Multiple-Access Design for Ad Hoc UWB Position-Location Networks

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Abstract— **In this paper, we analyze the problem of Medium Access Control (MAC) layer design for Impulse-based Ultra-Wideband (UWB) Position-Location Networks (PoLoNets). We focus our attention on PoLoNets in which stationary reference nodes are deployed in an ad hoc manner and determine their locations based on a small number of fixed anchors with known locations. Location and range information is propagated through the network of reference nodes in order to periodically estimate the locations of mobile nodes. The principal objective of our design is to minimize the localization error and convergence time of location estimates in the presence of node mobility and multipath. The properties of bounds on node localization accuracy in the presence range and location-estimation errors are derived. These results serve as a connection between the problem of multiple-access design and the accuracy of location estimates. A spread-spectrum based MAC protocol is developed that is shown to outperform the traditional Carrier-Sense Multiple-Access (CSMA) protocol in terms of accuracy and the convergence time of location-estimates.**

I. Introduction

Position-location is a desired feature in many commercial and military applications, such as inventory control, home automation, safety networks, tracking personal items, personnel monitoring, command-and-control in emergency and battlefield scenarios and the guidance of robots in remote or hazardous locations. In outdoor environments, accurate position information can be obtained via GPS. However, there are many situations where GPS is either unreliable (e.g., indoor scenarios), or impractical (e.g., where GPS receivers are too bulky or expensive), requiring the development of other solutions. For instance, consider the command-and-control of a fire-fighter operation [1] where multiple personnel are deployed in a building. For safety and efficiency purposes, it is desirable for a command-center located outside the building to establish not only communication but also personnel-position tracking. In such scenarios, we require *ad hoc* position location networks (PoLoNets) that are independent of GPS and not reliant on pre-existing infrastructure.

Impulse-based ultra-wideband (UWB) or Impulse-Radio is an excellent physical layer solution for the design of PoLoNets due to its robustness in harsh multipath environments, its ability to fuse accurate (on the order of tens of centimeters) position-location with low-data rate communication [2] and its covertness for tactical applications.

An overview of the application of UWB position-location networks to emergencies can be found in [1]. UWB PoLoNets based on established infrastructure are discussed in [3]. A PoLoNet can be viewed as a generic sensor network where the physical parameter being "sensed" is the location of the mobile nodes. However, unlike sensor networks, the lifetime required of such networks may be much shorter (e.g., in emergency scenarios) and thus energy efficiency is not always the primary focus, and in most cases, accuracy, reliability and scalability could take priority over energy efficiency. The accuracy and convergence time of mobile location estimates are key metrics that can be used to evaluate MAC protocols for such networks.

In this paper, we discuss the MAC design problem for ad hoc UWB PoLoNets from the perspectives of localization accuracy and convergence-time of location-estimates in these networks. Bounds on the accuracy of location-estimation [4] using TOA-based ranging [5] are used to derive results linking the performance of location-estimators with the number and accuracy of range-estimates and relative geometries of the nodes. These results are used to establish a link between the throughput of the applied MAC protocol and the resulting localization accuracy. It should be emphasized that the derived connection between throughput and localization accuracy is general and not specific to a UWB physical layer. The details of a spread-spectrum MAC protocol based on the Common-Transmitter protocol [6] that is applicable to UWB PoLoNets is discussed. We demonstrate via simulations that such a spread-spectrum protocol based on time-hopping (or direct-sequence spreading) [7] is a suitable multiple-access design from the perspectives of accuracy and convergence of location-estimates.

This paper is organized as follows: in section II, we describe the network architecture of ad hoc UWB position-location networks and the ranging mechanism which allows distributed location estimation by the nodes of the network. A model for the localization error which connects this mechanism to the design of MAC protocols for this application is also discussed in section II. The problem of multiple-access design for these networks is introduced in Section III and the details of a spread-spectrum MAC protocol are provided. Simulation results comparing the performance of the proposed protocol with CSMA in terms of the accuracy and convergence time of mobile location estimates are presented in Section IV. We conclude in Section V.

II. Network architecture

In this section, a specific network model that is applicable to PoLoNets and relies on minimal fixed infrastructure, is

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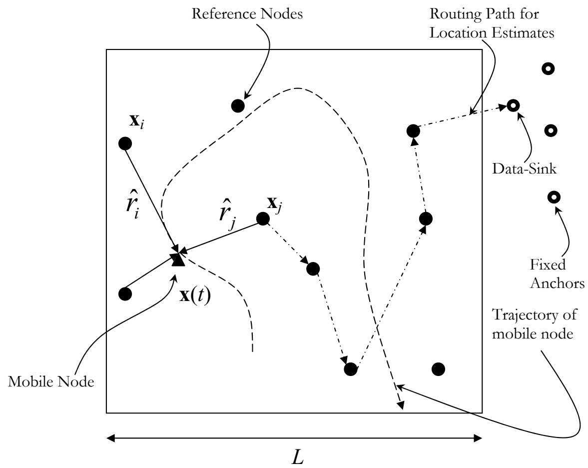

Fig. 1. Proposed Position-Location Network Architecture

presented. As shown in Figure 1 the network consists of a small number of *fixed anchors* located outside the area of interest whose locations are known a priori (e.g., a local coordinate system), *reference nodes* whose locations are not known a priori, and *mobile nodes*. The goals of this network are: (i) to allow any mobile nodes entering the area of interest to estimate their locations at regular intervals, and (ii), to route these location-estimates to a remote command-and-control (C&C) data-sink that receives and monitors these mobile-node location-estimates. In order to achieve these goals, the network proceeds in two phases:

a) Phase 1: Reference nodes are deployed in the region of interest in an ad hoc manner[1]. A subset of the deployed reference nodes that achieve connectivity with anchor nodes, triangulate their locations using time-of-arrival (TOA) based range-estimates from anchor nodes. Such “localized” reference nodes subsequently provide range-estimates to other “unlocalized” reference nodes, thereby “propagating” location-awareness among all reference nodes.

b) Phase 2: After the reference nodes have estimated their own locations by ranging to one another or to fixed anchors, the second phase of the network involves assisting any mobile node that enters the area of interest by providing a framework to estimate its own location. Mobile nodes, depending on their location and available connectivity, communicate with a subset of fixed anchors and/or reference nodes whose locations have been estimated, in order to determine their own locations via triangulation of range estimates. Additionally, in this phase, reference nodes provide a multi-hop communication network to relay the mobiles’ position information to the C&C. In this manner, position information is *propagated* from the fixed anchors outside the area of interest to the mobile nodes through the network of reference nodes.

The network model comprises a total of N nodes distributed randomly over an area[2] of dimension $L \times L$ (meters^2): N_M mobile nodes, N_R reference nodes whose locations are not known *a priori* and N_A (fixed) anchor nodes whose locations are known beforehand. It is clear that $N = N_M + N_R + N_A$, where typically $(N_M, N_A) \ll N_R$. The location of the ith node of the network is represented by the vector of x and y coordinates $\boldsymbol{x}_i = [x_i \ y_i]^T$, and $R_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ represents the distance or “range” between nodes i and j.

In the absence of network-wide synchronization, the mechanism of ranging between two nodes is via a packet-handshake as discussed in [2]. When an unlocalized mobile or reference node needs to estimate its current location, it broadcasts a *range-initiate (RI) packet*. Every node whose location is known or has been estimated, responds (provided it successfully decodes the RI packet) to this broadcast after a certain delay (see [2] or section III-B), with a *range-response (RR) packet*, which contains its own coordinates and possibly other data. Based on the delays between the transmission and reception times [2] of RI and RR packets, the unlocalized node can estimate the distances to the localized nodes which responded with RR packets. A useful feature of this scheme is that any data, and in particular, the coordinates of the localized nodes, can be exchanged between the nodes in the packet-handshake described above, since only the arrival-times of the packets are used for estimating range.

When an unlocalized mobile or reference node needs to estimate its current location, denoted by $\boldsymbol{x}$, it broadcasts an RI packet with a certain transmit power P_t. The effective received signal-to-noise-ratio (SNR) at localized node i, during the RI broadcast by the unlocalized node is modeled by $\xi_i = K_P P_t R_i^{-\beta}$, where β is the path-loss exponent, $R_i = \|\boldsymbol{x} - \boldsymbol{x}_i\|$ is the distance between the nodes, and K_P is a constant that subsumes the effects of other physical layer parameters. If multiple uncoordinated transmitting nodes are present, then the SNR would have to be replaced by signal-to-interference-and-noise ratio (SINR). We assume that the minimum SINR required for a packet to be received successfully is given by $\xi_{\min}$, resulting in a *maximum* radius of coverage $R_{\max}(P_t) = \left(\frac{K_P P_t}{\xi_{\min}}\right)^{\frac{1}{\beta}}$ around $\boldsymbol{x}$. Let $N(P_t)$ represent the number of localized (reference or anchor) nodes present within a radius $R_{\max}(P_t)$ from the mobile node. Consequently, for a transmit power P_t and in the absence of collisions, an unlocalized node can obtain $m \leq N(P_t)$ estimates $\{r_i\}$ of the true ranges $\{R_i\}$, $i = 1, 2, \cdots, m$, from the arrival times of the responses.

We assume that the range errors arising from the two-way ranging scheme described above are zero-mean Gaussian random variables:

$$r_i = R_i + n_i, \ n_i \sim \mathcal{N}(0, \sigma_i^2), \ i = 1, 2, \cdots, m. \qquad (1)$$

This assumption is fairly common in the literature on position-

[1]Deployment options are not considered here but reference nodes could either be deployed manually as in a fire-fighter scenario, via tiny robots, dispersed via UAV, or launched into the area of interest.

[2]We restrict our attention to the problem of two-dimensional position-location. However, the development is easily extended to the three-dimensional case.

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location [4], [8], although experimental results [9] indicate that range estimates may not be Gaussian due to the presence of multipath. However, in the presence of well-designed Time-Hopping (TH) codes [5] and ML estimators, which are asymptotically Gaussian and efficient [10], the range errors can be modeled as Gaussian random variables. It is further assumed that bias errors due to hardware, clock jitter etc., are eliminated via calibration. The presence of non-line-of-sight (NLOS) links can result in positive bias errors in UWB TOA-based ranging schemes [2], [9]. The impact of NLOS range errors on the performance of indoor PoLoNets is currently being investigated, but for simplicity, we restrict our attention to the case where the LOS path is not completely obstructed.

The Cramer-Rao Lower Bound (CRLB) for the estimation of the TOA of a single-multipath component in AWGN has been shown [5] to be

$$CRLB(\tau) = \frac{1}{N_p \gamma_p} \frac{\int_T p^2(t-\tau)dt}{\int_T \dot{p}^2(t-\tau)dt} = \frac{K_T}{\xi_{\text{eff}}},$$

where N_p is the number of pulses used for TOA-estimation, γ_p is the ratio of the energy per pulse to the noise power spectral density, $p(t)$ and $\dot{p}(t)$ are respectively the transmitted pulse and its time-derivative, T is the pulse-repetition period, $\xi_{\text{eff}} = N\gamma_p$ is the effective SNR and K_T is a constant that subsumes the other physical layer parameters. Once again, in the presence of multiple-access interference, the effective SNR above is replaced by the effective SINR. With the use of well-designed TH-codes and ML estimators, the CRLB for TOA-based range estimation in a multipath environment [5] can be approximated by

$$\sigma_R^2 \approx \frac{K_R}{\xi_{\text{eff}}}, \tag{2}$$

where ξ_{eff} is the effective SINR and $K_R = C_R c^2 K_T$, where $C_R > 1$ is a constant that takes the impact of multipath on range estimation into account, and c is the speed of light.

A. Triangulation of Unbiased Gaussian Range Estimates

The CRLB for the estimation of a node's location given unbiased Gaussian range estimates $\{r_i\}$ from known locations $\boldsymbol{x}_i$, $(i = 1, 2, \cdots, m)$ has been derived previously in [4], [8]. We define the *localization error* of a location-estimate in terms of the variances of the estimate of $\boldsymbol{x}$ in the x and y coordinates:

$$\Omega_{\boldsymbol{x}} = \sigma_x^2 + \sigma_y^2$$

The CRLB for the estimation of a location $\boldsymbol{x}$ of a node given m noisy Gaussian ranges $r_i \sim \mathcal{N}(R_i, \sigma_i^2)$, from nodes with known locations $\boldsymbol{x}_i$, $i = 1, 2, \cdots, m$, in terms of the localization error is given by:

$$\Omega_{\boldsymbol{x},m} = \frac{\sum_{i=1}^{m} \frac{1}{\sigma_i^2}}{\sum_{i=1}^{m} \sum_{j=1,j>i}^{m} \frac{\sin^2(\alpha_i - \alpha_j)}{\sigma_i^2 \sigma_j^2}}, \tag{3}$$

where α_i is the orientation (angle) of the ith localized node relative to the node whose location is being estimated: $\alpha_i = \measuredangle(\boldsymbol{x}_i - \boldsymbol{x})$. From (3), the localization error is a function of (i) the number of range estimates (m), (ii) the accuracy of the range estimates $(\sigma_i^2,\ i = 1, 2, \cdots, m)$ and (iii) the geometry of localized nodes $(\alpha_i,\ i = 1, 2, \cdots, m)$.

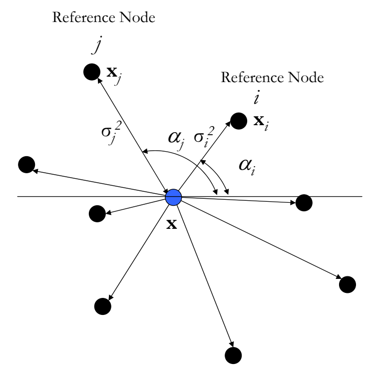

Fig. 2. The problem of node location estimation: given range estimates from localized nodes located at $\boldsymbol{x}_i$, $i = 1, 2, \cdots, m$, the goal is estimate the location $\boldsymbol{x}$. The orientation of the localized nodes relative to $\boldsymbol{x}$ are defined using $\alpha_i = \measuredangle(\boldsymbol{x}_i - \boldsymbol{x})$.

Using the notation

$$\gamma_m = \sum_{i=1}^{m} \frac{1}{\sigma_i^2}, \quad \psi_m = \sum_{i=1}^{m} \sum_{j=1,j>i}^{m} \frac{\sin^2(\alpha_i - \alpha_j)}{\sigma_i^2 \sigma_j^2},$$

we define the *Generalized Geometric Dilution of Precision* (GGDOP) as

$$\Gamma_m = \frac{\psi_m}{\gamma_m^2} \;\Rightarrow\; \Omega_{\boldsymbol{x},m} = \frac{\gamma_m}{\psi_m} = \frac{1}{\gamma_m \Gamma_m}. \tag{4}$$

Note that this is a generalized version of the classical GDOP definition [4], which constitutes the special case $\sigma_i^2 = \sigma^2,\ \forall i$. For a given value of γ_m, as the GGDOP Γ_m increases, the localization error decreases. It must be noted that Γ_m is not purely a function of the geometry of unlocalized nodes, but also depends on the range estimate variances. However, scaling all the range estimate variances by a common factor while maintaining the same relative orientations does not alter the value of Γ_m. It can be shown (see Appendix I) that $\forall\ \{\sigma_i^2\}, \{\alpha_i\}$: $0 \le \Gamma_m \le \frac{1}{4}$.

The localization error of a mobile node depends upon the number of range-estimates available for localized reference or anchor nodes through m. Suppose we have an initial geometric configuration of localized nodes $\{\alpha_i\}$, $i = 1, 2, \cdots, m$ with corresponding range variances $\{\sigma_i^2\}$, $i = 1, 2, \cdots, m$. As shown in Appendix II, the introduction of an additional range-estimate from a node with orientation and variance $\left(\alpha_{m+1}, \sigma_{m+1}^2\right)$ results in a reduction of the localization error. Specifically,

$$\Omega_{\boldsymbol{x},m+1} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{\gamma_m}{2\sigma_{m+1}^2} - \frac{\sqrt{\gamma_m^2 - 4\gamma_m}\cos(2\alpha_{m+1} - 2\nu)}{2\sigma_{m+1}^2}} \le \Omega_{\boldsymbol{x},m}, \tag{5}$$

where ν is given by (10). For the special case: $\alpha_{m+1} = \alpha_k$, $\sigma_{m+1}^2 = \sigma_k^2$, where $k \in \{1, 2, \cdots, m\}$, the improvement

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in localization error can be viewed as a repeated range measurement followed by averaging of range estimates, which reduces the range variance and the localization error. From the above property, except when all the localized anchor nodes are collinear, increasing the number of range estimates always improves localization accuracy defined through the CRLB.

B. A Simple Least-Squares (LS) estimator

The CRLB localization error given by (3) provides a benchmark for comparing the performance of location-estimation, but does not explicitly describe the estimator [10] that achieves it. In order to evaluate the different MAC schemes from a practical standpoint, an estimator that is easily implemented would be valuable. The Least-Squares (LS) estimation approach is known to be suitable if the statistical distribution of the available data is not known [10], and is briefly described below.

Let $\boldsymbol{x} = [x\ y]^T$ denote the coordinates of the node whose location is to be estimated. Let the coordinates of m localized nodes $\boldsymbol{x}_i$, $(i = 1, 2, \cdots, m)$ and the estimates r_i of the ranges R_i be given. Therefore, we have m equations of the form

$$\|\boldsymbol{x} - \boldsymbol{x}_i\|^2 = \|\boldsymbol{x}\|^2 + \|\boldsymbol{x}_i\|^2 - 2\boldsymbol{x}_i^T\boldsymbol{x} = r_i^2,\ i = 1, 2, \cdots, m.$$

Taking the difference of each of the above m equations, this system reduces to a set of $\binom{m}{2}$ equations of the form

$$2(\boldsymbol{x}_i - \boldsymbol{x}_j)^T\boldsymbol{x} = (R_{i0}^2 - R_{j0}^2) - (r_i^2 - r_j^2),$$

$i = 1, 2, \cdots, m-1,\ j > i$, where $R_{i0} = \|\boldsymbol{x}_i\|$. The above set of linear equations is of the form $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{A}$ is a $\binom{m}{2} \times 2$ matrix whose rows are from the set $\{2(\boldsymbol{x}_i - \boldsymbol{x}_j)^T\}$, $(i = 1, 2, \cdots, m-1),\ j > i$ and $\boldsymbol{b}$ is a column vector of length $\binom{m}{2}$ whose components are from $\{(R_{i0}^2 - R_{j0}^2) - (r_{is}^2 - r_{js}^2)\}$. The LS estimate of the node's location [10] is given by

$$\hat{\boldsymbol{x}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b} \tag{6}$$

The localization error for the LS estimator is defined as $\Omega_{\boldsymbol{x},LS} = \|\hat{\boldsymbol{x}} - \boldsymbol{x}\|^2$.

In Figure 3, the performance of the LS estimator is compared with the CRLB for location-estimation (see figure caption for simulation parameters). The average localization error for both cases in compared, as m increases. In the LS case, the data (range estimates) cannot be expressed as a linear model in terms of $\boldsymbol{x}$, and therefore, this estimator is not the minimum-variance unbiased estimator [10]. Consequently, the LS localization error is always larger than the localization error given by (3). Figure 3 also shows the localization error of the LS location estimator when the range estimates *and* the localized node location-estimates are noisy. We see that even with noisy location estimates of the localized nodes $\hat{\boldsymbol{x}}_i \sim \mathcal{N}(\boldsymbol{x}_i, \sigma_P^2\boldsymbol{I})$, increasing the number of available range estimates decreases the localization error, which was identical to the trend observed through the CRLB.

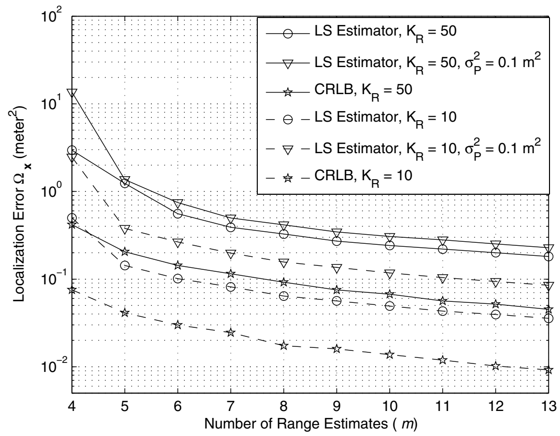

Fig. 3. Localization error versus the number of range estimates available at an unlocalized node using an LS estimator with (a) noisy range estimates $r_i \sim \mathcal{N}(R_i, \sigma_i^2)$, $\sigma_i^2 = \frac{K_R R_i^\beta}{K_P P_t}$, $\beta = 2$, $P_t = 1$ mW, and exact localized node coordinates $\hat{\boldsymbol{x}}_i = \boldsymbol{x}_i$ and (b), noisy range estimates $r_i \sim \mathcal{N}(R_i, \sigma_i^2)$ and noisy localized node coordinates $\hat{\boldsymbol{x}}_i \sim \mathcal{N}(\boldsymbol{x}_i, \sigma_P^2\boldsymbol{I})$. The localized nodes are assumed to be uniformly distributed over a 20×20 meter^2 area.

III. MAC Design for UWB PoLoNets: Localization Accuracy and Convergence-Time

It is clear from the preceding analysis and Figure 3 that on the average, the accuracy of the LS location estimate of an unlocalized node is determined by the number of ranges successfully received from other localized nodes. It was shown using the CRLB that, except when all localized nodes were collinear, increasing the number of range estimates results in reduction of the localization error. Even when connectivity with localized nodes is limited, repeated range measurements allow averaging of range estimates, which reduces their variance and, hence, the localization error.

Therefore, a MAC protocol that allows each unlocalized node to accumulate several range estimates in a short duration, increases the likelihood that an accurate estimate of the node's location is computed at the end of that duration. In the case of reference nodes, these accurate location estimates subsequently translate to accurate estimates of the location of the mobile nodes, since mobile nodes utilize these coordinates to triangulate their own locations. In the case of mobile nodes, the lower latency in obtaining accurate location estimates allows mobile nodes to obtain estimates of their locations more frequently, which is especially important at high mobile node speeds.

In terms of the ranging scheme discussed, this implies that a MAC protocol which provides a *higher effective throughput* of range estimates (RI and RR packets) allows faster convergence of location-estimates of mobile and reference nodes to their true locations. Similarly, in the second phase of the network location-estimation, where the estimates of locations of mobile nodes are routed back to a monitoring station, a higher effective throughput reduces the lag between the estimation of the mobile's location and the availability of this information at a remote monitoring location, provided the MAC protocol treats range-packet handshakes and data exchanges in the same manner.

A multi-channel spread-spectrum protocol allows simultaneous transmissions by the nodes of the network at the cost of incurring multi-access interference; a "single-channel" approach such as CSMA on the other hand does not allow nodes in the same vicinity to transmit simultaneously. Additionally, due to the high amount of spreading (high transmission bandwidth to data-rate ratio) inherent in the use of UWB signals for this low-rate application, a "single-channel" approach such as CSMA appears wasteful. This suggests that a time-hopping (TH) (or direct-sequence) spread-spectrum MAC protocol could provide a much higher location estimate convergence rate than the CSMA protocol. The details of such a protocol, designed specifically for UWB PoLoNets are described in the following section.

A. Proposed Solution: TH-CDMA MAC Protocol

As mentioned previously, in the absence of network-wide synchronization, the process of ranging between two nodes is performed through an exchange [2] of range-initiate (RI) and range-response (RR) packets. An efficient MAC protocol from the perspective of localization accuracy (i) provides higher rates of range packet exchanges within the network between unlocalized and localized nodes, and (ii) allows an arbitrary number of packet exchanges between several unlocalized nodes and localized nodes, in order to ensure scalability (in terms of the number of nodes in the *ad hoc* PoLoNet). The MAC protocol we propose for ad hoc UWB position-location networks is based on the Common Transmitter Spread Spectrum Multiple Access protocol (C-T SSMA) [6], modified to in order meet the above requirements.

The details of the scheme are as follows: at an arbitrary instant of time, nodes would ideally like to obtain range-estimates from every other localized node in the network, when their location-estimates are either unknown or not up-to-date (in the case of mobile nodes). The contention between such unlocalized nodes for broadcasting RI packets to localized nodes occurs on a "common" [6] spreading code C_0 and all nodes, when idle, are listening on the common code C_0. However, each RI packet specifies a "private" code C_i, unique to the node broadcasting it, on which RR packets from localized nodes are to be received. As discussed below, this allows several range exchanges to proceed simultaneously in all regions of the PoLoNet, at the cost of increased multiple-access interference.

B. Multiple-Access Ranging

The RI packet broadcast by a node i at $t = t_1$ contains[3] a new TH-code C_i. As soon as node i transmits the RI packet on code C_0, it begins to listen on code C_i for a window of time from $t = t_1$ to $t = t_1 + T_W$. The window-length T_W is much larger than the duration T_{RR} of an RR packet, $T_W \gg T_{RR}$, which allows multiple RR packets to be gathered within the duration of the window[4].

In order to allow an arbitrary number of range estimates to be gathered within this window, the following scheme, illustrated in Figure 4 is used: Suppose node i broadcasts the RI packet on C_0 at time $t = t_1$. In the absence of collisions, this packet reaches a localized node j at $t = t_1 + \tau_0$, where $\tau_0 = \frac{R_{ij}}{c}$ represents the propagation delay between nodes i and j. Node j synchronizes itself to the TOA of the received packet. Node j then responds with an RR packet on the private code C_i after a random integer multiple of a known delay Δ_T (at $t = t_1 + \tau_0 + k\Delta_T$), where k is a random positive integer in the interval $\left[1, \left\lfloor \frac{T_W}{\Delta_T} \right\rfloor\right]$. Provided there are no collisions, node i receives the RR packet from node j and determines its TOA, which is $t_2 = 2\tau_0 + t_1 + k\Delta_T$. Node A then computes the difference $\Delta t = t_2 - t_1 = 2\tau_0 + k\Delta_T$, from which the known delay Δ_T can be eliminated, without any ambiguity in interpreting the range, by ensuring that $\tau_0 \ll \Delta_T$. The range r_{ij} can then be computed using $r_{ij} = \frac{c(t_2 - t_1 - k\Delta_T)}{2}$.

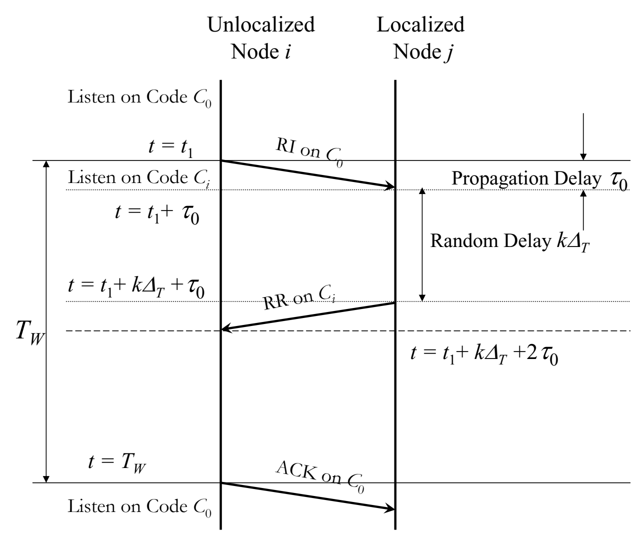

Fig. 4. Timing Diagram for the Proposed MAC protocol.

The use of the random delay $k\Delta_T$, $k \in \left[1, \left\lfloor \frac{T_W}{\Delta_T} \right\rfloor\right]$, in the above mechanism is for collision-avoidance between RR packets transmitted on the private code of the node that transmitted the RI packet. Once node j transmits its RR packet, it continues listening on code C_0. The unlocalized node i decodes data on code C_i and collects the packets from all localized nodes that transmitted an RR packet. An RR packet is assumed to be successfully received if the SINR at node i

[3]In order to avoid large overheads, the RI packet from node i could specify a parameter that can be used to uniquely compute C_i. For instance, in the case of random codes, the seed used to generate the random TH (or DS) sequences can be specified. Further, the use of random codes does not place a stringent limitation on the number of available private codes.

[4]The duration of the window T_W can be adapted based on the desired localization-accuracy and acceptable latency, since a longer window-length will likely result in a larger number of received RR packets. However, this discussion is outside the scope of this work.

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during the time of reception exceeds the threshold $\xi_{\min}$. Node i thereby estimates the distance r_j to every localized node j that successfully sends an RR packet, and computes an estimate $\hat{\boldsymbol{x}}_i$ of its location. The accuracy of the range estimate r_j is also determined by the SINR at node i through (2). At $t = T_W$, node i assumes that all other localized nodes are out of range and broadcasts an ACK packet on C_0 declaring the newly acquired range estimates and (newly computed) coordinates.

If node i receives no RR packets (due to collision between RI packets on the common code C_0, collisions between RR packets on the private code C_i, or because there are no localized nodes within range) within T_W seconds of the RI broadcast, it times out and (exponentially) backs-off for $T_W \cdot 2^{n-1}$ seconds, where n represents the index of the retransmission attempt. Additionally, node i can increase the transmit power from P_i to $P_i + \Delta P$ in its subsequent retransmission attempt. If node i receives more RR packets within the window than necessary to maintain the localization accuracy, it then decreases the transmit power from P_i to $P_i - \Delta P$ in its subsequent range-initiate transmission, which reduces power consumption. Such power-control algorithms, based on localization accuracy, are the subject of [11].

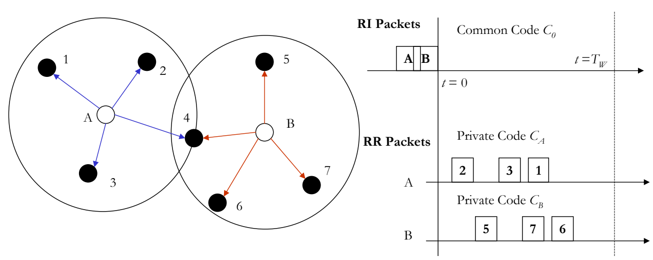

Fig. 5. An illustration of the multiple access ranging procedure. In this situation, two nodes A and B simultaneously wish to obtain range estimates from localized nodes around them (nodes 1 through 7). The RI packets broadcast by nodes A and B on the common code C_0 collide at node 4, and as a result node 4 does not respond with RR packets. The remaining nodes respond with RR packets on the private codes C_A and C_B, each offset by a random delay $k\Delta_T$. Nodes A and B each collect $m = 3$ range estimates at the end of this process.

An example scenario based on the above protocol mechanism is illustrated in Figure 5, where two unlocalized nodes simultaneously attempt to receive range estimates. The benefits of the proposed multiple-access mechanism are as follows: (1) It allows independent ranging to proceed independently and simultaneously, achieving efficient spatial reuse in different parts of the network and therefore reduces latency; (2) it is scalable, with a larger number of nodes being accommodated at the price of higher multiple-access interference leading to graceful degradation in performance; (3) Network-wide synchronization is not required, and (4) the hardware complexity of a node is kept small by ensuring that it is not necessary for a node to listen and transmit at the same time, or listen simultaneously on multiple TH-codes.

Due to the ability of the discussed ranging mechanism to "piggyback" data on RI and RR packets, a node that transmits a RI packet can transmit all the range and coordinate information it has along with the time-stamps for each of these values. It must be noted that the same MAC protocol can be used for data transmissions in the second phase of the network, to route the coordinates of the nodes to a data sink, by making a distinction between the data and ranging packets. In such a case, the high throughputs promised by the proposed approach result in a lower latency in the arrival of this data at the data-sink that monitors the node locations.

IV. Simulation Results

In order to evaluate the performance of the proposed multiple-access scheme, we compare its performance with carrier-sense multiple access (CSMA), since CSMA forms the basis of most MAC protocols typically investigated for mobile ad hoc and sensor networks. The performance of the proposed scheme was compared with CSMA in terms of the average convergence rate of the total localization error of mobile and reference node location estimates, as well as the average number of RR packets successfully received in an interval. The total localization error of the PoLoNets at any instant t is defined as

$$\Omega(t) = \left(\overbrace{\sum_{i=1}^{N_M} \|\boldsymbol{x}_i(t) - \hat{\boldsymbol{x}}_i(t)\|^2}^{mobile\ nodes} + \overbrace{\sum_{k=1}^{N_R} \|\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k(t)\|^2}^{stationary\ reference\ nodes} \right)$$

In the CSMA case, RI and RR packets are transmitted on a single TH-code C_0. Nodes "sense" the channel by listening on C_0 before transmitting. A transmission by a node j is sensed at node i if the SNR corresponding to that transmission exceeds a threshold ξ_L at node i. A node that transmits an RI packet awaits RR packets from localized nodes within a window of duration T_W. If no RR packets are received within that window (or if an ongoing transmission was sensed before broadcasting the RI packet), the node exponentially backs-off for $T_W \cdot 2^{n-1}$ seconds, where n represents the index of the retransmission attempt, before re-broadcasting the RI packet.

In these simulations, the reference and mobile nodes are deployed at $t = 0$ and the mobile nodes move linearly with speed $v_M = 0.25\ m/s$. It was assumed that $N_M = 2$, $N_R = 10$ and $N_A = 5$. The stationary reference nodes were uniformly distributed over the $L \times L$ area, defined by the lines $x = 0$, $x = L$, $y = 0$, $y = L$. The fixed anchor nodes were assumed to located outside the area of interest at $\{(L/2 \pm L/4, -L/2 \pm L/4), (L/2, -L/2)\}$.

The LS estimator, defined by (6), was used to estimate node locations with the total localization error of unlocalized nodes arbitrarily set at a large value ($\sim 2000\ m^2$) at $t = 0$. The value of K_R used to generate the range estimate variances in (2) was $K_R = 100$, and the transmit power of all nodes was fixed at $P_t = 1\ mW$. The probability of an unlocalized node transmitting an RI packet in any given time-slot was set at $p = 0.15$, and the duration of the window $T_W = 20T_s$, where the width of a slot was $T_s = 50\ ms$, and the durations of RI and RR packets and the delay Δ_T are assumed to be equal: $T_{RI} = T_{RR} = \Delta_T = 20\ ms$.

The SINR at a node j corresponding to a signal from node i is computed using the relation

$$\xi_{j \leftarrow i} = \frac{K_P P_t R_{ij}^{-\beta}}{N_0 + \sum_{k \neq i} g_k K_P P_t R_{ik}^{-\beta}} \tag{7}$$

where k represents the index of simultaneous transmissions, N_0 represents the noise-power spectral density and g_k incorporates the effect of spreading gain if multiple TH codes are used. If the kth simultaneous transmission occurs on the same code as node i, then $g_k = 1$, else, $g_k = 1/64$. The threshold SINR and SNR respectively determining the successful decoding of packets and sensing of ongoing packet transmissions are $\xi_{\min} = 20$ dB and $\xi_L = 10$ dB respectively.

The performance of the two protocols in terms of the average number of RR packets received successfully within the network versus time, for different values of β and L, was compared. Figure 6 depicts the performance for $\beta = 2$ and different values of L. We see that the proposed approach has a much higher effective throughput of RR packets than CSMA. The average total localization error $E\{\Omega(t)\}$ was computed by averaging $\Omega(t)$ over a large number of simulation runs. Figures 7 and 8 compare the performance of the two protocols in terms of the average total localization error of mobile and reference nodes versus time for different values of β. We see that the average total localization error $E\{\Omega(t)\}$ decreases much faster with t for the proposed approach than for the CSMA scheme, which is expected since the localization error and the average effective RR packet throughputs are strongly (negatively) correlated.

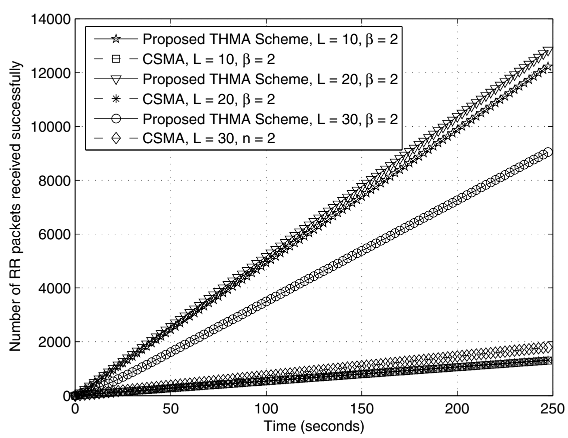

Fig. 6. Comparison of CSMA and the Proposed Scheme in terms of the average number of successful RR packet transmissions versus time for $\beta = 2$.

In the initial phase after deployment after $t = 0$, relatively few localized nodes (only N_A fixed anchors at $t = 0$) can provide range information to unlocalized (reference and mobile) nodes. Consequently, location estimates are inaccurate, and can even be outside the area of interest[5] leading to large localization errors. As time progresses, more reference nodes estimate their locations and consequently, and therefore more range estimates are available to the mobile nodes and unlocalized reference nodes. At this point, the total average localization error decreases sharply.

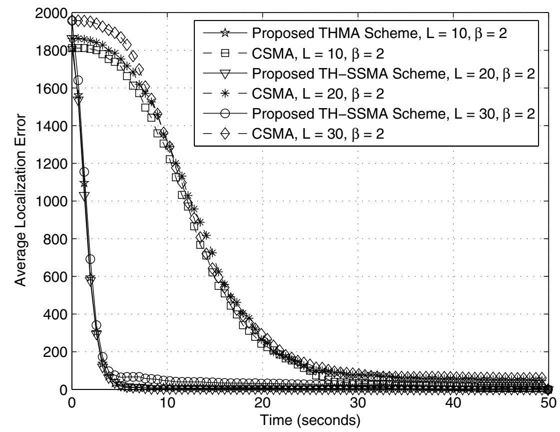

Fig. 7. Comparison of CSMA and the Proposed Scheme in terms of the average total localization error $E\{\Omega(t)\}$ versus time (t) for $\beta = 2$.

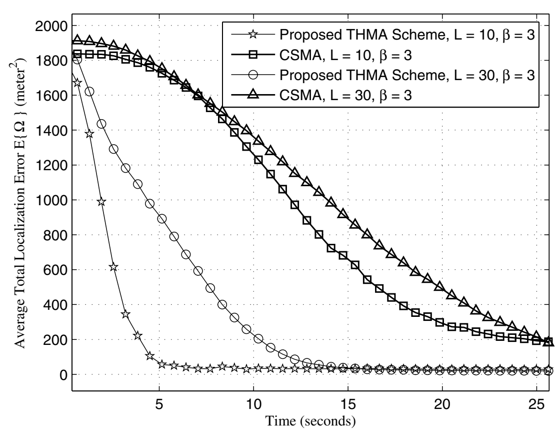

Fig. 8. Comparison of CSMA and the Proposed Scheme in terms of the average total localization error $E\{\Omega(t)\}$ versus time (t) for $\beta = 3$.

With $\beta = 2$, an increase in the area (L^2) of the network does not have a significant impact on performance. For the $\beta = 3$ case[6], increasing the area of the network reduces the effective throughput of RR packets. There are two opposing factors at work: as the area increases, the average distance between nodes increases, thereby decreasing the probability

[5]It is assumed that the nodes are not aware of the boundaries of the area of interest, precluding the possibility of eliminating such egregious location-estimates.

[6]Although we assume a LOS model for the range estimation process, this scenario represents the case where the LOS multipath component is severely attenuated (soft-NLOS).

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of successful detection of RI and RR packets. However, as the area increases, the inherent spatial reuse increases; if the density of nodes is kept constant as the area of the network is increased, the benefits of spatial reuse in terms of increased effective throughput are observed.

V. Conclusions

This paper discussed the problem of multiple-access design for ad hoc UWB PoLoNets from the perspectives of localization error and convergence time of location-estimates. We looked at the characterization of the accuracy of node location estimates, through Cramer-Rao lower bound analysis of the problem of location estimation, given unbiased Gaussian range information from location-aware nodes. A novel characterization of the properties of these bounds provided insight into the problem of MAC design and indicate that the problem of minimizing localization error in PoLoNets is equivalent to the problem of maximizing effective throughput of range information within the network.

We then described a spread-spectrum protocol based on time-hopping for UWB PoLoNets, which is based on the common-transmitter spread-spectrum multiple-access approach. Network simulations were performed to compare the performance of the proposed protocol with CSMA, which is one of the most commonly investigated protocols for ad hoc and sensor networks. We showed that the proposed protocol is superior to CSMA in terms of the localization accuracy and convergence time of the location estimates.

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Appendix I: Bounds on Γ_m

The generalized GDOP is defined by

$$\Gamma_m = \frac{\psi_m}{\gamma_m^2} = \frac{\sum_{i=1}^{m}\sum_{j=1,j>i}^{m}\frac{\sin^2(\alpha_i-\alpha_j)}{\sigma_i^2\sigma_j^2}}{\left(\sum_{i=1}^{m}\frac{1}{\sigma_i^2}\right)^2} = \frac{1}{4} - \frac{\left(\sum_{i=1}^{m}\frac{\cos 2\alpha_i}{\sigma_i^2}\right)^2 + \left(\sum_{i=1}^{m}\frac{\sin 2\alpha_i}{\sigma_i^2}\right)^2}{4\left(\sum_{i=1}^{m}\frac{1}{\sigma_i^2}\right)^2}$$

When $\alpha_i = \alpha_j\ \forall i,j$, we see that we obtain the lower bound, $\Gamma_m = 0$. The upper limit $\Gamma_m = \frac{1}{4}$ is achieved when $\sum_{i=1}^{m}\frac{\cos 2\alpha_i}{\sigma_i^2} = 0$ and $\sum_{i=1}^{m}\frac{\sin 2\alpha_i}{\sigma_i^2} = 0$. A similar albeit less general result can be found in [8].

Appendix II. Proof of Inequality (5)

The localization error with $(m+1)$ range estimates is:

$$\Omega_{\boldsymbol{x},m+1} = \frac{\sum_{i=1}^{m+1}\frac{1}{\sigma_i^2}}{\sum_{i=1}^{m+1}\sum_{j=1,j>i}^{m+1}\frac{\sin^2(\alpha_i-\alpha_j)}{\sigma_i^2\sigma_j^2}} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{1}{\sigma_{m+1}^2}\sum_{i=1}^{m}\frac{\sin^2(\alpha_i-\alpha_{m+1})}{\sigma_i^2}} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{\zeta}{\sigma_{m+1}^2}}, \quad (8)$$

where $\zeta = \sum_{i=1}^{m}\frac{\sin^2(\alpha_i-\alpha_{m+1})}{\sigma_i^2} \geq 0$. After some manipulation, we can show that

$$\zeta = \frac{\gamma_m}{2} - \frac{\sqrt{\gamma_m^2 - 4\psi}\cos\left(2\alpha_{m+1} - 2\nu\right)}{2}, \quad (9)$$

where the angle ν is defined as

$$\nu = \frac{1}{2}\arctan\left(\frac{\sum_{i=1}^{m}\left(\frac{\sin 2\alpha_i}{\sigma_i^2}\right)}{\sum_{i=1}^{m}\left(\frac{\cos 2\alpha_i}{\sigma_i^2}\right)}\right). \quad (10)$$

From (8) and (9),

$$\Omega_{\boldsymbol{x},m+1} = \frac{\gamma + \frac{1}{\sigma_{m+1}^2}}{\psi + \frac{\gamma_m}{2\sigma_{m+1}^2} - \frac{\sqrt{\gamma_m^2-4\psi}\cos\left(2\alpha_{m+1}-2\nu\right)}{2\sigma_{m+1}^2}}. \quad (11)$$

From (4) and (11), followed by simplification,

$$\Omega_{\boldsymbol{x},m} - \Omega_{\boldsymbol{x},m+1} = \left[\frac{\frac{1}{2} - \frac{\sqrt{1-4\Gamma_m}\cos\left(2\alpha_{m+1}-2\nu\right)}{2} - \Gamma_m}{\gamma_m\left(\sigma_{m+1}^2\psi + \frac{\gamma_m}{2} - \frac{\sqrt{\gamma_m^2-4\psi}\cos\left(2\alpha_{m+1}-2\nu\right)}{2}\right)}\right] = \left[\frac{\frac{\left(\sqrt{1-4\Gamma_m}-\cos\left(2\alpha_{m+1}-2\nu\right)\right)^2+\sin^2\left(2\alpha_{m+1}-2\nu\right)}{2}}{\gamma_m\left(\sigma_{m+1}^2\psi + \frac{\gamma_m}{2} - \frac{\sqrt{\gamma_m^2-4\psi}\cos\left(2\alpha_{m+1}-2\nu\right)}{2}\right)}\right].$$

Since the numerator and denominator are always positive, $\Omega_{\boldsymbol{x},m} \geq \Omega_{\boldsymbol{x},m+1}$.

References

[1] S. J. Ingram, D. Harmer, and M. Quinlan, "Ultra-wideband Indoor Positioning Systems and their Use in Emergencies," in *Position Location and Navigation Symposium, 2004. PLANS 2004*, Rome, Italy, April 2004.

[2] J.-Y. Lee and R. A. Scholtz, "Ranging in a dense multipath environment using an UWB radio link," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 9, pp. 1677–1683, December 2002.

[3] I. Oppermann, L. Stoica, A. Rabbachin, Z. Shelby, and J. Haapola, "UWB wireless sensor networks: UWEN - a practical example," *IEEE Communications Magazine*, vol. 42, no. 12, pp. S27–S32, December 2004.

[4] N. Patwari, A. O. Hero, M. Perkins, N. S. Correal, and R. J. O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 51, no. 8, pp. 2137–2148, Aug. 2003.

[5] J. Zhang, R. R. A. Kennedy, and T. D. Abhayapala, "Cramer-Rao lower bounds for the time delay estimation of UWB signals," in *2004 IEEE International Conference on Communications (ICC 2004)*, vol. 6, 20-24 June 2004, pp. 3424 – 3428.

[6] E. S. Sousa and J. A. Silvester, "Spreading code protocols for distributed spread-spectrum packet radio networks," *IEEE Transactions on Communications*, vol. 36, no. 3, pp. 272–281, February 1988.

[7] L. D. Nardis, G. Giancola, and M.-G. D. Benedetto, "Power-aware design of MAC and routing for UWB networks ," in *2004 IEEE Global Telecommunications Conference Workshops*, 2004.

[8] C. Chang and A. Sahai, "Estimation bounds for localization," in *IEEE SECON 2004*, 4-7 Oct. 2004, pp. 415 – 424.

[9] B. Denis, J. Keignart, and N. Daniele, "Impact of NLOS propagation upon ranging precision in UWB systems," in *2003 IEEE Conference on Ultra Wideband Systems and Technologies*, 2003.

[10] S. M. Kay, *Fundamentals of Statistical Processing, Volume I : Estimation Theory*, 1993, 2nd Edition.

[11] S. Venkatesh and R. M. Buehrer, "Power-Control for UWB PoLoNets," in *Accepted for Publication at the International Conference on Communications (ICC 2006)*, Istanbul, Turkey, 11-15 June 2006.

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