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Mitigation of the propagation of localization error using multi-hop bounding

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***Abstract*—In ad hoc position-location networks, location information is obtained through the sequential estimation of node locations. An unlocalized node can estimate its location based on range and location estimates of nearby localized ("anchor") nodes, and subsequently provide range and location information to other unlocalized nodes in its vicinity. In such distributed location-estimation scenarios, the accuracy of node location estimates is degraded due to the propagation of localization errors, particularly in NLOS propagation environments. This paper discusses the use of a novel method of mitigating the propagation of localization error using the linear programming framework proposed by the authors in [1]. This method utilizes multi-hop distance estimates to create additional constraints on the feasible region for a node's location, thereby limiting the propagation of error even in NLOS propagation environments.**

Index Terms **—** ad hoc localization, Ultra-wideband (UWB), position-location networks, TOA-based ranging, NLOS environments.

I. Introduction

The envisioned applications for ad hoc wireless networks often depend on the automatic and accurate location of deployed nodes. Accurate node location estimation enables applications such as inventory management [2], intrusion detection [3], traffic monitoring, and locating emergency workers in buildings [4]. In outdoor situations with a clear view of the sky, the Global Positioning System (GPS) is a practical means of obtaining position information. However, if GPS receivers are too bulky for the sensor application or GPS is not available (e.g., indoors or in dense forestation), network-based positioning becomes a reasonable alternative. In infrastructure-based position-location networks (PoLoNets), "anchor" nodes are extensively deployed within the area of interest and programmed with their location coordinates. This allows direct tracking of the locations of unlocalized or mobile nodes through ranging and triangulation. In an ad hoc network architecture without pre-deployed fixed infrastructure, anchor nodes can be deployed inside or outside the area of interest with connectivity to a relatively small subset of nodes. The network then relies on those nodes to propagate location information into the area of interest, thereby allowing node localization.

In this paper, we focus our attention on location-estimation in ad hoc PoLoNets which must propagate location information throughout the network from a limited number of anchors. In this type of location-aware network, unlocalized nodes that obtain a sufficient number of range estimates from the fixed anchors can estimate their own locations in a distributed fashion, and then can act as anchors for other unlocalized nodes. Thus, location information is sequentially propagated from the "true" anchors to the unlocalized nodes. In such networks, the resulting location estimation error can grow as the information propagates through the network. This is referred to as *error propagation* and can severely degrade localization accuracy at locations distant from the true anchors. This propagation of localization error limits the area over which a desired localization accuracy can be attained. As an example, consider a group of fire-fighters entering a building without position-location infrastructure. By rapidly deploying sensors into the environment, an ad hoc sensor network could be established to provide position location information to the fire-fighters. For such an application, it is desirable that the requisite localization accuracy be maintained throughout the network. Thus, techniques which mitigate the impact of error propagation on localization accuracy are needed. In addition, as we will show in this work, in networks which are based on time-of-arrival (TOA) or received signal strength, error propagation can be exacerbated by non-line-of-sight (NLOS) connections between nodes. Since NLOS propagation environments are common in indoor or urban scenarios where traditional position location techniques may not be directly applicable [1], it is important that NLOS conditions be considered.

The propagation of localization error problem and its mitigation are discussed in [5], and briefly in [6]. In [5], radio implementation constraints on the maximum angle-of-arrival (AOA) errors and range estimation errors from one-hop neighbors of a given node are used to constrain the set of possible locations for the node. The main drawback to such an approach is that additional information such as AOA information or radio limits may not be available in the absence of multiple antennas. Due to the complexity constraints, it is unlikely that multiple antennas will be available on many typical sensors. The approach proposed in this paper does not require such information. Additionally, if the information were available, the proposed method can incorporate AOA information as well and potentially improve performance. In [6] the covariance matrix of the anchor position errors is relayed from node to node to provide a better estimate of the weighting matrix in the weighted least-square (WLS) formulation. However, estimating and feeding back the connectivity matrix for WLS estimation may prove to be impractical in many cases.

The method presented here is based on a novel NLOS mitigation method which uses linear programming and was first presented in [1] for mitigating the adverse effect of

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NLOS range estimates. In the current work, we extend the technique to provide a more general method which (in addition to mitigating the impact of NLOS connections) limits the propagation of localization error. Although we will apply the technique to Ultrawideband (UWB) based ranging data, the approach is more general and can be applied to any range-based position location system that allows for LOS/NLOS classification.

This paper is organized as follows: in Section II, we discuss the propagation of localization error, its implications, and methods of mitigation. Section III discusses a novel approach of mitigation based on linear programming. Simulation and measurement results that demonstrate the efficacy of the proposed method are presented in Section IV. Section V concludes this paper.

II. Propagation of Localization Error

For a given network of nodes, there are two basic classes of location estimation approaches: fully-distributed approaches and centralized approaches. In fully-distributed approaches, each node can determine its own location based on range estimates to at least three (for two-dimensional location-estimation) anchor nodes (i.e., nodes which have location awareness). Centralized location-estimation approaches consider all connectivity and range information simultaneously and use a centralized position solver to determine the location of all nodes. These location estimates are then routed back to the individual nodes. The former approach requires that all nodes have connectivity to at least three anchor nodes, and may be impractical in many situations, especially indoors. Centralized approaches require fewer anchor nodes, but require centralized processing which has a very high communication and routing cost, and places a computational burden on the centralized solver.

A compromise approach is one which uses distributed location estimation but requires any node which is able to determine its location to become a *virtual* anchor, as opposed to *true* anchors whose locations are known apriori, to aid in localizing other nodes in the network. As a result, location estimation information propagates through the network. We will refer to methods which must propagate location information in such a manner as *sequential estimation* techniques.

In this work, we assume that each unlocalized node is able to obtain range estimates to a minimum of three localized neighbors. These range estimates could be either LOS or NLOS. For an unlocalized node with coordinate $\mathbf{x} = (x, y)^T$, its jth LOS range estimate can be modeled as [7]:

$$r_{Lj} = R_{Lj} + n_{Lj}, \quad j = 1, 2, \ldots, m_L \tag{1}$$

where m_L is the total number of LOS range estimates. $R_{Lj} = ||\mathbf{x} - \mathbf{x}_{Lj}||$ is the true inter-node distance where $\mathbf{x}_{Lj}$ is the coordinate of the jth localized LOS node. n_{Lj} is a zero-mean Gaussian noise with a variance $\sigma^2_{Lj} = K_E R^{\beta}_{Lj}$, where β is the path loss exponent, and K_E is a constant that represents the combined effects of the physical layer communication parameters. Similarly, the jth NLOS range estimate can be modeled as [8]

$$r_{Nj} = R_{Nj} + n_{Nj} + b_{Nj}, \quad j = 1, 2, \ldots, m_N \tag{2}$$

where m_N is the total number of NLOS range estimates. R_{Nj} and n_{Nj} are defined similarly as in the LOS case, and b_{Nj} is an exponentially distributed bias term due to NLOS propagation, which is assumed to be much larger than the range noise.

A least-square (LS) estimator has been proposed to estimate node locations based on minimizing the sum of the square residuals from the unlocalized node to the lines formed by subtracting two circular equations defined by two distinct range estimates [9]. As shown in [1], blindly incorporating NLOS range estimates into the LS estimator may degrade the localization accuracy, and a safe bet would be to utilize only LOS range estimates. Based on this, we formulate the LS estimator using only LOS range estimates as follows. First, the jth LOS range estimate defines

$$||\mathbf{x} - \mathbf{x}_{Lj}||^2 = r^2_{Lj} = (x - x_{Lj})^2 + (y - y_{Lj})^2. \tag{3}$$

Subtracting (3) with the ith circular equation, we obtain

$$a_{ij}x + b_{ij}y = c_{ij}, \quad i, j = 1, 2, \ldots, m_L, \ i < j \tag{4}$$

where

$$\begin{aligned} a_{ij} &= x_{Li} - x_{Lj}, \quad b_{ij} = y_{Li} - y_{Lj} \\ c_{ij} &= \frac{1}{2}\left[(x^2_{Li} - x^2_{Lj}) + (y^2_{Li} - y^2_{Lj}) - (r^2_{Li} - r^2_{Lj})\right] \end{aligned} \tag{5}$$

In the presence of noise, equations defined in (4) can not be satisfied simultaneously. By minimizing the sum of the square residuals

$$\widehat{\mathbf{x}} = \arg\min_{\mathbf{x}} Q = \arg\min_{\mathbf{x}} \sum_{i} \sum_{j, j>i} e^2_{ij} \tag{6}$$

where

$$e_{ij} = a_{ij}x + b_{ij}y - c_{ij}, \quad i, j = 1, 2, \ldots, m_L, \ i < j, \tag{7}$$

the LS location estimate is then given by [9]

$$\widehat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{c} \tag{8}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{12} & a_{13} & \ldots & a_{1m_L} & a_{23} & \ldots & a_{(m_L-1)m_L} \\ b_{12} & b_{13} & \ldots & b_{1m_L} & b_{23} & \ldots & b_{(m_L-1)m_L} \end{bmatrix}^T$$

$$\mathbf{c} = \begin{bmatrix} c_{12} & c_{13} & \ldots & c_{1m_L} & c_{23} & \ldots & c_{(m_L-1)m_L} \end{bmatrix}^T$$

Note that in 2D localization, (8) requires $m_L \geq 3$ to form an unambiguous solution.

In Figs. 1 and 2, we demonstrate the effect of the propagation of localization error when the LS estimator is applied in the sequential manner we mentioned early. We consider an $L \times L$ square area, and N sensor nodes are randomly deployed in this area, with $N_A = 5$ anchors located around the origin. If the distance between two nodes is less than the transmission radius $R_{\max}$, they can estimate ranges to each other. For brevity, we only present the results in a pure LOS range estimate scenario. In fact the localization error in an LOS/NLOS scenario becomes far more worse. Fig. 1 plots example location estimates of one network realization, while Fig. 2 plots the average localization error versus distance. We observe that as the distance from the true anchors increases,

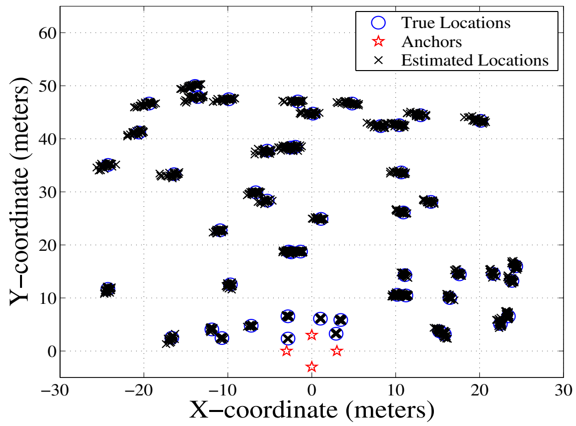

Fig. 1. Illustration of the propagation of error for one network realization when $N = 50$, $L = 50\,\text{m}$, $K_E = 0.001$, $\beta = 2$ and $R_{\max} = 30\,\text{m}$, and the LS estimator is applied sequentially. The estimated locations for 30 noise realizations are shown.

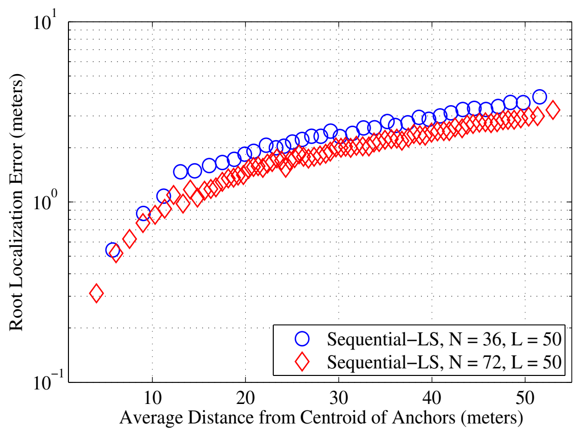

Fig. 2. The root localization error (meters) versus the distance from nodes to the centroid of the anchors, for $K_E = 0.001$, $\beta = 2$, $R_{\max} = 30\,\text{m}$, and different values of N and L

the root localization error (i.e., the Euclidean distance between the true and estimated distances) increases.

In LOS scenarios, several methods can be used to reduce the propagation of localization error, such as (i) increasing transmit power; this can increase the number and accuracy of range estimates [10], (ii) increasing the deployed node density; this reduces the average inter-node distances, which reduces the variance of range estimates, (iii) deploying a larger number of true anchors around the area of interest; this reduces the geometric dilution of precision [7], (iv) improving the accuracy of location estimators; it was shown in [11] that reducing the bias in practical location estimators can significantly reduce the propagation of error. However, in ad hoc NLOS scenarios, such modifications may not be viable, and we need other methods to mitigate the impact of the propagation of localization error.

III. Multi-hop propagation of error mitigation

A. Linear programming method for NLOS mitigation

The linear programming (LP) method developed in [1] provides a computationally efficient means of incorporating (biased) NLOS range estimates into location estimation without degrading the localization accuracy. This method assumes that an NLOS identification has been performed so that we know which range estimates are LOS and which are NLOS. For instance, in UWB-base ranging, this can be achieved by examining the received signal characteristic [12]. With this knowledge, LOS range estimates are used to define a linear objective function, which is realized by replacing e_{ij}^2 in (6) by $|e_{ij}|$, then substituting the unconstrained variable e_{ij} by constrained variables $e_{ij}^+ - e_{ij}^-$ where $e_{ij}^+ \geq 0, e_{ij}^- \geq 0$ [13]. Finally, the objective function is linearized and the position solution is given by

$$\widehat{\mathbf{x}} = \arg\min_{\mathbf{z}} Q = \sum_i \sum_{j,j>i} \left(e_{ij}^+ + e_{ij}^-\right) \tag{9}$$

Then, by considering the constraints given by (7) (substituting e_{ij} by $e_{ij}^+ - e_{ij}^-$), an LP can be properly formulated and solved using standard techniques. The mathematical details of the LP defined by LOS range estimates are omitted here due to space limitation.

On the other hand, NLOS range estimates are not incorporated into the formulation of (9). Instead, they are only utilized to define additional constraints based on the fact that in indoor environments, the NLOS bias errors are typically much larger than the range estimation noise. Therefore, the true location of the unlocalized node generally lies within the circle defined by each NLOS range estimate. This can be shown by the following circular inequality,

$$||\mathbf{x} - \mathbf{x}_{Nj}||^2 = (x - x_{Nj})^2 + (y - y_{Nj})^2 \leq r_{Nj}^2, \tag{10}$$

for $j = 1, 2, \ldots, m_N$. Then, we can linearize each of these circular constraints into linear one by relaxing it to four rectangular inequalities,

$$\begin{aligned} x - x_{Nj} \leq r_{Nj}, \quad -x + x_{Nj} \leq r_{Nj} \\ y - y_{Nj} \leq r_{Nj}, \quad -y + y_{Nj} \leq r_{Nj} \end{aligned} \tag{11}$$

These *linear* inequalities can be incorporated into the LP formed by the LOS range estimates to constrain the location estimate. It has been shown that this *complete* LP achieves similar performance as the LS estimator when only LOS range estimates are present, while it outperforms the LS estimator when both LOS and NLOS range estimates are provided. Due to space limitation, we omit the full description of the method and refer interested readers to [1] for details.

B. Mitigating the propagation of localization error

Our proposed approach of partially mitigating the propagation of localization error is based on similar principles as the way of utilizing NLOS range estimates in our proposed LP method. Consider a scenario where an unlocalized node can obtain range estimates from its one-hop localized neighbor A, but not from its two-hop localized neighbor B. However, A and B are one-hop neighbors, and A can estimate its distance

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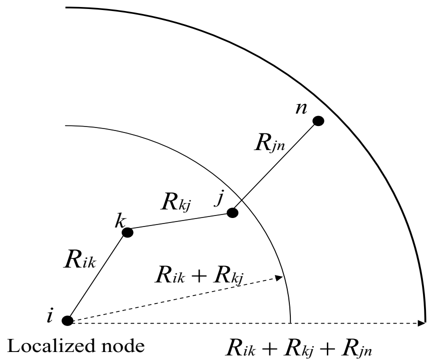

Fig. 3. Illustration of bounds in the multi-hop case.

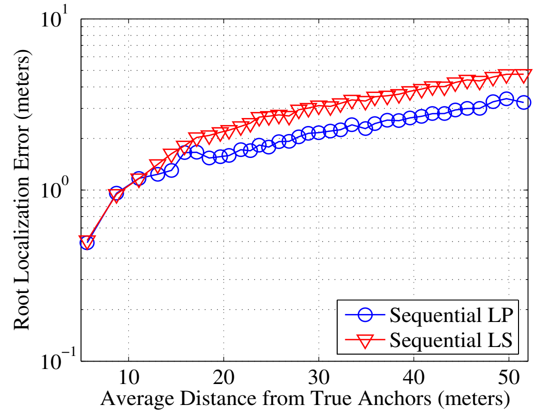

Fig. 4. Improvements in the average root localization error when the sequential LP with 2-hop bounding is applied with all LOS range estimates. Here, $N = 36$, $N_A = 5$, $L = 50$m, $K_E = 0.001$, $\beta = 2$, $\alpha = 1.1$. Full connectivity is assumed.

to B. If we define the distance between the unlocalized node and one-hop neighbor as $\|\mathbf{x} - \mathbf{x}_A\| = a$, the distance between A and B as $\|\mathbf{x}_A - \mathbf{x}_B\| = b$, and the distance between the unlocalized node and B as $\|\mathbf{x} - \mathbf{x}_B\| = c$, then clearly

$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$

where θ is angle between the two distances a and b. From the above equation (or the Triangle inequality),

$$c \leq a + b,$$

with equality achieved when $\theta = \pi$. This implies that when anchor nodes A and B are collinear, the sum of the distances a and b is also equal to the distance between the unlocalized node and B. Otherwise, the sum of the distances a and b is always larger than the true distance between the unlocalized node and B. This implies that we can bound the location of the unlocalized node using the sum of single-hop distances from a localized node to the unlocalized node. In the presence of range measurement noise, we can ensure that the unlocalized node lies *within* this distance by using a multiplicative factor $\alpha > 1$:

$$c < \alpha(a + b).$$

The above upper bound becomes tighter as the angle θ increases and tends toward π. The bound can be extended to the general multiple two-hop localized nodes case. Specifically, if $\mathbf{x}_i$ is the location of a localized node, the location of an unlocalized node is defined as $\mathbf{x}_j$, and $c_{ij} = \{0, 1\}$ where $c_{ij} = 1$ if nodes i and j are one-hop neighbors, $c_{ij} = 0$ if they are not. Then

$$\|\mathbf{x}_i - \mathbf{x}_j\| \leq R_{ik} + R_{kj} < \alpha(R_{ik} + R_{kj}), \quad (12)$$
$$c_{ij} = 0,\ c_{ik} = 1,\ c_{kj} = 1,\ \alpha > 1$$

where R_{ij} denotes the distance between nodes i and j. This can also be extended to the 3-hop case:

$$\|\mathbf{x}_i - \mathbf{x}_n\| \leq R_{ik} + R_{kj} + R_{jn} < \alpha(R_{ik} + R_{kj} + R_{jn}), \quad (13)$$
$$c_{in} = c_{ij} = c_{nk} = 0, c_{ik} = c_{kj} = c_{jn} = 1, \alpha > 1.$$

The above bounds can be illustrated (by the dash lines) in Fig. 3. Each of the constraints in (12) or (13) can be used to limit the feasible region for localizing the unlocalized node. For example, if a location solution for node j happens to fall outside the circle defined by (12), the inclusion of the 2-hop bounding given by (12) can rule out this solution and try to improve it. In fact, the constraints in (12) or (13) are similar to the NLOS constraints discussed in [1], which implies that the above multi-hop constraints can be linearized as in (11) and combined with the NLOS constraints in the LP approach, constraining the feasible region for a given node's location. This results in the straightforward formulation of a *sequential* LP scheme that uses LOS range estimates to define the objective function and constraints, with both NLOS and multi-hop constraints defining the feasible region. As shown later, this effectively reduces the propagation of localization error, at the expense of additional computational complexity. Although the technique can be extended to include an arbitrary number of hops, typically using more than three hops has been observed to not provide significant additional gains. Since the computational complexity required to track n-hop neighbors increases exponentially with n, and their bounding power decreases (due to the fact that in an increasing number of cases the triangle inequality becomes loose) leading to diminishing returns, it is not generally useful to consider more than 2-3 hops in the bounding. Therefore, in the following section, we only present simulation and measurement results corresponding to 2-hop bounding.

IV. Simulation and Measurement Results

To test the usefulness of the algorithm, we examined its performance through both simulation and measurements. In the following sections we describe both studies.

A. Simulation Results

The first set of tests use simulation to examine the efficacy of the proposed approach. Specifically, we compare the performance of the sequential LS estimator with the sequential LP approach with 2-hop bounding when all range estimates are LOS. The network setup is the same as mentioned in Section II, with $N = 36$, $N_A = 5$, $L = 50$m, $\alpha = 1.1$.

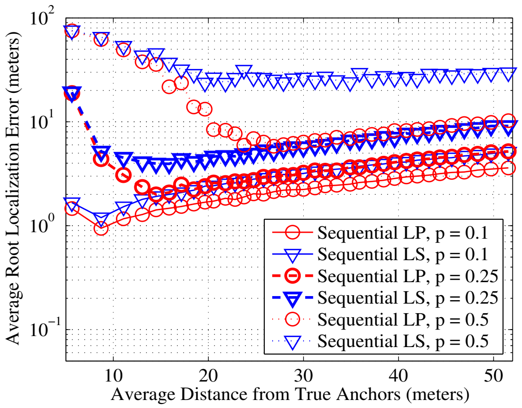

Fig. 5. Comparison of the performance of the sequential LP with 2-hop bounding and sequential LS approaches, when NLOS range estimates are present (with probability p) and partial connectivity (with $R_{\max} = 30$m). As before, $N = 36$, $N_A = 5$, $K_E = 0.001$, $\beta = 2$, $\alpha = 1.1$.

Note that the choice of $\alpha = 1.1$ does not guarantee (12) to be valid in the presence of measurement noise. The larger α is, the more likely (12) is to be valid, but the looser the multi-hop bound becomes. Based on these simulation parameters, Fig. 4 plots the root localization error versus the distance from the origin for the two localization algorithms. For both algorithms we find that due to error propagation, the localization error increases with distance from the anchor nodes. If we define $\tilde{L}(\Omega_0)$ as the max distance at which an average root localization error of Ω_0 or better is achieved, we find $\tilde{L}_{\rm LS}(2) = 20$m is less than $\tilde{L}_{\rm LP}(2) = 30$m, which means the proposed approach improves the localization accuracy. Note that the improvements are due entirely to the multi-hop bounding, because the LP approach without 2-hop bounding performs similar to the LS estimator with pure LOS range estimates when localizing a single node.

The previous example showed that performance gains can be achieved in LOS measurement scenarios. However, a larger advantage is possible in NLOS environments. Fig. 5 compares the performance of the sequential LS estimator with the sequential-LP approach with 2-hop bounding when both LOS and NLOS range estimates are present. The simulation parameters are the same as those used in Fig. 4, with the addition of partial connectivity ($R_{\max} = 30$m) and a probability p that a given range estimate is NLOS. Only 2-hop constraints were considered in the sequential LP approach. The sequential LS estimator uses only LOS range estimates, since blindly using a mixture of LOS and NLOS range estimates in the LS estimator without any NLOS mitigation can severely degrade its performance [1]. From Fig. 5, we see that as p increases, the localization accuracy degrades, and the advantage of the sequential LP approach with 2-hop bounding over the sequential LS estimator progressively increases. When $p = 0.1$, we find $\tilde{L}_{\rm LS}(2) = 15$m as compared to $\tilde{L}_{\rm LP}(2) = 25$m, which again demonstrates the efficacy of our proposed approach in mitigating the propagation of localization error. The error for a small number of nodes is large, as the links to the true anchors may be NLOS, and only a small number of LOS range estimates may be available. Another observation is that the average localization error of the sequential LS estimator when at a distance of 50m from the true anchors is 5m, 10m, and 30m when the probability of NLOS connectivity is 0.1, 0.25 and 0.5 respectively. This is in contrast to our proposed approach which achieves an average localization error of 3m, 5m and 10m respectively.

B. Measurement Results

In addition to simulation tests, we examined the performance of the algorithm in an indoor environment using UWB measurement results. The schematic in Fig. 6 shows the floor plan for the 4th floor of Durham Hall at Virginia Tech. Specifically, we collect the received signal measurements from $N = 71$ unlocalized nodes communicating with $N_A = 5$ anchors in the corridors of a typical office building. The five anchors, represented by triangles, are located around the left side of the schematic. The 71 unlocalized node locations were sequentially selected (starting from the left) along the trajectory shown by the dots, and separated by approximately 1.2m. The received signal at each of these points from the transmitter placed at different anchor locations was measured, provided the signal could be captured. A bicone antenna connected to a 30ps pulse generator was used as the transmitter, and the receiver consisted of a second bicone antenna connected to a digital sampling oscilloscope that was triggered by the pulser. The pulse repetition frequency was set at 200 kHz, and the effective sampling frequency of the oscilloscope was 20 GHz. The averaged received signals from the oscilloscope were extracted, and subsequently bandpass-filtered with lower and upper cutoffs of 3.1 GHz and 10.6 GHz respectively. For each measurement location, the state of the channel (LOS or NLOS) was noted. The "soft NLOS" cases, where the LOS path was present albeit attenuated, were classified as LOS scenarios, as the TOA of the first path could still be estimated. The channel was stationary for the duration of each measurement.

The inter-node distance was estimated using the corresponding received signal, if available (i.e., if the signal was not blocked). The range estimation algorithm was based on energy thresholding of the received multipath profile discussed in [14], [15], [16]. The range estimation method was calibrated using a reference measurement taken at a distance $d_0 = 1$ meter. For each measurement location, based on the range estimates obtained from the set of available localized nodes, the location can be estimated using LS or LP approaches. These location estimates can then be compared to the physically measured locations. The errors arising from the physical measurement and placement of the Bicone antennas were within ± 5cm. Due to the nature of the measurement locations, 10% of the measurements were NLOS, i.e., $p = 0.1$. It must be pointed out that the probability of NLOS links can be significantly higher in indoor scenarios.

Fig. 7 compares the performance of the sequential LS estimator using LOS range estimates only with that of the sequential LP approach which uses both NLOS range estimates and 2-hop bounding. We see that the proposed method considerably limits the propagation of error, and displays significant gains over the sequential LS estimator. Specifically, the

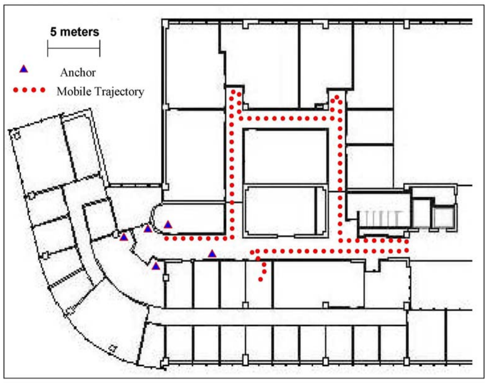

Fig. 6. Floor-plan [17] of 4th floor, Durham Hall, Virginia Tech.

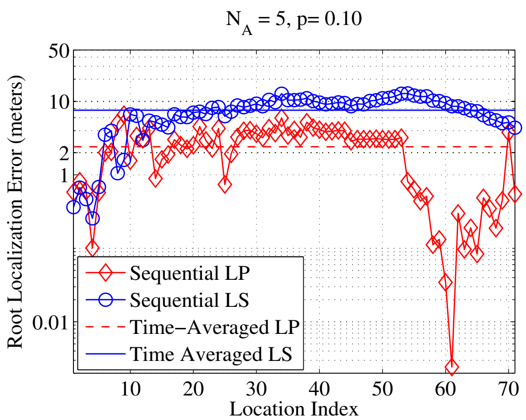

Fig. 7. Performances of the sequential LS and the sequential LP approach using measurement results.

average root localization error decreases from approximately 10m when the LS estimator is used to just over 2m when the proposed method is utilized. Although the root localization error achieved by the sequential LP method is larger than 1 meter (a typical target accuracy), further constraints can be added to improve the localization accuracy at the cost of computational complexity. In general, as the local map of node locations remains intact, creating a feasible region for node locations can considerably limit the propagation of error by "guiding" the local map to the true locations. In addition, physical constraints of the area of interest can be used to further reduce the size of the feasible region.

V. Conclusion

In this paper we have examined the ability of a linear programming-based method for position location to mitigate the propagation of localization error problem in multi-hop position location networks which use a sequential estimation approach. It was shown that the approach provides modest gains (30% localization error reduction) when localization is based on all line-of-sight connections. However, in the more practical case where non-line-of-sight connections represent a substantial portion of the connections (10%-50%), the improvement in localization error was much more dramatic. Specifically, the localization error can be reduced by a factor of 2-3 at distances of 10-50m from the anchors (i.e., those nodes with absolute position information). Additionally, the technique can easily be extended to include other constraints.

Acknowledgements

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