A Game Theory Perspective on Interference Avoidance

James E. Hicks, Allen B. MacKenzie, James A. Neel, Jeffrey H. Reed
Mobile and Portable Radio Research Group
Virginia Polytechnic Institute and State University
432 Durham Hall, Blacksburg VA, 24060
{jahicks,mackenab,janeel,reedjh}@vt.edu

Abstract – We show that the fixed power, synchronous Interference Avoidance (IA) scheme of [3] employing the (greedy) eigen-iteration can be modeled as the recently developed *potential game* of [10]. Motivated by the fact that receivers can make small mistakes, we consider the convergence of the eigen-iteration when noise is added in a manner similar to [2]. Further, we restrict ourselves to a class of signal environments that we call *levelable environments*. Applying game-theory, we obtain a convergence result similar to that of [2] for *levelable environments*: arbitrarily small noise assures that the eigen-iteration almost surely converges to a neighborhood of the optimum signature set.

I. INTRODUCTION

An Interference Avoidance (IA) scheme is a distributed waveform adaptation mechanism by which multi-access interference can be reduced in a wireless communication system. The earliest IA scheme was presented in [1], which considers the uplink of a fixed power, single cell, synchronous CDMA system whose transmitters employ fully adaptive code-on-pulse signatures. User's receivers take turns sensing their signal environment, calculating improved signatures using a prescribed *MMSE-update* and the updated signatures are sent back to their respective receivers. It was later recognized by many authors that this IA scheme can easily be extended to any peer-to-peer multi-access communication system in which receivers are co-located. The surprisingly fast convergence results experimentally found in [1] have motivated more detailed investigations. Most notably, in [2], it was shown that arbitrarily small perturbations of the MMSE-*update* guarantee almost sure convergence to a neighborhood of the optimum code set. In [3], an alternative update scheme, the (greedy) *eigen-iteration* is considered and, in order to assure convergence, an intuitively appealing variation dubbed *class-warfare* is introduced in [4]; however, this scheme requires some coordination among receivers. Other progress has been made, including an extension to asynchronous channels [5], frequency selective channels [6,7], and multi-carrier systems [8]; improved feedback channels [7,9]; an excellent overview of existing work [18]; and an extension to multi-cell systems [20]. In this paper, we revisit the synchronous IA problem in light of Game Theory with the hopes of paving the way for IA's application to a broader range of wireless systems. We show that the various forms of [1]'s IA scheme are all examples of the recently emergent *potential game* [10-12], a game in which players can serve the greater good by serving their own best interests. We also show that the (negated) generalized total squared correlation function developed in [4] is a potential function for a large class of IA schemes. In the process, we clarify to what extent the convergence result in [2] applies to updates other than the MMSE-update, including the eigen-iteration. Our work differs from [20], which considers the *existence* of Nash Equilibria in multi-cell systems employing IA.

II. POTENTIAL GAME THEORY

Consider a normal form game [13] expressed as the following tuple

$$\Gamma = \left\langle N, \{A_i\}_{i\in N}, \{u_i\}_{i\in N} \right\rangle, \tag{1}$$

where $N = \{1,\ldots,|N|\}$ is the set of players. For a given player $i \in N$, A_i, is the set of actions available, and u_i is its utility function. If $A = \times_{i\in N} A_i$ then $u_i : A \to \mathbb{R}$. Player $i \in N$ prefers $x \in A$ over $x' \in A$ if $u_i(x) \ge u_i(x')$. We will call an element, $x \in A$ an *action profile*. We assume that A_i is a compact metric space, which implies that A is a compact metric space when endowed with the product metric. In this paper we will adopt the following notation. $A_{-i} = \times_{j\neq i} A_j$, $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{|N|})$. Given $x_0 \in A$, $N_d(x_0) = \{x \in A : d(x, x_0) < d\}$ is *a neighborhood of* x_0. $N_e^*(x) = N_e(x) - \{x\}$ is the so called *deleted neighborhood*. Given $B \subseteq A$, $\bar{B}$ denotes its closure, and $P(B)$ denote its *power set* (i.e. set of all subsets). Given $V : A \to \mathbb{R}$, $V(B) = \{V(x) : x \in B\}$. Finally, for a matrix $\mathbf{R} \in \mathbb{C}^{m\times n}$, $\mathbf{R}^\dagger$ denotes the Penrose pseudo-inverse and $\mathbf{P}_\perp(\mathbf{R}) = \mathbf{I} - \mathbf{R}\mathbf{R}^\dagger$ is the orthogonal projection matrix onto $Range(\mathbf{R})^\perp$.

A fundamental concept for normal form games is the so called Nash Equilibria (NE). An action profile, $x \in A$ is a *Nash Equilibrium* if[1] $\forall k \in N$,

$$u_k(x) \ge u_k(x_k', x_{-k}), \forall x_k' \in A_k \tag{2}$$

In simple terms, an NE is an action profile where no player can improve its utility with a unilateral deviation. If a game-master wanted to suggest an agreement to players, she would

---

[1] With some of abuse of notation, let $V(x_i', x_{-i})$ denote $V(x_1, \ldots, x_{i-1}, x_i', x_{i+1}, \ldots, x_{|N|})$ where $x_i' \in A_i$ and $x_{-i} \in A_{-i}$.

want to choose an action profile from which, in the absence of coordination, no player would want to deviate. In other words, she would choose an NE.

An important question in game theory is how the players might reach an NE without the help of a game-master. One possibility is to consider what would happen if the game were played repeatedly. When one considers how a given player might react to other players' actions, it is interesting to investigate better response and best response dynamics. Given an action profile, $x \in A$, a *better response for player* $i \in N$ is any $x_i' \in A_i$ such that $u_i(x_i', x_{-i}) \geq u_i(x)$, and a *best response* for player $i \in N$ is any $x_i' \in \operatorname{argmax}\{u_i(z_i, x_{-i}) : z_i \in A_i\}$. With this definition in hand, an NE is an action profile $x \in A$ such that x_i is a best response for every player $i \in N$. Two games with the same sets N and $\{A_i\}$ are said to be *best response equivalent* if $\forall x \in A$ and every player, the best response of both games coincide. *Better response equivalence* has a corresponding definition. For a game, Γ, a function $V : A \to \mathbb{R}$ is said to be:

1) an *exact potential function* if $\forall i \in N,\ x \in A, x_i' \in A_i$

$$u_i(x) - u_i(x_i', x_{-i}) = V(x) - V(x_i', x_{-i}) \tag{3}$$

2) an *ordinal potential function* if $\forall i \in N,\ x \in A, x_i' \in A_i$

$$u_i(x) \geq u_i(x_i', x_{-i}) \Leftrightarrow V(x) \geq V(x_i', x_{-i}) \tag{4}$$

3) a *best response* (BR) *potential function* if $\forall i \in N,\ x_{-i} \in A_{-i}$,

$$\operatorname*{argmax}_{x_i \in A_i} u_i(x_i, x_{-i}) = \operatorname*{argmax}_{x_i \in A_i} V(x_i, x_{-i}) \tag{5}$$

A game is an exact, ordinal, or BR potential game if, for the game, there exists an exact, ordinal, or BR potential function, respectively. In addition, a game is a *transformable ordinal potential game* if there exists an ordinal transformation[2], $f : u(A) \to \mathbb{R}^{|N|}$, such that the game $\left\langle N, \{A_i\}_{i \in N}, \{\tilde{u}_i\}_{i \in N} \right\rangle$ with $\tilde{u}_i(x) = f_i(u_i(x))$, is an exact potential game. A game is a *potential game* if it is any of these games.

The importance of potential games hinges on its relation to the following so called coordination game

$$\Gamma' = \left\langle N, \{A_i\}_{i \in N}, \{V\}_{i \in N} \right\rangle \tag{6}$$

where each player's utility function is replaced by the (player-independent) potential function. Exact and ordinal potential games are better response equivalent to this coordination game, while BR potential games are only best-response equivalent. In both cases, when the potential function is also a measure of social welfare, this equivalence says that players can serve the greater good by following their own best interest. In a *round robin best-response dynamic*, players take turns in a round robin fashion choosing a best response until there are no more improving solutions[3]. In such a case, the best response dynamic enters a Nash Equilibrium. Since a potential game is best-response equivalent to its corresponding coordination game any result for the best-response dynamic of a coordination game also applies to a potential game. Let $\Phi : A \to P(A)$ map an action-tuple $x \in A$ to the set of all possible best-responses after one round-robin iteration. Since, in general, there may be more than one such best response, Φ must be a correspondence (i.e. set valued function) [17].

We conclude this section with the *noisy best response*, a generalization of the noisy MMSE iteration considered in [2]. In this case, the best response of each player is perturbed by bounded noise, with bound $d > 0$. The motivation behind the noise is that in the process of determining their actions, players may make random mistakes, however small, in a manner whose nature may depend upon the best response itself. For each $c \in A$, let $z(c)$ be a random vector with arbitrary joint probability density function $p_z(z; c, d)$. We require that $p_z(z; c, d)$ be positive almost everywhere on $\overline{N_d(c)}$ and zero outside $\overline{N_d(c)}$. For instance, this noise may be uniformly distributed on $\overline{N_d(c)}$.

Noisy Best Response Iteration (NBRI)

```
Given noise bound $d > 0$, and $x[0] \in A$
For each $t \in \mathbb{N}_0$,
    1. Choose $c[t] \in \Phi(x[t])$
    2. $x[t+1] = z(c[t])$
```

Here, $\{c[t]\}$ is the sequence of chosen best-responses, $\{x[t]\}$ is the sequence of action profiles actually taken (including the mistakes). The index $t \in \mathbb{N}_0$ indexes one round robin iteration. The choice in step 1 is a convenient representation of all the choices that players make on one round-robin iteration. In the event that there is not a unique best response (Φ is not singleton valued), the NBRI does not specify the mechanism of choice. However, the coming Theorem 1 holds for any sequence of choices. For convenience, given a $d > 0$, let d-NBRI denote the NBRI algorithm with noise bound d.

Let Φ_F denote the Nash Equilibria of Γ; this set is so denoted since the set of NE is exactly the set of fixed points of Φ, i.e. $\Phi_F = \{x \in A : x \in \Phi(x)\}$. A continuous function, $V : A \to \mathbb{R}$, on compact A is *Nash separable* if there are no suboptimal local maxima on A, it's maximum is isolated from the image of other fixed points (i.e. $\exists\ e_m > 0,$ s.t. $N^*_{e_m}(V_{\max}) \cap V(\Phi_F) = \varnothing$), and best response iterations are strictly improving in a neighborhood of the maximum (i.e. $\forall e > 0,$ with $e < e_m$ $\Phi(V^{-1}([V_{\max} - e, V_{\max}])) \subset V^{-1}((V_{\max} - e, V_{\max}])$. A potential

---

[2] An ordinal transformation is a transformation $f : \mathbb{R}^{|N|} \to \mathbb{R}^{|N|}$, $f(u) = (f_1(u_1), \ldots, f_{|N|}(u_{|N|}))$, $u \in \mathbb{R}^{|N|}$ such that $\forall i \in N$, f_i is a monotonically increasing function.

[3] For brevity, henceforth we will drop the round-robin distinction.

game is *Nash Separable* if its potential is Nash Separable. The following result is a generalization of Theorem 8 in [2] for the NBRI and is proven in [14-15]. It says that even if a Nash Separable potential game has sub-optimal NE, arbitrarily small noise will cause the NBRI to almost surely converge to a neighborhood of the global optima.

Theorem 1 Consider a Nash Separable potential game with the form of Equation (1) and potential function V. Then, $\forall e > 0$, $\exists d_0 > 0$, such that $\forall d$ with $0 < d < d_0$ and $\forall x[0] \in A$, the d-NBRI with iterates $(x[t])$ obeys $\liminf_{t \to \infty} V(x[t]) \underset{a.s.}{\geq} \max V - e$.

III. INTERFERENCE AVOIDANCE

Consider the following signal model for a synchronous, peer-to-peer, code-on-pulse CDMA system in which all receivers are co-located, and hence experience the same received signal. The signal model after chip-level matched filtering and preconditioning is

$$\begin{aligned} \underline{r} &= \mathbf{SP}^{1/2}\underline{b} + \underline{z} \\ &= \sqrt{p_k}\,\underline{s}_k b_k + \mathbf{S}_{-k}\mathbf{P}_{-k}^{1/2}\underline{b}_{-k} + \underline{z} \qquad (7) \\ &= \sqrt{p_k}\,\underline{s}_k b_k + \underline{i}_k \end{aligned}$$

Here, $\underline{r} \in \mathbb{C}^{m \times 1}$ is the received vector over one symbol interval and has auto-correlation $\mathbf{R}_{\underline{rr}}$ (m is the number of chips per symbol). The k^{th} element of the vector, $\underline{b} \in \mathcal{A}^{n \times 1}$, is the symbol transmitted by the k^{th} transmitter and is drawn equally likely from the alphabet, $\mathcal{A}$ (n is the number of peer-to-peer links). The symbols sent by each transmitter are assumed to be independent, zero mean and of unit variance, $s_{b_k}^2 = 1$. The matrix $\mathbf{P}$ is the diagonal matrix whose k^{th} diagonal element, p_k, is the fixed received power of the k^{th} transmitter; this paper will assume that $\mathbf{P}$ is fixed. Signatures, $\underline{s}_k$, are constrained to the set $\Omega = \left\{ \underline{s} \in \mathbb{C}^m : \| \underline{s} \|^2 = 1 \right\}$. The $m \times n$ matrix $\mathbf{S} = [\underline{s}_1 \cdots \underline{s}_n]$ and with some abuse of notation, we say $\mathbf{S} \in \Omega^n$. The vector $\underline{z}_k \in \mathbb{C}^{m \times 1}$ is additive complex Gaussian noise with covariance matrix $\mathbf{R}_{\underline{zz}} \triangleq E[\underline{z}\,\underline{z}^H]$. So, $\mathbf{R}_{\underline{rr}} \triangleq E[\underline{r}\,\underline{r}^H] = \mathbf{SPS}^H + \mathbf{R}_{\underline{zz}}$. Let the matrix, $\mathbf{S}_{-k} \in \mathbb{C}^{m \times (n-1)}$ be the matrix $\mathbf{S}$ with the k^{th} column removed and let both $\mathbf{P}_{-k} \in \mathbb{C}^{(n-1) \times (n-1)}$ and $\underline{b}_{-k} \in \mathbb{C}^{(n-1) \times 1}$ have corresponding definitions. The $m \times 1$ vector $\underline{i}_k \triangleq \mathbf{S}_{-k}\mathbf{P}_{-k}^{1/2}\underline{b}_{-k} + \underline{z}$ contains all of the interference for transmitter k and has auto-covariance matrix, $\mathbf{R}_{\underline{ii}}[k] = \mathbf{S}_{-k}\mathbf{P}_{-k}\mathbf{S}_{-k}^H + \mathbf{R}_{\underline{zz}}$. In the subsequent discussion we will allow $\mathbf{R}_{\underline{zz}}$ to be singular, and hence $\mathbf{R}_{\underline{ii}}[k]$ may also be singular for some k. It will be convenient to let $\mathcal{R}_i[k] = Range\left\{ \mathbf{R}_{\underline{ii}}[k] \right\}$. Implicit in Equation (7) are the following assumptions: the system can achieve perfect symbol-timing and carrier frequency (but not necessarily phase) synchronization; received signals incur no frequency selective multi-path. Although, these assumptions are not realistic, it is generally thought that progress toward understanding synchronous IA systems will eventually help understand non-ideal systems.

The receiver for each peer-to-peer link has the responsibilities of both estimating the link's transmitted symbols and calculating its transmitter's signature. It is assumed that the k^{th} link's receiver has the ability to perfectly estimate $\mathbf{R}_{rr}$ or $\mathbf{R}_{ii}[k]$. The receivers are not allowed to directly communicate their choice of signature with other receivers, even though they are co-located. For instance, the receivers cannot collaborate to compute an orthogonal set of codes on the first iteration. Instead, the links are only permitted to interact indirectly through estimates of $\mathbf{R}_{rr}$ and $\mathbf{R}_{ii}[k]$. Nonetheless, these receivers can coordinate enough to take turns updating their signatures in a round-robin fashion. We will assume that each receiver employs a linear symbol estimate of the following form

$$\hat{b}_k = a\,\underline{w}_k^H \underline{r} \qquad (8)$$

where $\underline{w}_k \in \mathbb{C}^{m \times 1}$ is the k^{th} receiver's linear processor and $a \neq 0$ is some convenient scaling obtained, perhaps, through automatic gain/phase control. There are several natural choices of $\underline{w}_k$. Among these are the correlation receiver (Corr), in which case

$$\underline{w}_k = \underline{s}_k \qquad (9)$$

and the maximum signal to interference and noise ratio (MSINR) receiver, in which case $\underline{w}_k$ is any solution to

$$\mathbf{R}_{\underline{ii}}[k]\underline{w}_k = \underline{s}_k \qquad (10)$$

when $\underline{s}_k \in \mathcal{R}_i[k] = Range\left\{ \mathbf{R}_{\underline{ii}}[k] \right\}$ and

$$\underline{w}_k = \mathbf{P}_{\perp}(\mathbf{R}_{\underline{ii}}[k])\underline{s}_k \qquad (11)$$

when $\underline{s}_k \notin Range\left\{ \mathbf{R}_{\underline{ii}}[k] \right\}$. Another candidate, the minimum mean squared error (MMSE) receiver is well known to also be an MSINR receiver [2] and hence, will not be considered separately.

Motivated by the information theory developed in [16], it is argued in [1-4] that the best choice of signatures is the one that maximizes the following so called negated generalized total squared correlation function ($\mathrm{NTSC_g}$)

$$V(\mathbf{S}) = -\| \mathbf{SPS}^H + \mathbf{R}_{\underline{zz}} \|_F^2 \qquad (12)$$

where $\| A \|_F \triangleq \sqrt{\Sigma_{i,j} | a_{i,j} |^2}$ is the Frobenius matrix norm. Algorithms are given in [2] for finding $V_{\max} = \max\{V(\mathbf{S}) : \mathbf{S} \in \Omega^n\}$ and global optimum solutions. In many circumstances, optimum signatures are those that whiten the spectrum of $\mathbf{R}_{\underline{rr}}$ [18], in which case we say that the signal environment is *levelable*. This, for example, occurs in an overloaded IA system ($n \geq m$) when all signals are received with equal power in AWGN [1]. In the parlance of Game Theory, $V(\mathbf{S})$ is a *social welfare function* that measures the benefit of all signature selections for the greater good. Now compare the coordination game $\left\langle N, \{\Omega\}, \{V\} \right\rangle$ to the, so called, *primitive game*, $\left\langle N, \{\Omega\}, \{\tilde{u}_k(\mathbf{S})\} \right\rangle$ with $\tilde{u}_k(\mathbf{S}) = -2p_k \underline{s}_k^H \mathbf{R}_{\underline{ii}}[k] \underline{s}_k$. Expanding our potential function in terms of $\underline{s}_k$, we have,

$$\begin{aligned} V(\mathbf{S}) &= -\| \mathbf{SPS}^H + \mathbf{R}_{\underline{zz}} \|_F^2 \\ &= -\| \mathbf{R}_{\underline{ii}}[k] + p_k \underline{s}_k \underline{s}_k^H \|_F^2 \\ &= -\| \mathbf{R}_{\underline{ii}}[k] \|_F^2 + p_k^2 \\ &\quad -2 p_k \underline{s}_k^H \mathbf{R}_{\underline{ii}}[k] \underline{s}_k . \end{aligned}$$

Hence, the $\left\langle N, \{\Omega\}, \{\tilde{u}_k(\mathbf{S})\} \right\rangle$ is an exact potential game, $V(\mathbf{S})$ is its potential function, and the best response for the k^{th} player is to choose a normalized inferior eigen-vector of $\mathbf{R}_{\underline{ii}}[k]$. In fact many other IA games also have $V(\mathbf{S})$ as a potential function. Table 1 lists 4 potential games with the following naming convention: in game A/B each player measures its signal quality with criterion A on the output of receiver B. For brevity, we list the resulting utility functions without proof. The games for which players use correlation receivers are ordinal potential games because they are within an ordinal transformation of the primitive game.

A. CONVERGENCE

The round robin best response iteration, Φ, is better known as the *eigen-iteration*, and is known to *converge in class*, e.g. to a set of (not necessarily optimal) eigen- vectors of $\mathbf{R}_{\underline{rr}}$ [18]. It is also known that a variant of the (greedy) best response iteration called *class warfare* asymptotically maximizes NTSC_g; however, this IA scheme requires coordination between receivers[4]. Nevertheless, as the following theorem shows, IA coordination games in levelable signal environments are Nash Separable, and hence, the NBRI asymptotically converges to a neighborhood of an optimum signature set without *class warfare* or any other form of coordination.

Theorem 2: The IA coordination game, $\left\langle N, \{\Omega\}, \{V\} \right\rangle$ in a levelable signal environment is Nash Separable.

Proof: In [2], it is shown that $V(\mathbf{S})$ has no suboptimal local minima and a straightforward adaptation of Theorem 2 in [2] shows that $V(\Phi_F)$ is a finite set. Use this fact to choose a $0 < e_m < \min_{k \in N} p_k^2 / 2$ such that $\mathbf{S} \in \Phi_F$ implies $V(\mathbf{S}) = V_{\max}$ on $W_{e_m} = V^{-1}(N_{e_m}(V_{\max}))$.

Now, given $0 < e < e_m$, $\mathbf{S} \in W_e$, if $\inf V(\Phi(\mathbf{S})) = V(\mathbf{S})$, then there is a non-improving sequence of best-responses on a round. If $V(\mathbf{S}) = V_{\max}$, $V(\mathbf{S}) > V_{\max} - e$. Therefore, consider $V(\mathbf{S}) < V_{\max}$. Note, $\mathbf{S} \notin \Phi_F$ by choice of e_m. Hence, $\exists j \in N$ s.t. $V(\Phi_j(\mathbf{S})) > V(\mathbf{S})$. Choose the smallest such j. This can only occur if player j's improving response is blocked by a player $k < j$ who changed its response. Consider the smallest such k. However, by the choice of j, player k's signature in $\mathbf{S}$ was already a best response. Therefore, the inferior eigen-space of $\mathbf{R}_{\underline{ii}}[k]$ has dimension ≥ 2. Let $l_{\min}[k] = \min l(R_{\underline{ii}}[k])$ and

$$\mathbf{R}_{ii}[k] = \mathbf{Q} \begin{bmatrix} \Lambda & \mathbf{0} \\ \mathbf{0} & l_{\min} \mathbf{I}_2 \end{bmatrix} \mathbf{Q}^H \qquad (13)$$

be the eigen-decomposition of $\mathbf{R}_{\underline{ii}}[k]$. Now, choose an optimum signature set, $\mathbf{S}_{opt} \in \Omega^n$. Since the IA game is levelable, $\exists k > 0$ s.t. $\mathbf{R}_{\underline{rr}opt} \triangleq \mathbf{S}_{opt} \mathbf{PS}_{opt}^H + \mathbf{R}_{\underline{zz}} = k\mathbf{I}$. Thus, we have,

$$\begin{aligned} & V_{\max} - V(\mathbf{S}) \\ &= -\| \mathbf{R}_{\underline{rr}opt} \|_F^2 + \| \mathbf{SPS}^H - \mathbf{S}_{opt} \mathbf{PS}_{opt}^H + \mathbf{R}_{\underline{rr}opt} \|_F^2 \\ &= \| \mathbf{SPS}^H - \mathbf{S}_{opt} \mathbf{PS}_{opt}^H \|_F^2 + 2k \operatorname{Re} Tr\{ \mathbf{SPS}^H - \mathbf{S}_{opt} \mathbf{PS}_{opt}^H \} \\ &= \| \mathbf{R}_{\underline{ii}}[k] + p_k \underline{s}_k \underline{s}_k^H - k\mathbf{I} \|_F^2 \\ &\geq \| \, ? \, - k\mathbf{I} \|_F^2 + (l_{\min}[k] + p_k - k)^2 + (l_{\min}[k] - k)^2 \\ &\geq p_k^2 / 2 > e , \end{aligned}$$

a a

contradiction. This completes the proof.

Direct application of Theorems 1 and 2 imply that the NBRI for a levelable IA coordination game is convergent (of the type in Theorem 1). Since all of the games in Table 1 are best response equivalent to the coordination game, $\left\langle N, \{\Omega\}, \{V\} \right\rangle$, the NBRI converges in the sense of Theorem 1 for a game combining any of the utilities in Table 1.

IV. CONCLUSION

For a variety of end-metric/receiver-types, a synchronous IA scheme can be modeled as a potential game. In addition, when the IA system's signal environment is levelable, the noisy best response iteration almost surely converges for a game in which links independently choose any of the metrics in Table 1. This enables Game Theory to extend the convergence result in [2] to the case of the eigen-algorithm, for which on some iterations, a link may not have a unique best response. For instance, this can happen when $\mathbf{R}_{\underline{zz}}$ is singular, a case that is important in interference limited systems when $\mathbf{R}_{\underline{zz}} \approx \mathbf{0}$ and $\mathbf{R}_{\underline{ii}}[k]$ can be ill-conditioned. We have also shown that when all links use correlation receivers, the IA game is an ordinal potential game, and hence, is better-response equivalent to the NTSC_g coordination game. A more thorough understanding of better response algorithms such as those considered in [19] could reveal IA schemes with less burdensome feedback channels and comparable convergence speed.

Table 1: Some IA Games, their utility functions, and their type of potential function.

Game Name	**Utility Function,** $u_k(\mathbf{S})$	**Potential Game Type**	**Ordinal Transformation**
SINR/Corr.	$p_k / \underline{s}_k^H \mathbf{R}_{\underline{i\,i}}[k]\underline{s}_k$	Ordinal	$-1/x, x>0$
MSE/Corr.	$-\underline{s}_k^H \mathbf{R}_{\underline{i\,i}}[k]\underline{s}_k / \underline{s}_k^H \mathbf{R}_{\underline{r\,r}}\underline{s}_k$	Ordinal	$x/(x+1), x>-1$
SINR/MSINR	$\left\{\begin{array}{ll} p_k \underline{s}_k^H \mathbf{R}_{\underline{i\,i}}^{\dagger}[k]\underline{s}_k & \underline{s}_k \in \mathcal{R}_i[k] \\ \infty & else \end{array}\right]$	Best Response	N/A
MSE/MSINR	$\left\{\begin{array}{ll} \dfrac{-1}{p_k \underline{s}_k^H \mathbf{R}_{\underline{i\,i}}^{\dagger}[k]\underline{s}_k + 1}, & \underline{s}_k \in \mathcal{R}_i[k] \\ 0, & else \end{array}\right]$	Best Response	N/A

ACKNOWLEDGEMENTS

We thank the Office of Naval Research (ONR) Grant# N000140310629, the Motorola Universities Partnership in Research (UPR) Program, and the MPRG Affiliates program for the support of this work.

REFERENCES

[1] S. Ulukus, R.D. Yates, "Iterative Construction of Optimum Signature Sequence Sets in Synchronous CDMA systems," *IEEE Trans. On Information Theory*, vol. 47, no. 5, July 2001, pp. 1989-1998.

[2] P. Anigstein, V. Anantharam, "Ensuring Convergence of the MMSE Iteration for Interference Avoidance to the Global Optimum," *IEEE Trans. On Information Theory*, vol. 49, no. 4, April, 2003.

[3] C. Rose, S. Ulukus, R. D. Yates, "Wireless Systems and Interference Avoidance," *IEEE Trans. On Wireless Communications*, vol. 1, no. 3, July 2002, pp. 415-427.

[4] C. Rose, "CDMA Codeword Optimization: Interference Avoidance and Convergence Via Class Warfare," *IEEE Trans. On Information Theory*, vol. 47, no. 6, Sept. 2001, pp. 2368-2382.

[5] S. Ulukus, R. D. Yates, "Signature Sequence Optimization in Asynchronous CDMA Systems," *Proc. IEEE Int. Conf. Communications*, vol. 2, June 2001, pp. 545-549.

[6] J. I. Concha, S. Ulukus, "Optimization of CDMA Signature Sequences in Multipath Channels," *Proc. IEEE Vehic. Tech. Conf.* vol. 3, May 2001, pp.1978-1982.

[7] G. J. Rajappan, M. L. Honig, "Signature Sequence Adaptation for DS-CDMA with Multipath," *IEEE JSAC*, vol. 20, no. 2, Feb. 2002, pp. 384-395.

[8] D. C. Popescu, C. Rose, "Interference Avoidance Applied to Multiaccess Dispersive Channels," *Proc. 35th Asilomar Conf. on Signals, Systems, and Computers, vol. II, pp. 1200-1204,* Pacific Grove, California, Nov. 2001.

[9] D. C. Popescu, C. Rose, "Codeword Quantization for Interference Avoidance," *ICASSP 2000*, vol. 6, Istanbul, June 2000, pp. 3670-3673.

[10] D. Monderer, L. Shapley, "Potential Games," *Journal of Games and Economic Behavior*, vol. 14, no. 0044, pp. 124-143, 1996.

[11] M. Voorneveld, *Potential Games and Interactive Decisions with Multiple Criteria*, PhD Dissertation, Tilburg University, September 1999.

[12] J. Neel, J. H. Reed, R. Gilles, "Game Theoretic Analysis of a Network of Software Radios," *SDR Forum Conf. 2002.*

[13] P. K. Dutta, *Strategies and Games, Theory and Practice*, MIT Press, Cambridge Massachusetts, © 1999.

[14] J. Hicks, A. B. MacKenzie, "A Convergence Result for Potential Games," submitted to *11th Int'l Symp. on Dynamic Games and Applications*, Tuscon, Arizona, Dec. 18-21, 2004.

[15] J. Hicks, A. B. MacKenzie, "A Convergence Result for Potential Games," *MPRG Technical Report,* downloadable from www.mprg.org/people/gametheory.

[16] P. Viswanath, V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Trans. on Info. Theory*, vol. 45, pp. 1984-1991, Sept. 1999.

[17] R. Sundaram, *A First Course in Optimization Theory*, Cambridge University Press, Cambridge, U.K. Ó 1996.

[18] D. Popescu, C.Rose, *Interference Avoidance Methods for Wireless Systems*, Kluwer Academic, NY, NY, © 2004.

[19] J. W. Friedman, C. Mezzetti, *Learning in Games by Random Sampling*, Journal of Economic Theory, vol. 98, pp. 55-84 (2001).

[20] C. W. Sung and K. K. Leung, "On the Stability of Distributed Sequence Adaptation for Cellular Asynchronous DS-CDMA Systems", *IEEE Trans. on IT,* 49(7): 1828-1831, July 2003.