because of the potential advantages gained by the quicker decision maker. Or in a more mundane circumstance, anyone who has met someone head on in a hallway and proceeded to repeatedly block each others' attempts to pass knows the effects that there is a significant difference between synchronous and asynchronous decision timings. To capture the cognitive radio equivalent of these conditions, our model requires addition of the following symbols and conventions.

- T_j – The times at which radio j can update its decision (a radio may have a time allocated for updating, but choose not to update its decision). Unless stated otherwise, we assume that each T_j is infinite, i.e., $T_j = \{t_j^0, t_j^1, \ldots, t_j^m, \ldots\}$. As we are ultimately modeling interactive software processes, we always assume that T_j is a discrete set.
- T – The set of all times where decision updates can occur, i.e., $T = T_1 \cup T_2 \cup \cdots \cup T_n$, where $t \in T$ denotes a particular updating time. For convenience, we treat t^k as the k^{th} element of T arranged chronologically.

Further, when appropriate, we also use the notation d^t to denote the *network decision rule* at time t where in general d^t captures the adaptations of the subset of radios that update their decisions at time t, i.e., $d^t = \times_{k \in M} d_k$, $M \subset N$. While it is also possible that a radio bases its decisions on past observations and predictions about the future state of the network, this text assumes that d_j^t is only a function of cognitive radio j's most recent observation.

Additionally, we make use of the following terms in describing the timing of the decision update process: *synchronous decision processes*, *round-robin decision processes*, *random decision processes*, and *asynchronous decision processes*.

Definition 2.1: *Synchronous decision process*
If $\forall t \in T$, $d^t = \times_{k \in N} d_k$, then we say that the network has a *synchronous decision process* and write $a^{t_{k+1}} = d^{t_k}\left(a^{t_k}\right)$.

Definition 2.2: *Round-robin decision process*
If $t_1^m < t_2^m < t_3^m < \cdots < t_n^m < t_1^{m+1}$, then we say that the network is updating its decisions in *round-robin* order.

Definition 2.3: *Random decision process*
If $\forall t \in T \;\; d^t = \underset{k \in N}{rand}\{d_k\}$, then we say that the network is updating its decisions in *random* order

Definition 2.4: *Asynchronous set decision process*
If $\forall t \in T \;\; d^t = \underset{k \in 2^N}{rand}\{d_k\}$, then we say that the network is updating its decisions in *random* order

For asynchronously updating networks, there may be some points in time where $t_i^m = t_j^k$ (the m^{th} update of radio i occurs at the same time as the k^{th} update of radio j) is satisfied for two or more radios. As we will see in subsequent sections, these different decision update timings – synchronous, round robin, random, and asynchronous – can have a significant impact on the analysis of our cognitive radio network.

Systems with synchronous timings are most frequently encountered in centralized systems and thus will be rarely encountered in an interactive cognitive radio decision process as an interactive decision process implies some degree of distributed decision timings. A round-robin scheme can occur in centralized systems with distributed decision making with scheduling (as might occur in a hybrid ARQ scheme). Without a synchronizing agent and assuming an arbitrary fineness in the time scale, every distributed cognitive radio algorithm will be a randomly decision process. However, because real-world observations necessitate processing data collected over non-infinitesimal intervals and because of signal propagation delays, a system with random timings will behave more like an asynchronous system.

Summarizing this discussion, the basic model of cognitive radio interaction consists of a collection of a collection of cognitive radios, N, an action space A, a utility function for each cognitive radio $j \in N$ which is a function of the actions of each radio and the passive operating environment, a decision rule for each cognitive radio, and a set of times at which these decisions occur. This can be compactly represented as the 5-tuple shown in (2.1).

$$\left\langle N, A, \left\{u_j\right\}, \left\{d_j\right\}, T \right\rangle \quad (2.1)$$

The following chapters illustrate how this model can be applied to networks of procedural and ontological radios. For procedural radios, we place increased modeling emphasis on the decision rules; for ontological radios, we place increased emphasis on the radios' goals. If we ignore the mapping from actions to outcomes, our model is implementation independent, though not particularly useful for analysis. With the mapping from actions to outcomes in place, our model is implementation specific – useful for analysis, though difficult to generalize.

Example 2.1: Example: Modeling a Cognitive Radio Algorithm

Consider two cognitive radios, {1,2}, with actions (waveforms) $\left\{w_{1_a}, w_{1_b}\right\}$ and $\left\{w_{2_a}, w_{2_b}\right\}$, respectively, that are communicating with a common receiver which reports to each cognitive radio that radio's *signal-to-interference ratio* (SIR). In this scenario, there are four different possible elements in *A*, which form the set $\left\{\left(w_{1_a}, w_{2_a}\right), \left(w_{1_a}, w_{2_b}\right), \left(w_{1_b}, w_{2_a}\right), \left(w_{1_b}, w_{2_b}\right)\right\}$. However, there are an infinite number of possible observations due to the infinite number of passive operating environments.

In this case, the passive operating environment is defined by the gains from each cognitive radio to the common receiver, g_1 and g_2. We'll consider the interference that one waveform induces on the other to be given by the absolute value of the correlation of their signal space representations, $\left|r\left(w_j, w_{-j}\right)\right|$ where w_j is the waveform chosen by radio $j \in \{1,2\}$ and w_{-j} is the waveform chosen by the other radio. In such a system, the observed outcome for each radio *j* is given by the SIR equation given in (2.2) where $j \in \{1,2\}$, g_j is the observed SIR for radio *j*, g_j is the link budget gain of radio *j* to the common receiver, and g_{-j} is the gain of the other radio to the common receiver.

$$o_j = g_j = \frac{g_j}{g_{-j}\left|r\left(w_j, w_{-j}\right)\right|} \tag{2.2}$$

A reasonable goal or a utility function for a cognitive radio operating in this system would be to maximize (2.2) so that the greater the SIR the radio achieves, the higher the value the radio assigns to the outcome. Note that this goal incorporates both the relevant information from the passive operating environment (in this case, the link gains), the potential actions that could be taken by the radios, and the interactive nature of those actions.

Particularly as each radio only has two waveforms to choose from, it seems reasonable to assume that whether procedurally or ontologically each radio implements a locally optimal decision rule or more formally as given in (2.3).

$$d_j(a) = \underset{w_j \in \{w_{ja}, w_{jb}\}}{\operatorname{argmax}} \frac{g_j}{g_{-j}\left|r\left(w_j, w_{-j}\right)\right|} \tag{2.3}$$

Finally, by controlling when observations are returned to the cognitive radios, the common receiver could conceivably enforce any decision timing scheme. However, this example will assume that adaptations occur in a round robin fashion with one adaptation permitted each half second, e.g.., $T_1 = n$ sec and $T_2 = n + 0.5$ sec where $n \in \mathbb{N}$. Based on this discussion, these various modeling parameters can be compactly summarized as shown in Table 2.1.

Table 2.1: Parameters for Example Model

General Model Symbols	Modeled System Parameters
N (cognitive radio set)	{1,2}
A (action space)	$\left\{(w_{1_a}, w_{2_a}), (w_{1_a}, w_{2_b}), (w_{1_b}, w_{2_a}), (w_{1_b}, w_{2_b})\right\}$
$\{u_j\}$ (utility functions)	$u_j(a) = \dfrac{g_j}{g_{-j}\left\lvert r(w_j, w_{-j})\right\rvert}$
$\{d_j\}$ (decision rules)	$d_j(a) = \underset{w_j \in \{w_{ja}, w_{jb}\}}{\operatorname{argmax}}\ u_j(a)$
T_j (decision timings)	$T_2 - 0.5\text{s} = T_1 = \mathbb{N}$

2.2 Analysis Objectives

By using these modeling parameters and considering another example of cognitive radio interaction, we can begin to formalize our analysis objectives. Consider a network of three radios where each radio, $j \in \{1,2,3\}$, can choose actions from a convex action set, A_j, according to its decision update rule d_j. Starting at any initial action vector, the repeated application of the decision update rules trace out *paths* in the action space.

Definition 2.5: *Path*
A path is a sequence of action vectors, $\{a^{t_k}\}$ formed by the recursion $a^{t_{k+1}} = d^{t_k}(a^{t_k})$.

Note that even if the same network decision rule and the same passive operating environment are used, different paths result from different initial points, a^0.

Conceptually, a path may terminate in a stable point, but under different conditions a path may enter an infinite loop. There may also be points in the action space that are fixed points of the decision update rule but are unstable so that any small perturbation in initial conditions drives the network away from the point. Each of these concepts is illustrated in the example interaction diagram shown in Figure 2.2 where paths are shown by the arrows and fixed points are labeled as “NE” for reasons that will become clear in Chapter 4.

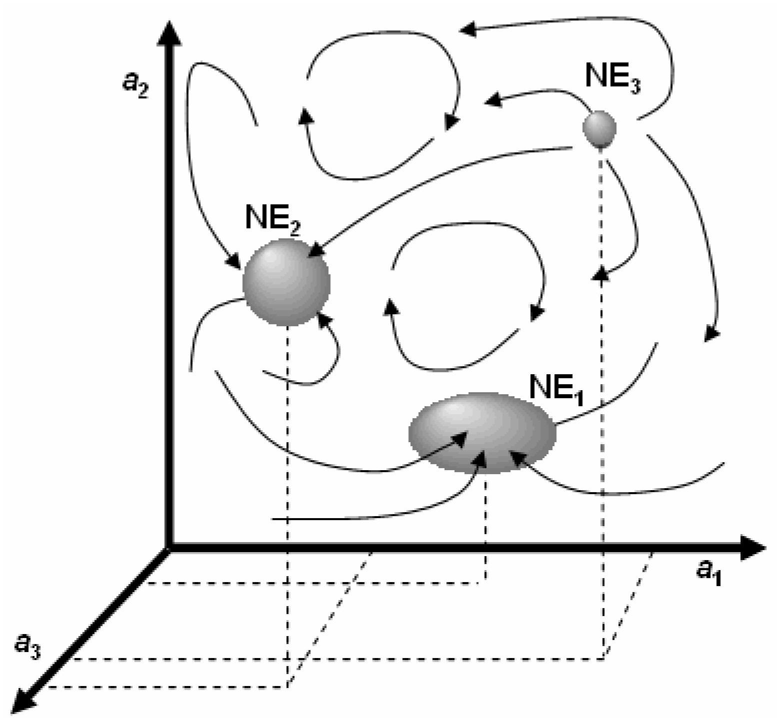

Figure 2.2: A three radio interaction diagram with three steady states (NE_1, NE_2, and NE3) and adaptation paths.

This conceptual interaction diagram illustrates the four different analysis questions we identified in Chapter 1 that we would like to answer when analyzing the interactions of a network of cognitive radios.

1. What is the expected behavior of the network?
2. Does this behavior yield desirable performance?
3. What conditions must be satisfied to ensure that adaptations converge to this behavior?
4. Is the network stable?

The following formalizes the analysis objectives underlying each of these questions and previews some of the techniques introduced later in this text.

2.2.1 Establishing Expected Behavior

As is the case for many systems, the analysis in this report assumes that expected behavior of a cognitive radio network is equivalent to its steady-state behavior. Accordingly, establishing expected behavior is concerned with addressing the following issues:

- *Existence* – Does the system have a steady state?
- *Identification* – What are the specific steady states for the system?

In general, we will consider an action vector, $a^* \in A$, to be a steady state for a network if it is a fixed point of the decision rule, a condition that is expressed more formally in Definition 2.6.

Definition 2.6: *Steady state*
An action vector a^* is a steady state for the cognitive radio network $\langle N, A, \{u_j\}, \{d_j\}, T \rangle$ if there is some $t^* \in T$ such that for all $t \geq t^*$, $d^t\left(a^*\right) = a^*$.

Subsequent chapters will introduce a number of different techniques for demonstrating that a steady state exists and for identifying the steady states of the network. These include showing that the network decision rule is a variant of a *contraction mapping*, that the network can be modeled as an *absorbing Markov chain*, and that the network obeys certain game theoretic properties.[1]

2.2.2 Desirability of Expected Behavior

Of course, determining a cognitive radio network's steady states tells us nothing about whether or not we should implement the algorithm under study. We also need to address whether or not those steady states are "good" steady states of "bad" steady states and if there are other action vectors that would be preferable from a network designer's perspective. Again, there are two specific issues that we would like to address:

- *Desirability* – How "good" are the steady states of the algorithm?
- *Optimality* – Does an optimal action vector exist and how close do the steady states come to achieving optimal performance?

There are many different ways of identifying whether or not an action vector is a "good" steady state, but we will make the assumption that the network designer has some objective function, $J : A \rightarrow \mathbb{R}$ that he/she wishes to maximize or minimize (perhaps total system goodput or spectrum utilization). Assuming we wish to maximize J, we'll treat

[1] Contraction mapping and absorbing Markov chain are defined in Chapter 3. The associated game theoretic techniques are defined in Chapters 4 and following.

action vector a^2 as more desirable than a^1 if $J\left(a^2\right) > J\left(a^1\right)$. To determine if an optimal action vector exists and if our steady states are indeed optimal, subsequent chapters will introduce gradient techniques and *Pareto optimality* criteria.

2.2.3 Convergence Conditions

Even if we demonstrate that a cognitive radio network has desirable steady states, it is important to identify the conditions (decision rules, passive operating environments, initial conditions) under which paths *converge*, a concept formalized in Definition 2.7.

<u>Definition 2.7</u>: *Convergence*
Given path $\left\{a^{t_k}\right\}$, we say that the path *converges* to some action vector $a^* \in A$ if for every $e > 0$, there is a $t^* \in T$ such that $t \geq t^*$ implies $\left\| a^{t^*}, a^* \right\| < e$.

In other words, path $\left\{a^{t_k}\right\}$ converges to a^* if for every arbitrarily small region around a^* that we might define, there is a time after which $\left\{a^{t_k}\right\}$ remains "trapped" in that region.

For convergence, this research addresses the following issues:

- *Rate* – Given $\left\{a^{t_k}\right\}$ and $e > 0$, what is the value of t^* such that the path converges?
- *Sensitivity* – How do changes in the value of a^0, slight variations in d^t (perhaps asynchronous instead of round-robin timings) or changes in the passive operating environment impact the paths and the network's steady states?

Frequently when assessing convergence this text considers a time-independent decision rule, *d*, coupled with varying timings for implementing decision rule. For example, this text considers time-independent decision rules corresponding to locally optimal decisions, directional improvement, and randomly selected better responses coupled with synchronous, asynchronous, random, or round robin decision timings. This approach allows us to establish the sensitivity of the decision rules to timing variations more precisely.

2.2.4 Network Stability

Implicitly, the preceding analysis objectives assume the radios have perfect knowledge of their operating environment and behave deterministically. However, wireless networks are stochastic, not deterministic. Accordingly, the cognitive radios' observations will not be the deterministic functions and instead will be estimates of their operating environment. Because these are only estimates, the radios will frequently make adaptations that appear to be mistakes to the analyst. While this research assumes the radios' estimates and errors are unbiased, there is the concern of stability as small perturbations could potentially lead to undesirable behavior. Because of this concern, this research addresses the following analytical issues with respect to a network decision rule's steady state(s):

- *Lyapunov stability* – After a small perturbation, will stay the system within a bounded region about the steady state?
- *Attractivity* – After a small perturbation, will the network converge back to the steady state?

2.3 Summary

This chapter has presented a generalized model of cognitive radio interactions and identified important analysis objectives. This model is defined by the tuple $\langle N, A, \{u_i\}, \{d_i\}, T \rangle$ where the associated symbols are summarized in Table 2.2. Subsequent chapters provide application-specific refinements of this model and introduce techniques for determining steady states, desirability of those steady states, convergence criteria, and stability.

Table 2.2: Symbol Summary

Symbol	Meaning	Symbol	Meaning
N	Set of cognitive radios	i, j	Particular cognitive radios
A_j	Adaptations for j	a_j	Adaptation chosen by j
a_{-j}	Adaptation vector excluding a_j	u_j	Goal of j
O	Set of outcomes	O_j	Outcome observed by j
d_j	Decision rule for j	T_j	Times when j adapts
T	Adaptation times $\forall j \in N$	t	An element of T

In general we will seek to design cognitive radio algorithms such that all of their steady-states maximize the design objective for the particular application, are converged to and are stable under the broadest possible conditions, and require a minimal amount of signaling overhead and device resources to realize the algorithm. While this seems to be an impossible order to fill, by leveraging the analysis insights of the subsequent chapters, Chapters 6, 7, and 8 present several algorithms that meet all of these objectives.

2.4 References

[Mitola_00] J. Mitola III, "Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio," Ph.D. Dissertation Royal Institute of Technology, Stockholm, Sweden, May 2000.

[Neel_06] J. Neel. J. Reed, A. MacKenzie, "Cognitive Radio Network Performance Analysis" in **Cognitive Radio Technology**, B. Fette, ed., Elsevier August 11, 2006.