A Collaborative Quasi-Linear Programming Framework for Ad Hoc Sensor Localization

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***Abstract*—In this paper, we propose a collaborative localization scheme which utilizes, in addition to the range estimates to nodes with known locations (anchors), the range estimates between unlocalized nodes to estimate their positions. The proposed collaborative method is incorporated into the linear programming (LP) framework proposed in [13] and is implemented in a distributed fashion. We also adopt a constrained nonlinear optimization method to deal with the degenerate cases in the LP approach, i.e., when an unlocalized node has less than three line-of-sight (LOS) range estimates to localized nodes. Simulation results suggest that this collaborative quasi-LP framework can improve the localization accuracy as well as increase the percentage of nodes that are able to be localized, as compared to the standard sequential least-squares (LS) estimator.**

Keywords: collaborative localization, wireless sensor networks (WSN), linear programming (LP), least-squares (LS).

I. Introduction

Automatic and accurate sensor localization is critical for the successful deployment of wireless sensor networks (WSN) in applications such as environment monitoring, precision agriculture, and emergency-rescue personnel tracking. In traditional infrastructure-based localization techniques, a mobile device derives its location by making measurements to several known locations, e.g., satellites in GPS or base stations in E-911. However, in WSNs, due to complexity and cost constraints, only a small portion of sensor nodes are aware of their locations (referred to as anchor nodes or simply anchors), achieved by either equipping them with expensive GPS receivers or intentionally distributing them to pre-determined locations. In this situation, it is most likely that most sensor nodes may not have enough anchors within their communication range in order to determine their locations. Instead, they can only make measurements with other sensor nodes whose locations are not yet determined (referred to as unlocalized nodes or unknowns). Consequently, how to utilize these measurements becomes crucial for localizing sensor nodes while achieving an acceptable localization accuracy. This has triggered research on collaborative localization, i.e., techniques where unlocalized nodes collaborate with each other to compute their locations [1]-[3].

The benefits from node collaboration have been well acknowledged. In [1], the authors proved that adding more unlocalized nodes into a network will strictly lower the Cramer-Rao bound (CRB) for the localization error as long as the newly-added nodes satisfy a simple connectivity condition. However, despite many existing works on collaborative localization [4]-[11], two important questions remain to be answered completely, namely, *whether* and *how* to collaborate? Specifically, the first question is concerned with determining which nodes should participate in the collaboration, which generally is equivalent to node localizability. In [4], the authors adddressed this by formulating sensor network localization as a graph realization problem and applying graph rigidity theory to derive the conditions for unique network localizability. However, a sufficient and necessary condition for generic individual node localizability is yet to be found. In addition, how the presence of measurement noise affects the localizability is still an open problem. The second question is pertinent to the nature of the computation involved in node collaboration. Existing positioning algorithms can largely be classified into two categories: centralized algorithms [1], [5]-[8] and distributed algorithms [9]-[11]. Compared to centralized algorithms which are intrinsically collaborative since the central solver has knowledge of the entire network, distributed algorithms spread the computation load over many nodes, and are therefore more scalable and robust. However, finding an efficient way of incorporating node collaboration into distributed algorithms remains a challenging issue. In [11], the authors attempted to address both of the above two issues. The proposed collaborative multilateration scheme starts with establishing a collaborative subtree which includes those uniquely-localizable nodes and then estimates their positions using an iterative least-squares (LS) method. However, the derived conditions for unique localizability are sufficient but not necessary. In addition, the LS method suffers in the presence of non-line-of-sight (NLOS) propagation [13], which is a major source of error for indoor localization.

In this paper, we focus on how to incorporate node collaboration into a distributed localization algorithm. Specifically, we develop a collaborative quasi-linear programming (CQLP) framework for localizing a network of sensor nodes in a distributed fashion. The proposed framework utilizes each unlocalized node's range estimates with other unlocalized nodes by extending them to reach anchors that are multiple hops away from the unlocalized node. By doing this, we can create additional *virtual* NLOS range estimates, which can then be incorporated into our LP approach proposed in [13] to improve the localization accuracy. One limitation of the LP approach is that it requires at least three LOS range estimates to properly set up the objective function. We overcome this limitation by adopting the range scaling algorithm (RSA) proposed in [16]. Because the RSA involves nonlinear optimization, we term our framework as *quasi-linear*. This complete CQLP framework

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can be applied to the general network localization problem and is demonstrated to provide improved performance. The main contributions of this paper can be summarized as the following:

- We propose a collaborative localization scheme to effectively utilize the range estimates between unlocalized nodes to increase location coverage and improve localization accuracy;
- We propose a method to deal with degenerate cases, i.e., when an unlocalized node that has less than 3 LOS range estimates to localized nodes, in the LP approach;
- We provide detailed performance results which demonstrate how the number of hops in the node collaboration impacts the localization performance.

The rest of the paper is organized as follows. In Section II, we briefly describe the overall network model, the LP approach and the RSA. The proposed collaborative localization scheme is elaborated in Section III. Simulation results and performance comparisons are presented in Section IV. Concluding remarks are given in Section V.

II. LP APPROACH AND RSA

We consider a 2D network of N unlocalized nodes and M anchors. Denote the position of the ith anchor as $\mathbf{A}_i = (x_{Ai}, y_{Ai})^T$ and the position of the nth unlocalized node as $\mathbf{U}_n = (x_{Un}, y_{Un})^T$. We use R_{in} to denote the distance between the ith anchor and the nth unlocalized node and r_{nm} to denote the distance between the nth and mth unlocalized nodes, i.e.,

$$R_{ni} = ||\mathbf{U}_n - \mathbf{A}_i|| \quad (1)$$
$$r_{nm} = ||\mathbf{U}_n - \mathbf{U}_m|| \quad (2)$$

We assume that each unlocalized node can obtain range estimates to the nodes within its communication range, e.g., by measuring round-trip time-of-filght of a packet handshake [1]. To model the indoor environments, range estimates may be NLOS or LOS with certain probabilities. Range estimates, when they exist, are modeled as,

$$\hat{R}_{ni} = R_{ni} + \mathcal{N}(0, \sigma_{ni}^2) + I_{ni}\, \mathcal{U}(0, B_{\text{max}}) \quad (3)$$
$$\hat{r}_{nm} = r_{nm} + \mathcal{N}(0, \sigma_{nm}^2) + I_{nm}\, \mathcal{U}(0, B_{\text{max}}) \quad (4)$$

where $\mathcal{N}(0, \sigma_{ni}^2)$ represents the zero-mean Gaussian measurement noise with variance $\sigma_{ni}^2 = K_E R_{ni}^{\beta}$, β is the path loss exponent, and K_E is a constant representing the combined effects of the physical layer communication parameters. $\mathcal{U}(0, B_{\text{max}})$ is an NLOS bias uniformly distributed over $[0, B_{\text{max}}]$ where B_{max} is usually much larger than the measurement noise term. $I_{ni} \in \{0, 1\}$ is an indicator variable, where $I_{ni} = 1$ (with probability p) means $\hat{R}_{ni}$ is an NLOS range estimate, and $I_{ni} = 0$ (with probability $1 - p$) indicates $\hat{R}_{ni}$ is an LOS range estimate. Similar parameter definitions hold for $\hat{r}_{nm}$.

A. LS estimator and LP approach

We first describe the LS estimator, then the LP approach naturally follows. In non-collaborative localization, only range estimates to anchors are used to estimate an unlocalized node's location. Using these range estimates, the LS estimator [15] first generates intersection lines by subtracting a pair of circular equations defined by the range estimates to two localized nodes. For the nth unlocalized node, its ith and jth range estimates are

$$\hat{R}_{ni}^2 = (x_{Un} - x_{Ai})^2 + (y_{Un} - y_{Ai})^2, \quad (5)$$
$$\hat{R}_{nj}^2 = (x_{Un} - x_{Aj})^2 + (y_{Un} - y_{Aj})^2. \quad (6)$$

Subtracting (5) from (6), we obtain

$$a_{ij} x_{Un} + b_{ij} y_{Un} = c_{ij}, \quad i, j = 1, 2, \ldots, K_n,\ i < j \quad (7)$$

where K_n is the total number of range estimates to anchors and there are $\binom{K_n}{2}$ such linear equations,

$$a_{ij} = x_{Ai} - x_{Aj}, \quad b_{ij} = y_{Ai} - y_{Aj},$$
$$c_{ij} = \frac{1}{2}\left[(x_{Ai}^2 - x_{Aj}^2) + (y_{Ai}^2 - y_{Aj}^2) - (\hat{R}_{ni}^2 - \hat{R}_{nj}^2)\right]. \quad (8)$$

In the presence of noise, the equations defined in (7) can not be satisfied simultaneously. By minimizing the sum of the squared residuals

$$\hat{\mathbf{U}}_n = \arg\min_{\mathbf{U}_n} \sum_{i=1}^{K_n} \sum_{j=1, j>i}^{K_n} e_{ij}^2 \quad (9)$$

where

$$e_{ij} = a_{ij} x + b_{ij} y - c_{ij}, \quad i, j = 1, 2, \ldots, K_n, i < j, \quad (10)$$

the LS location estimate of $\mathbf{U}_n$ is given by [15]

$$\hat{\mathbf{U}}_n = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{c} \quad (11)$$

where

$$\mathbf{A} = \begin{bmatrix} a_{12} & a_{13} & \ldots & a_{1K_n} & a_{23} & \ldots & a_{(K_n-1)K_n} \\ b_{12} & b_{13} & \ldots & b_{1K_n} & b_{23} & \ldots & b_{(K_n-1)K_n} \end{bmatrix}^T$$
$$\mathbf{c} = \begin{bmatrix} c_{12} & c_{13} & \ldots & c_{1K_n} & c_{23} & \ldots & c_{(K_n-1)K_n} \end{bmatrix}^T$$

From above, it is easy to see that $K_n \geq 3$ (at least 3 range estimates) is required to obtain the unlocalized node's location.

It has been observed that the localization accuracy of the LS estimator in (11) degrades when NLOS range estimates are present and blindly used [13]. In indoor environments, it is likely that the majority of available range estimates are NLOS. Therefore, we may not have the luxury of simply discarding NLOS range estimates. The LP approach, by treating LOS and NLOS range estimates differently, provides an efficient way of utilizing NLOS range estimates without degrading the localization accuracy. Specifically, for the nth unlocalized node, let us assume K_n range estimates are arranged such that the first K_n^L are LOS and the remaining K_n^N are NLOS range estimates, i.e., $K_n = K_n^L + K_n^N$, $I_{ni} = 0$ for $i = 1, 2, \ldots, K_n^L$ and $I_{ni} = 1$ for $i = K_n^L + 1, K_n^L + 2, \ldots, K_n$. We assume that NLOS identification has been performed so that we know which range estimates are NLOS and which are LOS. This can be achieved by examining the received signal statistics [14]. With this knowledge, the LP approach only uses the K_n^L LOS range estimates to define the objective function. Further, it linearizes the objective function by replacing e_{ij}^2 with $|e_{ij}|$, and then substituting for the unconstrained variable e_{ij} with $e_{ij}^+ - e_{ij}^-$, $e_{ij}^+ \geq 0$, $e_{ij}^- \geq 0$, leading to the following objective

$$\hat{\mathbf{U}}_n = \arg\min_{\mathbf{U}_n} \sum_{i=1}^{K_n^L} \sum_{j=1,j>i}^{K_n^L} \left(e_{ij}^+ + e_{ij}^-\right), \tag{12}$$

subject to $e_{ij}^+ - e_{ij}^- = a_{ij}x + b_{ij}y - c_{ij}$. Similar to the LS estimator, $K_n^L \geq 3$ (at least three LOS range estimates) is required to obtain the unlocalized node's location. On the other hand, the K_n^N NLOS range estimates are used only to define additional linear constraints, based on the fact that the NLOS bias is typically much larger than the noise, resulting in positively biased range estimates. In particular, each NLOS range estimate defines a circle within which the unlocalized node must lie, i.e.,

$$(x_{Un} - x_{Ai})^2 + (y_{Un} - y_{Ai})^2 \leq \hat{R}_{ni}^2, \tag{13}$$

for $i = K_n^L + 1, \ldots, K_n$. We can relax the circular inequality in (13) into four rectangular inequalities, given by

$$\begin{aligned} x_{Un} - x_{Ai} \leq \hat{R}_{ni}, \quad -x_{Un} + x_{Ai} \leq \hat{R}_{ni} \\ y_{Un} - y_{Ai} \leq \hat{R}_{ni}, \quad -y_{Un} + y_{Ai} \leq \hat{R}_{ni}, \end{aligned} \tag{14}$$

for $i = K_n^L + 1, \ldots, K_n$. The linearized constraints in (14) can be combined with the linear objective function in (12) to form a complete linear program which can be solved by standard techniques. Due to space limitations, we have omitted the mathematical details and refer interested readers to [13] for details.

B. Range scaling algorithm (RSA) for degenerate cases

As we have seen, the LP approach requires $K_n^L \geq 3$ to properly setup the linear objective function in (12). This is a more stringent requirement than what is required by the LS estimator (i.e., $K_n \geq 3$), and thus it is more likely fails to occur in ad hoc sensor networks. In the following, we refer to the cases corresponding to $K_n^L < 3$ as *degnerate* cases and develop a scheme utilizing the range scaling algorithm (RSA) [16] to handle the degenerate cases in the LP approach.

The RSA was originally proposed to deal with the NLOS problem when localizing mobile devices in a cellular system [16]. The basic idea is to view an NLOS positively-biased range estimate as an up-scaled version of the true distance. Then, three such NLOS range estimates can be used to setup a constrained nonlinear optimization problem which outputs three estimated scaling factors. By multiplying the three scaling factors with the correpsonding NLOS range estimates, three LOS range estimates are generated and finally used to estimate the unlocalized node's location.

We now describe how our proposed scheme leverages the RSA to handle the degenerate cases. Assume the nth unlocalized node has $K_n^L \leq 2$ LOS range estimates and $K_n^N \geq 3 - K_n^L$ NLOS range estimates to anchors. For each of the K_n^L LOS range estimates, we produce an *artificial* NLOS range estimate by adding a uniformly-distributed bias, written as

$$\hat{R}_{ni} \Rightarrow \hat{R}'_{ni} = \hat{R}_{ni} + \mathcal{U}(0, B_{\text{art}}), \quad 1 \leq i \leq K_n^L \tag{15}$$

where B_{art} is the maximum bias to create artificial NLOS range estimates. Then, we can select (e.g., the smallest) $3 - K_n^L$ from K_n^N NLOS range estimates and combine those with the K_n^L artificial NLOS range estimates in (15). Those three NLOS range estimates can be used to set up the constrained nonlinear optimization as in [16], which finally outputs three scaling factors. We then select the scaling factors corresponding to the $3 - K_n^L$ NLOS range estimates and generate $3 - K_n^L$ LOS range estimates accordingly. Now, we have 3 LOS range estimates, including the original K_n^L LOS and $3 - K_n^L$ newly-generated LOS range estimates, and can apply the LP approach as described. Our simulation results suggest the efficacy of this scheme in handling the degenerate cases, as our proposed framework using this modified LP approach can provide better localization accuracy than the LS estimator. Note that due to the constrained nonlinear optimization involved when handling the degenerate cases, we term the overall framework as quasi-linear. It is also possible to convert more NLOS into LOS range estimates. However, since the nonlinear optimization generally has higher computational complexity, we choose to use it only when it is necessary.

C. Sequential location estimation

As mentioned earlier, the LS estimator needs at least three range estimates to anchors in order to compute an unlocalized node's position unambiguously. The LP approach, on the other hand, needs at least three LOS range estimates to achors in order to set up the linear optimization. However, in the context of WSNs, where there are few anchors, these conditions will rarely be satisfied. To localize more nodes, a *sequential* location estimation scheme [12] is adopted here. In this scheme, an unlocalized node immediately updates itself as a new anchor to assist localizing other unlocalized nodes once its location has been computed. By doing this, the location information is *propagated* throughout the network. This appears to be a simple and practical way of extending positioning coverage, especially for large-area sensor deployment, although it suffers from the propagation of the localization error [17]. In this paper, we apply this sequential location estimation to both the LS estimator, which will be termed the *sequential LS estimator*, and our proposed CQLP framework.

In the following, we will simply refer to the original anchors and the updated anchors together as localized nodes.

III. Node Collaboration

So far, the location estimation methods described above only use range estimates to localized nodes, i.e., $\hat{R}_{ni}$. In this section, we describe how our proposed framework can be extened to utilize range estimates to other unlocalized nodes, i.e., $\hat{r}_{nm}$, to improve the localization accuracy. The basic idea is to create virtual NLOS range estimates by adding multihop range estimates. This is illustrated in Fig. 1, where the circles and squares represent localized and unlocalized nodes, respectively. The existence of a solid line between two nodes indicates that they are within communication range of each other and can obtain a range estimate. In Fig. 1(a), $\mathbf{U}_n$ and $\mathbf{U}_m$ are unlocalized nodes. Each of them has two range estimates to localized nodes, and one range estimate to an

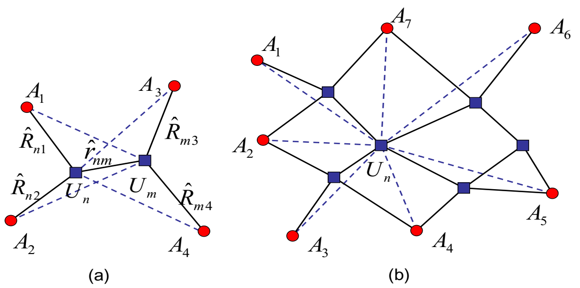

Fig. 1. Illustration of creating virtual NLOS range estimates by adding multihop range estimates.

```
while # of localized nodes increases
  for n = 1 : N
    if U_n is not localized and its # of range estimates ≥ 3
        create virtual NLOS range estimates to multi-hop localized nodes;
        if  # of range estimates to localized nodes ≥ 3
            if  # of LOS range estimates to localized nodes ≥ 3
                apply LP to obtain Û_n , update U_n as localized;
            else
                apply the RSA to generate sufficient LOS range estimates;
                apply LP to obtain Û_n , update U_n as localized;
            end
        end
    end
  end
end
```

Fig. 2. The complete CQLP framework.

unlocalized node. In a non-collaborative localization scheme, neither of them can be localized. However, in our proposed scheme, we utilize the following triangular inequalities for node $\mathbf{U}_n$

$$\begin{aligned} r_{nm} + R_{m3} &\geq R_{n3}, \\ r_{nm} + R_{m4} &\geq R_{n4} \end{aligned} \tag{16}$$

This suggests that $\mathbf{U}_n$ must lie *within* the circle centered at $\mathbf{A}_3$ with a radius of $r_{nm} + R_{m3}$. Similarly, it must lie *within* the circle centered at $\mathbf{A}_4$ with a radius of $r_{nm} + R_{m4}$. This resembles the role of NLOS range estimates in the LP approach. Specifically, we can view $r_{nm} + R_{m3}$ and $r_{nm} + R_{m4}$ as two *virtual* NLOS range estimates to two localized nodes $\mathbf{A}_3$ and $\mathbf{A}_4$, respectively. In practice, we can loosely use $\hat{r}_{nm} + \hat{R}_{m3}$ and $\hat{r}_{nm} + \hat{R}_{m4}$ instead of the exact distances. Although this does not guarantee the validity of (16) in the presence of range estimation noise, it works with high probability and its efficacy can be seen from our simulation results. Similar virtual NLOS range estimates can be created for node $\mathbf{U}_m$. The virtual NLOS range estimates are shown as dashed lines in Fig. 1(a). All these virtual NLOS range estimates can now be easily incorporated into the LP approach to assist node localization without degrading the localization accuracy. By doing this, unlocalized nodes utilize range estimates to each other and thus collaboratively determine their locations.

In addition, another benefit from our collaborative localization scheme is that the virtual NLOS range estimates tend to be geometrically diversified, as can be better shown in Fig. 1(b) where the central unlocalized node has virtual NLOS range estimates from many directions. This benefits the localization in the sense that it can mitigate the geomtrical dilution of precision (GDOP) [1]. It is to be noted that the above node collaboration can be extended to more than two hops. The more hops included, the more virtual NLOS range estimates that can be created, thus the more constraints we have for the LP approach and better localization accuracy is expected. However, the complexity associated with creating NLOS range estimates grows rapidly with the number of hops. In addition, as the number of hops increases, the looser the NLOS range estimate constraints will be. Therefore, in our scheme, we only consider two to three hops. It is also worth noting that if only one hop is considered, the CQLP is equivalent to the non-collaborative LP approach.

Combining all the aforementioned components in Sections II and III, the complete collaborative quasi-linear programming (CQLP) framework is as shown in Fig. 2. It should be noted that the major computational complexity with the CQLP framework lies in the nonlinear optimization involved in the RSA algorithm. Despite this, the proposed framework is a decentralized solution and does not require intense anchor deployment. The major component, LP approach, has lower complexity than other nonlinear processing.

IV. Simulation Results

In this section, we examine the performance of our proposed CQLP framework by comparing it with the sequential LS estimator. We also attempted to compare our scheme with the collaborative multilateration method proposed in [11]. However, we have observed that even for some basic collaborative subtree topologies, the initial location solution in the collaborative multilateration method suffers from the NLOS problem, thus resulting in an average localization error even larger than the sequential LS estimator. Therefore, in this paper, we will compare our CQLP framework with the sequential LS estimator. Specifically, we present a comparison of two important metrics for node localization in ad hoc sensor networks: coverage (the percentage of the nodes which are localizable) and accuracy (the localization error). In addition, we investigate how the number of anchors and the number of unlocalized nodes affect the two metrics.

We consider a $25 \times 25\,\text{m}^2$ area. Four anchors are fixed at the four corners. In this case, N unlocalized nodes and M anchors, besides the four placed at the corners, are randomly distributed within the network. The radio communication range is set to be R_{cr}, i.e., two nodes can obtain a range estimate to each other if their distance is less than R_{cr}. The range estimation noises and biases are assumed to be independent of each other. The path loss exponent $\beta = 2$ and the proportionality constant $K_E = 0.001$. The maximum NLOS bias $B_{\text{max}} = 0.5\,\text{m}$. The probability of a range estimate being NLOS is $p = 0.1$. When dealing with the degenerate cases, we have used $B_{\text{art}} = 2\,\text{m}$.

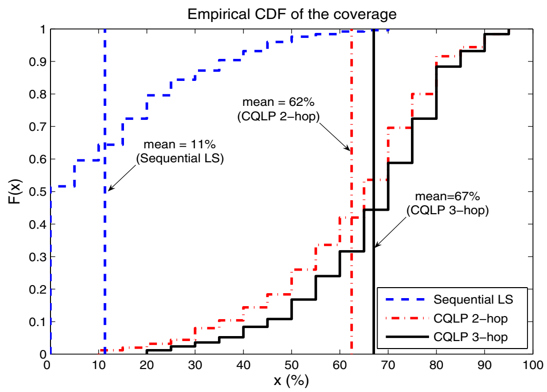

Fig. 3. CDF of the coverage of the sequential LS and the CQLP framework for 250 network realizations, when $N = 20$, $M = 5$, $R_{\text{cr}} = 6$.

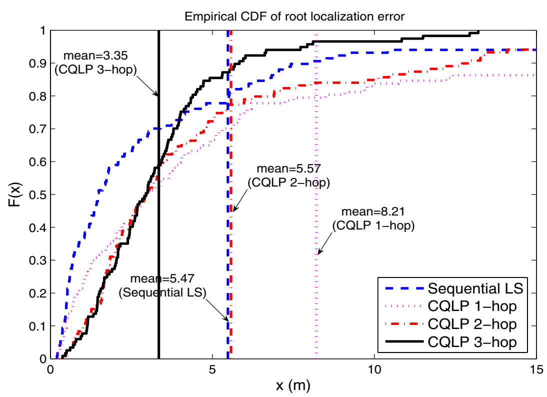

Fig. 4. CDF of root localization errors of the sequential LS and the CQLP framework for 250 network realizations, when $N = 20$, $M = 5$, $R_{\text{cr}} = 6$.

With different values of N, M and R_{cr}, we apply the sequential LS estimator and our proposed CQLP framework. The sequential LS estimator only utilizes range estimates to localized nodes to compute an unlocalized node's location, while the CQLP framework also utilizes range estimates to other unlocalized nodes by creating virtual NLOS connections to localized nodes that are originally more than one hop away. Both methods proceed until no additional nodes can be localized. For each set of parameters, we ran the simulation for 250 network realizations of unlocalized nodes, anchor's locations and NLOS indicators (i.e., I_{ni} and I_{nm} in (3) and (4)). For each network realization, we ran the simulation for 50 noise and bias realizations. For each network realization, the root localization error is defined as the Euclidean distance between the estimated and the true locations, averaged over all nodes that have been localized. The *average* root localization error refers to averaging the root localization error over all 250 network realizations.

In Fig. 3, we compare the coverage (i.e., the percentage of nodes that are able to be localized) of the proposed CQLP and the sequential LS estimator. The CDFs of the coverages over 250 network realizations of $N = 20$ unlocalized nodes and $M = 5$ anchors with $R_{\text{cr}} = 6$ m are plotted for both methods. As can be seen, the CQLP framework with both 2-hop and 3-hop node collaboration significantly increases the location coverage. For instance, the probability that the sequential LS estimator localizes less than 40% of the unlocalized nodes is about 0.9, while the probability of the CQLP framework with 2-hop and 3-hop node collaboration localizing less than 40% is about 0.05 and 0.1, respectively. Furthermore, the sequential LS estimator almost never localizes more than 70% of unlocalized nodes, while the CQLP framework achieves this coverage with a probabilty of greater than 0.55. The three vertical lines in Fig. 3 indicate the mean coverage based on 250 network realizations, which confirms that the proposed CQLP framework significantly increases the positioning coverage. Note that the 1-hop CQLP framework, i.e., without node collaboration, results in the same coverage as the sequential LS estimator. It is also observed that the coverage improvement decreases as the number of hops increases.

In Fig. 4, we plot the CDF of the root localization error of the two methods over 250 network realizations. To make a fair and meaningful comparison, we compare only the nodes that both methods can localize. As we can see from Fig. 4, the 1-hop CQLP actually performs worse than the sequential LS. The reason is that the LOS range estimates corrected by applying the RSA method are still likely to be biased. Therefore, using these biased range estimates to construct the objective function in (12) degrades the localization accuracy of the LP approach, and finally leads to the worse performance. This negative effect is more obvious in an ad hoc network, since in most cases, an unlocalized node does not have enough localized nodes within its one-hop radio range to provide LOS range estimates, and thus will have to resort to the RSA to generate a sufficient number of LOS range estimates. However, the CQLP with 2-hop node collaboration performs comparable to the sequential LS, while the CQLP with 3-hop node collaboration performs better than the sequential LS, in terms of average localization error. This is mostly attributed to the suppression of large localization errors, which can be seen by the crossover of the CDF curves of the CQLP 3-hop and the sequential LS. The performance improvement stems from the fact that 2-hop or 3-hop node collaboration adds more constraints by introducing more virtual NLOS range estimates. Together the improved localization coverage and the improved localization performance clearly demonstrate the benefit of node collaboration.

In Fig. 5, we compare the average localization coverage over 250 network realizations for the two localization methods with different numbers of unlocalized nodes N, i.e., node density. We observe that the CQLP with both 2-hop and 3-hop node collaboration consistently outperforms the sequential LS estimator by providing larger localization coverage. The more unlocalized nodes that are added into the network, the larger improvement we can obtain by using the CQLP framework, which suggests that the CQLP framework is scalable to dense sensor deployment. In addition, the coverage increment from 2-hop to 3-hop is not that significant, which coincides with our explanation that the more hops, the looser the constraints induced by the virtual NLOS range estimates, therefore leading to diminishing gains.

In Fig. 6, we compare the average root localization error

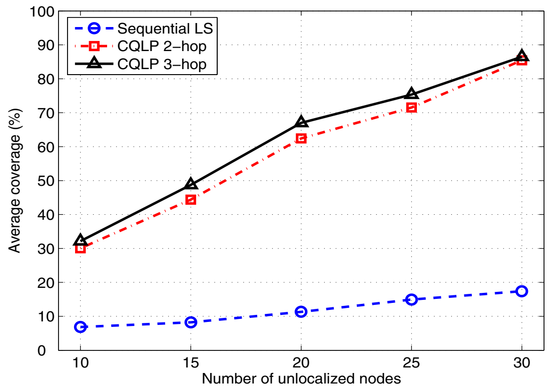

Fig. 5. Average localization coverage versus the number of unlocalized nodes, when $M = 5$, $R_{\text{cr}} = 6$.

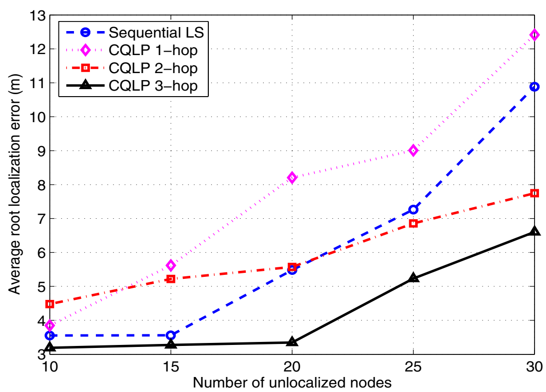

Fig. 6. Average root localization error versus the number of unlocalized nodes, when $M = 5$, $R_{\text{cr}} = 6$.

over 250 network realizations. We observe that the 1-hop CQLP performs worse than the sequential LS estimator. The CQLP with 2-hop node collaboration performs worse than the sequential LS estimator when the node density is low, while it performs a little bit better when the node density is high. This is easy to understand since as the node density increases, unlocalized nodes have a better chance to have more range estimates to localized nodes. On the other hand, the CQLP with 3-hop node collaboration performs consistently better than the sequential LS estimator. In addition, as the node density increases, the improvement of the CQLP with 3-hop node collaboration over the sequential LS estimator also increases. Figs. 5 and 6 together suggest that our proposed CQLP always provides significantly increased localization coverage and it is desirable to incorporate more than 2 hop node collaboration to provide improvement in the localization accuracy. On the other hand, for a high node density network, if the complexity is a concern, using 2-hop node collaboration is a reasonable choice, although its localization accuracy improvement over the sequential LS estimator is not as large as with 3-hop node collaboration.

V. Conclusion and Future Works

In this paper, we have proposed a complete collaborative quasi-linear programming framework for the general ad hoc sensor network localization problem. Our technique incorporates collaboration among unlocalized nodes by creating virtual NLOS range estimates. In addition, the range scaling algorithm is adopted to deal with the degenerate cases where an unlocalized node has less than three LOS range estiamtes. Simulation results show that the proposed method provides both increased localization coverage and improved localization accuracy. Our future work will include an examination of algorithms to improve localization accuracy even more as well as techniques to identify subnetworks where collaboration is likely to degrade performance.

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