A Linear Programming Approach to NLOS Error Mitigation in Sensor Networks

Swaroop Venkatesh and R. Michael Buehrer
Mobile and Portable Radio Research Group (MPRG), Virginia Tech,
Blacksburg, VA 24061, USA.
{vswaroop, buehrer}@vt.edu

ABSTRACT

In this paper, we propose a linear programming approach to the problem of non-line-of-sight (NLOS) error mitigation in sensor networks. The locations of sensor nodes can be estimated using range or distance estimates from location-aware "anchor" nodes. In the absence of line-of-sight (LOS) between the sensor and anchor nodes, e.g., in indoor networks, the NLOS range estimates can be severely biased. If these biased range estimates are directly incorporated into practical location estimators such as the Least-Squares (LS) estimator without the mitigation of these bias errors, this can potentially lead to degradation in the accuracy of sensor location estimates. On the other hand, discarding the biased range estimates may not be a viable option, since the number of range estimates available may be limited. We present a novel NLOS bias mitigation scheme, based on linear programming, that (i) allows us to incorporate NLOS range information into sensor location-estimation, but (ii) does not allow NLOS bias errors to degrade sensor localization accuracy.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Distributed networks - Wireless communication; G.1.0 [**Mathematics of Computing**]: Numerical Analysis - Numerical algorithms

General Terms

Algorithms, Design, Measurement, Performance

Keywords

line-of-sight, location estimation, NLOS environment, time-of-arrival estimation, wireless sensor networks.

1. INTRODUCTION

The envisioned applications for ad hoc wireless sensor networks often depend on the automatic and accurate location of deployed sensors. In numerous sensor networks, particularly for environmental applications [1], [2] such as water quality monitoring, precision agriculture, and indoor air quality monitoring, the available sensing data may be rendered useless by the absence of accurate sensor location estimates. The availability of accurate sensor location estimates can help reduce configuration requirements and device cost. Further, accurate sensor location estimation enables applications such as inventory management [3], intrusion detection [4], traffic monitoring, and locating emergency workers in buildings.

The design of ad hoc "location-aware" sensor networks requires the capability of peer-to-peer range or distance measurement. A sensor whose location is unknown, can estimate its location based on range measurements from location-aware sensors or "anchors", whose locations are known or estimated *a priori*. Range estimates from anchor nodes could be obtained using received signal strength (RSS) or time-of-arrival (TOA) estimation techniques [5], [6].

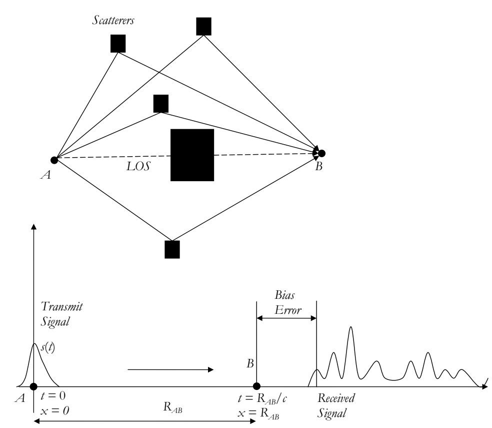


Figure 1: The bias introduced in the absence of LOS in TOA-based ranging between two sensors A and B.

In sensor networks, especially indoors, the LOS path between sensors may be obstructed as illustrated in Figure 1. As a consequence, TOA-based range estimates are positively biased with high probability, since the first multipath component travels a distance that is in excess of the true LOS distance. A similar effect is seen in the case of RSS-based range estimates, where the received signal power is reduced due to the obstruction of the LOS path. These effects re-

IPSN'06, April 19–21, 2006, Nashville, Tennessee, USA.

sult in range estimates that are often much larger than the true distances and as a consequence, in NLOS scenarios, the accuracy of sensor location estimates is adversely affected.

The problem of location-estimation with biased NLOS range estimates has been considered before, but mostly in the context of cellular communications [7], [8], where it was shown that the NLOS bias errors in the range estimates lead to large errors in the computation of a node's location. The literature on the NLOS problem typically falls in two categories: NLOS identification and NLOS mitigation. The former deals with the problem of distinguishing between LOS and NLOS range estimates, whereas the latter typically deals with the reduction of the adverse impact of NLOS range errors on the accuracy of location-estimates, assuming the NLOS range estimates have been identified. Several statistical NLOS identification techniques for cellular systems have been discussed previously [8], [9]. In this work, we focus on the problem of NLOS mitigation in two-dimensional sensor location estimation, assuming we are able to distinguish between LOS and NLOS range estimates. It must be pointed out that the results in this paper can easily be extended to three-dimensional sensor localization scenarios.

The Cramer-Rao Lower Bound (CRLB) analysis presented in [10] characterized the performance of the minimum variance unbiased estimator (MVUE) [11] of sensor location, given a mixture of (unbiased) LOS and (biased) NLOS range estimates. This analysis showed that the MVUE discards the biased NLOS range estimates and utilizes only LOS range information while estimating sensor locations. However, as will be demonstrated, in the case of practical non-efficient [11] estimators such as the commonly-used Least-Squares (LS) estimator [12], discarding NLOS range information does not necessarily improve performance. Additionally, for two-dimensional location-estimation, the LS estimator requires at least three range estimates in order to obtain an unambiguous solution. Consequently, in indoor sensor networks, limited connectivity with anchors may imply that we may not have the luxury of discarding any range estimates. This implies that in general, given a mixture of LOS and NLOS range estimates, we may be required to use the entire set of range information in order to compute a sensor's location, as illustrated in Figure 2.

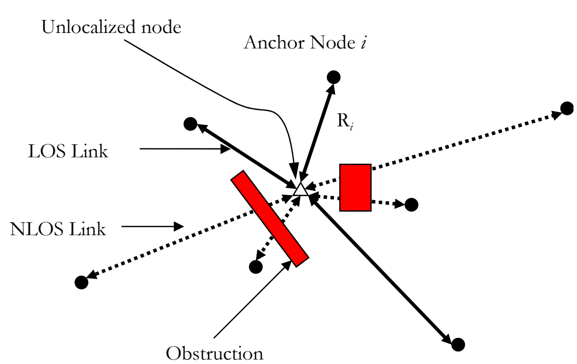


Figure 2: The NLOS Problem: In general, a mixture of unbiased LOS and biased NLOS range estimates needs to be used to compute a sensor's location.

A Semi-Definite Programming (SDP) approach to sensor localization based on connectivity information was investigated in [13], and a quadratic programming approach with NLOS range estimates was discussed in [14], but these approaches result in high computational complexity [15]. The Residual Weighting Algorithm (Rwgh) was proposed in [16]. The main advantage of this algorithm is that NLOS identification is not required *a priori*. However, this algorithm implicitly assumes that the range measurement noise is much smaller than the bias introduced, in order to distinguish between the LOS and NLOS range estimates. More importantly, it relies on the availability of a large number of range estimates, several of which are LOS, so that the set of range estimates finally selected to compute a node's location results in the smallest residual error. However, in indoor sensor networks, situations may arise where only NLOS range estimates are available while estimating a sensor's location.

In this paper, we present a novel linear programming (LP) approach that effectively incorporates both LOS and NLOS range information into the estimation of a sensor's location. A linear programming approach was briefly mentioned for the case of NLOS range estimates in [15], but was not pursued. We demonstrate that this low-complexity LP approach can be generalized to handle a mixture of LOS and NLOS range estimates (with the "only-LOS" and "only-NLOS" range information scenarios as sub-cases) without discarding any range information.

This paper is organized as follows: In section 2, we discuss the impact of NLOS bias errors on the accuracy of sensor location estimates. Section 3 discusses the LP approach to incorporating LOS range estimates, NLOS range estimates and a mixture of LOS and NLOS range estimates into sensor location estimation. We also discuss a series of sub-cases that need to be addressed in order generalize the proposed approach. Simulation results are presented in Section 5, where we evaluate the performance of the proposed method in terms of sensor localization accuracy. We conclude in Section 6.

2. IMPACT OF NLOS BIAS ERRORS

2.1 Notation, Models and Assumptions

Suppose the sensor's (unknown) location is $\boldsymbol{x} = [x\ y]^T$. Let $\mathcal{L}$ denote the set of anchors which provide LOS range estimates, with cardinality $m_L = |\mathcal{L}|$. The known locations of the LOS anchors are denoted by $\{\boldsymbol{x}_{Lj}\}$, $j = 1, 2, \cdots, m_L$. Similarly, $\mathcal{N}$ represents the set of anchors that provide NLOS range estimates, with $m_N = |\mathcal{N}|$, and the known locations of the NLOS anchors is represented by $\{\boldsymbol{x}_{Nj}\}$, $j = 1, 2, \cdots, m_N$.

The LOS range estimates $\{r_{Lj}\}$ are modeled as unbiased Gaussian [2], [17] estimates of the true ranges $R_{Lj} = \|\boldsymbol{x} - \boldsymbol{x}_{Lj}\|$:

$$r_{Lj} = R_{Lj} + n_{Lj}, \quad j = 1, 2, \cdots, m_L, \tag{1}$$

where n_{Lj} represents zero-mean Gaussian range measurement noise in the jth LOS range estimate: $n_{Lj} \sim \mathcal{N}(0, \sigma_j^2)$, where the range measurement noise variance σ_j^2 can be modeled as [18]

$$\sigma_j^2 = K_E R_{Lj}^{\beta}, \tag{2}$$

where β is the path loss exponent and K_E is a proportionality constant that determines the accuracy of range estimation. This above model for the accuracy of range estimates

[18] applies to both TOA and RSS-based range estimates when $\beta = 2$. The NLOS range estimates are assumed to be positively biased Gaussian estimates [8] of the true ranges:

$$r_{Nj} = R_{Nj} + n_{Nj} + b_{Nj}, \quad j = 1, 2, \cdots, m_N, \tag{3}$$

where $R_{Nj} = \|\boldsymbol{x} - \boldsymbol{x}_{Nj}\|$, $n_{Nj} \sim \mathcal{N}(0, K_E R_{Nj}^{\beta})$ and b_{Nj} are the NLOS bias errors. We assume that the bias errors are uniformly distributed: $b_{Nj} \sim \mathcal{U}(0, B_{\max})$, where $B_{\max}$ represents the maximum possible bias. Additionally, we assume that the maximum bias is much larger than the range measurement noise $B_{\max} \gg \sigma_j^2, \quad j = 1, 2, \cdots, m_N$. Finally, without loss of generality, we assume that the coordinate axes are selected such that $\boldsymbol{x} \geq \mathbf{0}$.

2.2 NLOS range estimates: To discard or not to discard?

As shall show in a later section, when we have at least three range estimates, the LS estimator can be used compute an estimate $\hat{\boldsymbol{x}}$ of the sensor's location $\boldsymbol{x}$. We define the *localization error*, a measure of the accuracy of the location-estimate $\hat{\boldsymbol{x}}$, as:

$$\Omega = \|\boldsymbol{x} - \hat{\boldsymbol{x}}\|^2 \quad \text{meter}^2 \tag{4}$$

It must be noted that Ω is a random variable, with different instances corresponding to different realizations of the range measurement noise, bias errors and anchor locations. Therefore, we characterize the accuracy of location-estimates through the mean μ_Ω and standard deviation σ_Ω of the localization error defined in (4); smaller values of both μ_Ω and σ_Ω indicate more accurate sensor location-estimates.

When $m_N = 0$ and $m_L \geq 3$, the LS estimator provides accurate estimates of a node's location [12]. However, when $m_N > 0$, we need effective ways of incorporating NLOS information into the estimation procedure. Figures 3 and 4 show the impact of directly (without mitigation of the bias errors) incorporating NLOS range estimates into the LS solution. For the specific distribution of anchors shown in Figure 3, directly incorporating the NLOS ranges into the LS solution can degrade localization accuracy. However, in some cases, and in particular for the example shown in Figure 4, introducing the NLOS range estimate directly into LS location estimation without any mitigation of the bias in the range estimate can *improve* performance in terms of μ_Ω and σ_Ω.

Generally speaking, discarding the NLOS range estimates does not result in poor performance when the geometry of LOS anchor nodes has certain properties, best described by the *geometric dilution of precision* (GDOP) [2], [19], where a larger GDOP (as defined in [2]) implies poorer localization accuracy. It has been observed that when the GDOP is very large, the presence of an additional NLOS range estimate results in an improvement in performance: the addition of a NLOS node reduces the GDOP and this compensates for the inaccuracy of the NLOS range estimate.

These two examples show that (i) directly incorporating NLOS range estimates into existing practical estimators without reducing the impact of bias errors can adversely affect localization accuracy. However, (ii) we do not wish to discard the NLOS range estimates, since their use could improve the performance of practical estimators under certain conditions. Indeed, in indoor sensor networks, we may have more NLOS range estimates than LOS range estimates. Therefore, what is desired is a method that allows

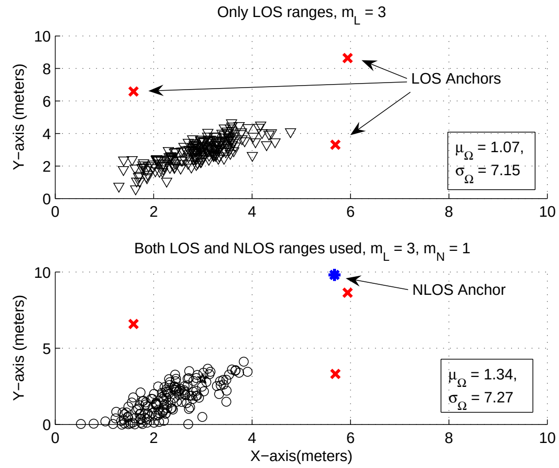


Figure 3: This example shows several instances of the LS location estimate $\hat{\boldsymbol{x}}$, one for each realization of the range estimates, with $\boldsymbol{x} = [3\ 3]^T$, $\beta = 2$, $K_E = 0.1$, $B_{\max} = 4$ meters, for (i) (top) Only $m_L = 3$ LOS estimates (ii) (bottom) including $m_L = 3$ LOS and $m_N = 1$ NLOS range estimates. The NLOS range estimate is treated exactly like an LOS range estimate and directly incorporated into the LS solution in the bottom figure. In this case, the addition of the biased NLOS range estimate *degrades* localization accuracy with respect to μ_Ω and σ_Ω.

the "soft-activation" of NLOS range information: the NLOS range estimates are not incorporated directly, but are used in conjunction with LOS range estimates when LOS range estimates alone do not guarantee accurate sensor location estimates. In the following section, an LP approach that achieves this goal is described.

3. A LINEAR PROGRAMMING APPROACH

In this section, we show that the problem of sensor location-estimation given LOS range information can be cast in the form of a linear program. We then modify the linear program to utilize NLOS range information, resulting in a method that utilizes a mixture of LOS and NLOS range estimates to compute a sensor's location accurately.

3.1 LOS range estimates

The LOS range estimates, which are modeled as unbiased estimates of the true ranges, can be used to define constraints on the unknown sensor location $\boldsymbol{x}$. We can write for $i = 1, 2, \cdots, m_L$:

$$\begin{aligned} \|\boldsymbol{x} - \boldsymbol{x}_{Li}\| &= r_{Li}, \\ \Rightarrow (x - x_{Li})^2 + (y - y_{Li})^2 &= r_{Li}^2. \end{aligned} \tag{5}$$

These relations are non-linear equations in x and y and represent the fact that $\boldsymbol{x}$ lies on a circle of radius r_{Li} whose center is $\boldsymbol{x}_{Li}$. These equations can be linearized by extracting the difference of each of these equations from the others,

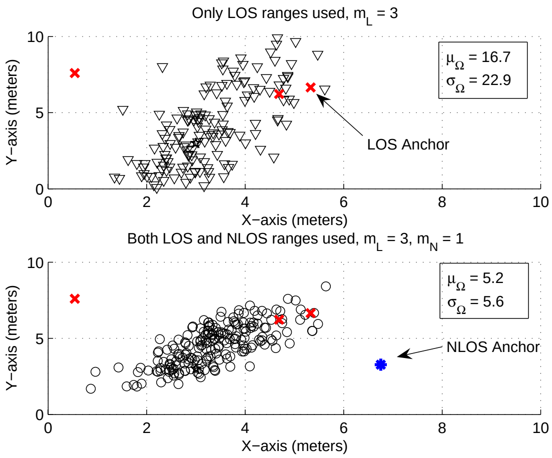


Figure 4: This example shows several instances of the LS location estimate $\hat{\boldsymbol{x}}$, one for each realization of the range estimates, with $\boldsymbol{x} = [3\ 3]^T$, $m_L = 3$, $m_N = 1$, $K_E = 0.1$, $B_{\max} = 4$ meters. In this case, the addition of the biased NLOS range estimate *improves* localization accuracy with respect to μ_Ω and σ_Ω.

forming a system of $M = \binom{m_L}{2}$ *distinct* equations:

$$\begin{aligned} a_{ij}x + b_{ij}y &= c_{ij}, \quad i,j = 1,2,3,\cdots,m_L,\ i<j \qquad (6)\\ \text{where } a_{ij} &= x_{Li} - x_{Lj},\ b_{ij} = y_{Li} - y_{Lj}, \\ c_{ij} &= \frac{\left(x_{Li}^2 - x_{Lj}^2\right) + \left(y_{Li}^2 - y_{Lj}^2\right) - \left(r_{Li}^2 - r_{Lj}^2\right)}{2} \end{aligned}$$

Each of these M equations can be viewed as representing the lines formed by connecting the intersection points (if any) of pairs of circular constraints defined in (5). From (1), since the range estimates are noisy, in general $r_{Li} \neq R_{Li}$, and solving these equations simultaneously may not yield a solution. Resorting to an error minimization approach, for every potential solution $\boldsymbol{x}$, and for every equation, we can define the residual error as:

$$e_{ij} = a_{ij}x + b_{ij}y - c_{ij}, \qquad i,j = 1,2,3,\cdots,m_L,\ i<j. \quad (7)$$

The final estimate $\hat{\boldsymbol{x}}$ can selected such that an objective function Z, such as the sum of the residual error squares, is minimized:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} Z = \arg\min_{\boldsymbol{x}} \sum_i \sum_{j,\ j>i} e_{ij}^2$$

This is the equivalent to the LS approach defined in [12]. It is important to note that (i) the objective function Z is non-linear in x and y, and (ii) we require $m_L \geq 3$ to form an unambiguous solution. The LS solution is given by

$$\hat{\boldsymbol{x}} = \left(\boldsymbol{A}^T\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\boldsymbol{c}$$

where

$$\boldsymbol{A} = \begin{bmatrix} a_{12} & a_{13} & \cdots & a_{1m_L} & a_{23} & \cdots & a_{(m_L-1)m_L} \\ b_{12} & b_{13} & \cdots & b_{1m_L} & b_{23} & \cdots & b_{(m_L-1)m_L} \end{bmatrix}^T_{2\times M} \quad (8)$$

and

$$\boldsymbol{c} = \begin{bmatrix} c_{12} & c_{13} & \cdots & c_{1m_L} & c_{23} & \cdots & c_{(m_L-1)m_L} \end{bmatrix}^T_{1\times M} \quad (9)$$

Looking at (7), we see that the set of variables $\{e_{ij}\}$ plays the role of unconstrained slack variables [20] in the system of M equations. This linear system of equations can be converted to a linear program if the objective function Z is linear. Specifically, if we define

$$Z \triangleq \sum_i \sum_{j,\ j>i} |e_{ij}|,$$

and then replace the unconstrained variable e_{ij} by $e_{ij}^+ - e_{ij}^-$, $e_{ij}^+ \geq 0$, $e_{ij}^- \geq 0$, we can write an alternative *linearized* objective function, that is to be minimized, as

$$Z \triangleq \sum_i \sum_{j,\ j>i} \left(e_{ij}^+ + e_{ij}^-\right). \qquad (10)$$

It must be noted that in the optimal solution that minimizes Z, only one of e_{ij}^+, e_{ij}^- will be equal to $|e_{ij}|$, with the other being zero [20]. The constraints are then given by

$$a_{ij}x + b_{ij}y = c_{ij} + e_{ij}^+ - e_{ij}^-, \qquad i,j = 1,2,3,\cdots,m_L,\ i<j. \quad (11)$$

Since there are now $2M$ non-negative slack variables, the vector $\boldsymbol{z}$ of $(2M{+}2)$ variables can be written as $\boldsymbol{z} = \left[x\ y\ \boldsymbol{\epsilon}^T\right]^T$, where :

$$\boldsymbol{\epsilon} = \left[e_{12}^+\ e_{12}^-\ e_{13}^+\ e_{13}^-\ \cdots\ e_{(m_L-1)m_L}^+\ e_{(m_L-1)m_L}^-\right]^T_{2M\times 1} \quad (12)$$

Thus, the linear program can be formulated in *standard form* [20] as

$$\begin{aligned} \min\ Z &= \boldsymbol{f}_L^T\boldsymbol{z} \quad \text{such that} \\ [\boldsymbol{A} \mid \boldsymbol{J}]\,\boldsymbol{z} &= \boldsymbol{c},\ \boldsymbol{z} \geq \boldsymbol{0}, \end{aligned} \qquad (13)$$

where $\boldsymbol{A}$ and $\boldsymbol{c}$ were respectively defined in (8) and (9),

$$\boldsymbol{J} = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}_{M\times 2M}, \quad (14)$$

and $\boldsymbol{f}_L = [\boldsymbol{0}_{2\times 1}^T\ \boldsymbol{1}_{2M\times 1}^T]^T$. Here $\boldsymbol{0}_{k\times l}$ represents a $k \times l$ matrix of zeros and $\boldsymbol{1}_{k\times l}$ represents a $k \times l$ matrix of ones.

Figure 5 compares μ_Ω for different values of K_E, using the (i) LS estimator and (ii) the LP approach, for sensor location-estimation with $m_L = 3$ LOS range estimates. We see that the linearization of the objective function has negligible impact on performance. Therefore, we now have a linear program that can be used to solve for a sensor's location given LOS range estimates. In this linear program, the objective function Z defined in (10) is a function of the distances of a point $\boldsymbol{x}$ to the straight lines given in (6). If we use NLOS range estimates in a similar manner, by incorporating them into the objective function we could potentially degrade the accuracy of the location estimate. Instead, as described in the following section, we can use the NLOS range estimates to constrain the feasible region for $\boldsymbol{x}$ without affecting the objective function defined using LOS range estimates, thereby limiting the possibility of large errors.

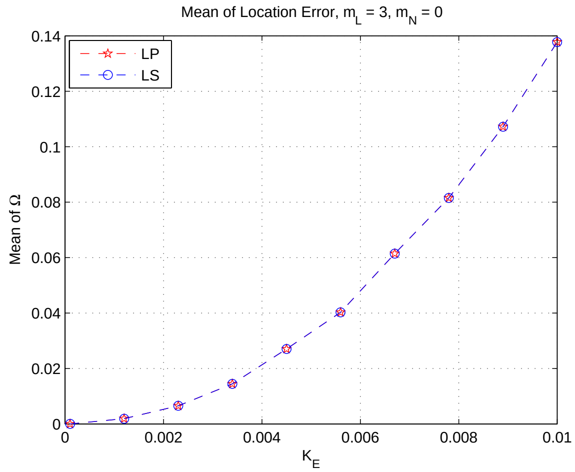


Figure 5: Mean of the Localization Error Ω, $m_L = 3$, $m_N = 0$ for a specific distribution of anchors.

3.2 NLOS range estimates

As the bias errors on the NLOS range estimates are always positive, and are assumed to be much larger than the range measurement noise, we know each NLOS range estimate r_{Ni} is, with high probability, larger than the true range R_{Ni}, $i = 1, 2, \cdots, m_N$. Based on this observation, we can convert the NLOS range estimates into inequalities for $i = 1, 2, \cdots, m_N$:

$$\begin{aligned} \|\boldsymbol{x} - \boldsymbol{x}_{Ni}\| &\leq r_{Ni}, \\ \Rightarrow (x - x_{Ni})^2 + (y - y_{Ni})^2 &\leq r_{Ni}^2. \end{aligned} \tag{15}$$

These inequalities imply that the feasible region for $\boldsymbol{x}$ lies *in the interior* of each of the circular constraints defined by (15). Note that this assumption cannot be made if the standard deviation of the zero-mean measurement noise and the positive bias are comparable in (3). Once again, these are non-linear constraints on x and y. However, these constraints can be relaxed to the following linear constraints, as suggested in [15]:

$$\begin{aligned} & x - x_{Ni} \leq r_{Ni}, \quad -x + x_{Ni} \leq r_{Ni}, \quad y - y_{Ni} \leq r_{Ni} \\ & -y + y_{Ni} \leq r_{Ni}, \quad i = 1, 2, \cdots, m_N. \end{aligned} \tag{16}$$

This essentially relaxes the circular constraints to rectangular constraints as shown in Figure 6. It is readily seen that the new rectangular feasible region contains the original (convex) feasible region formed by the intersection of three circular regions. We can now write the above four constraints for the ith NLOS range estimate in standard form [20]:

$$\begin{aligned} x - x_{Ni} + u_{1i} &= r_{Ni}, \quad -x + x_{Ni} + u_{2i} = r_{Ni} \\ y - y_{Ni} + v_{1i} &= r_{Ni}, \quad -y + y_{Ni} + v_{2i} = r_{Ni} \\ u_{1i},\ u_{2i},\ v_{1i},\ v_{2i} &\geq 0, \qquad i = 1, 2, \cdots, m_N. \end{aligned} \tag{17}$$

Defining $\boldsymbol{w}_i = [u_{i1}\ u_{2i}\ v_{i1}\ v_{2i}]^T$ and $\boldsymbol{z}_i = [x\ y\ \boldsymbol{w}_i^T]^T$ as the vectors of variables corresponding to the ith NLOS range estimate, we can express the above equations in matrix form as

$$\begin{aligned} [\boldsymbol{B}_1 |\ \boldsymbol{I}_{4\times4}]\, \boldsymbol{z}_i &= \boldsymbol{r}_i, \\ \boldsymbol{z}_i &\geq \boldsymbol{0}, \end{aligned}$$

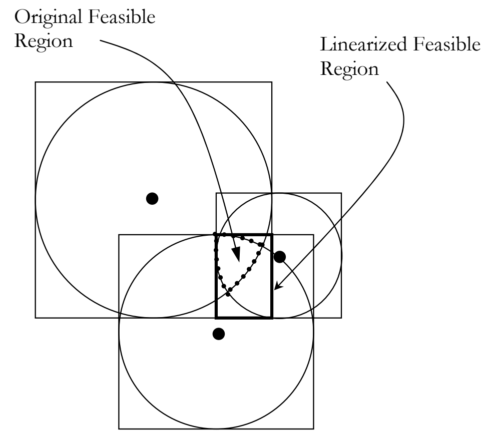


Figure 6: Linearization of $m_N = 3$ NLOS constraints: the NLOS circular constraints are converted to rectangular constraints.

where

$$\boldsymbol{B}_1 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad \boldsymbol{r}_i = \begin{bmatrix} r_{Ni} + x_{Ni} \\ r_{Ni} - x_{Ni} \\ r_{Ni} + y_{Ni} \\ r_{Ni} - y_{Ni} \end{bmatrix},$$

and $\boldsymbol{I}_{n\times n}$ denotes an $n \times n$ identity matrix. We can now stack the constraints due to each of the m_N NLOS range estimates to form a system of $N = 4m_N$ equations as follows:

$$\begin{aligned} [\boldsymbol{B}\ \boldsymbol{I}_{N\times N}]\, \boldsymbol{z} &= \boldsymbol{r}, \\ \boldsymbol{z} &\geq \boldsymbol{0}, \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{B} &= \left[\ \boldsymbol{B}_1^T \quad \boldsymbol{B}_1^T \quad \cdots \quad \boldsymbol{B}_1^T\ \right]_{(2\times N)}^T, \\ \boldsymbol{r} &= \left[\ \boldsymbol{r}_1^T \quad \boldsymbol{r}_2^T \quad \cdots \quad \boldsymbol{r}_{m_N}^T\ \right]_{(1\times N)}^T, \qquad (18) \\ \boldsymbol{w} &= \left[\ \boldsymbol{w}_1^T \quad \boldsymbol{w}_2^T \quad \cdots \quad \boldsymbol{w}_{m_N}^T\ \right]_{(1\times N)}^T, \qquad (19) \end{aligned}$$

with the vector of variables being defined as

$$\boldsymbol{z} = \left[x \quad y \quad \boldsymbol{w}^T\right]_{(1\times(N+2))}^T.$$

It is important to note that in the above analysis, no objective function was defined based on the NLOS range estimates, and we only constrained the feasible region for $\boldsymbol{x}$. The feasible region can further be constrained by including the tangents at the intersection points of the circular constraints defined in (15) to reduce the size of the feasible region, but these additional constraints add to the computational complexity of the linear program without providing substantial gains. In the following subsection, we integrate the constraints and objective function obtained using LOS range estimates with the NLOS constraints defined above, for the problem of sensor location-estimation, given a mixture of LOS and NLOS range estimates with $m_L \geq 3$, $m_N \geq 0$.

3.3 Combining the LOS and NLOS Range Information

Based on the above subsections, given $m_L \geq 3$ LOS range estimates and $m_N \geq 0$ NLOS range estimates, we can combine them into a single linear program. We define the vector of variables as

$$\boldsymbol{z} = [x \;\; y \;\; \boldsymbol{\epsilon} \;\; \boldsymbol{w}]^T_{(1\times(2M+N+2))},$$

where $\boldsymbol{\epsilon}$ and $\boldsymbol{w}$ are respectively defined in (12) and (19). The objective function Z is defined as

$$Z = \boldsymbol{f}^T\boldsymbol{z},$$

where $\boldsymbol{f}^T = [0 \; 0 \; \mathbf{1}_{2M\times 1} \; \mathbf{0}_{N\times 1}]_{1\times(2M+N+2)}$. The linear program is formulated as:

$$\begin{aligned} \min \; Z &= \boldsymbol{f}^T\boldsymbol{z}, \quad \text{such that} \\ \boldsymbol{D}\boldsymbol{z} &= \boldsymbol{g}, \; \boldsymbol{z} \geq \mathbf{0}, \end{aligned}$$

where

$$\boldsymbol{D} = \left[\begin{array}{c|cc} \boldsymbol{A} & \boldsymbol{J} & \mathbf{0}_{M\times N} \\ \boldsymbol{B} & \mathbf{0}_{2M\times N} & \boldsymbol{I}_{N\times N} \end{array} \right]_{(M+N)\times(2+2M+N)},$$

and

$$\boldsymbol{g} = \left[\begin{array}{c} \boldsymbol{c} \\ \boldsymbol{r} \end{array} \right]_{(2+2M+N)\times 1}.$$

In the above equations, the matrices $\boldsymbol{A}$, $\boldsymbol{J}$ and $\boldsymbol{B}$ are respectively defined in (8), (14) and (18), and the vectors $\boldsymbol{c}$ and $\boldsymbol{r}$ are defined in (9) and (18) respectively.

It must be pointed out that in the above linear program, LOS range information is used to define both the objective function and the feasible region, whereas the NLOS range information is used only to define the feasible region. This allows the NLOS range estimates to "assist" in improving the accuracy of location estimates by limiting the size of the feasible region, but does not allow the NLOS bias errors to adversely affect sensor localization accuracy, since the NLOS range information plays no part in defining the objective function. The efficacy of the proposed method is demonstrated through simulations in section 5. The above approach works for any mixture of LOS and NLOS range estimates, provided $m_L \geq 3$, $m_N \geq 0$. In the following section, we discuss some special sub-cases where there are insufficient LOS and NLOS range estimates to apply the approach described above.

4. SPECIAL CASES

4.1 Special Case: $m_L = 0$

If no LOS range estimates are available, then the above LP approach will not be applicable since the NLOS range estimates are not used to define an objective function, although they can be used to define a feasible region. An example of this situation with $m_N = 3$ is shown in Figure 7. In this case, we could either use the LS estimator (or a constrained LS estimator) without the mitigation of bias errors, or simply use the center of the rectangular feasible region as a location estimate:

$$\hat{\boldsymbol{x}} = \frac{1}{2}\left[\begin{array}{c} \min_i\{x_{Ni} + r_{Ni}\} + \max_i\{x_{Ni} - r_{Ni}\} \\ \min_i\{y_{Ni} + r_{Ni}\} + \max_i\{y_{Ni} - r_{Ni}\} \end{array} \right].$$

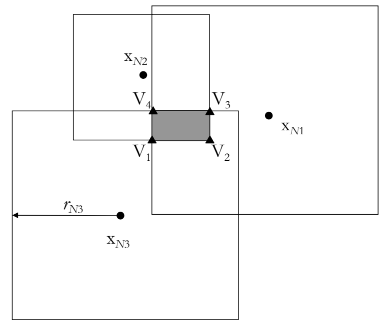
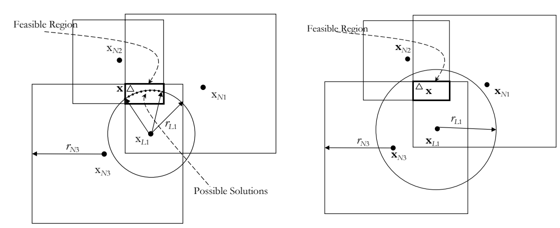


Figure 7: Special Case with $m_L = 0$ and $m_N = 3$. The vertices of the feasible region are denoted by V_1, V_2, V_3 and V_4.

4.2 Special Case: $m_L = 1$, $m_N \geq 2$

This situation is illustrated in Figures 8(a) and 8(b), which represent two sub-cases. Since $m_L = 1$, we have a single circular equality constraint. In the first sub-case (Figure 8(a)), the circle formed using the LOS range passes through the feasible region formed by the NLOS constraints. In such a case, there are infinitely many solutions. In the second sub-case, illustrated in Figure 8(b), the circular constraint formed using the LOS range estimate does not pass through the feasible region. In such a case we can pick the vertex of the feasible region that is closest to the circle as a potential solution.



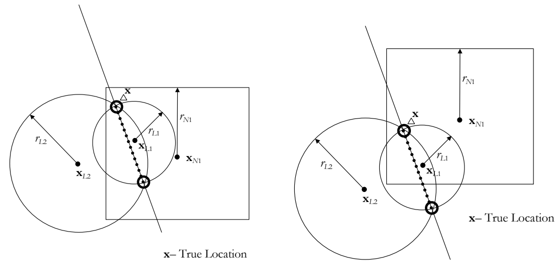
(a) $m_L = 1$, $m_N \geq 2$. Sub-case 1: the LOS constraint cuts through the feasible region generated using the NLOS constraints.

(b) $m_L = 1$, $m_N \geq 2$. Sub-case 1: the LOS constraint does not pass through the feasible region generated using the NLOS constraints.

Figure 8: Case: $m_L = 1$, $m_N \geq 2$.

4.3 Special Case: $m_L = 2$, $m_N \geq 1$

In this case, since $m_L = 2$, we have two circular constraints, and the linearization procedure is not particularly useful since a single linear constraint is generated using the difference. Therefore, instead of two potential solutions corresponding to the intersections of the two circles, we have an infinite number of solutions. It is easier to compute the two intersections of the circles, and if there are additional NLOS constraints, three sub-cases arise: (1) Neither of the intersection points lies inside the feasible region formed by the NLOS constraints, (2) both intersection points lie inside the feasible region formed by the NLOS constraints (Figure 9(a)), and (3) only one of the intersection points lies inside



(a) $m_L = 2$, $m_N \geq 1$, Sub-case 1: The feasible region contains both intersection points of the circle.

(b) $m_L = 2$, $m_N \geq 1$, Sub-case 2: The feasible region contains one of two intersection points of the circle.

Figure 9: Case: $m_L = 2$, $m_N \geq 2$.

the feasible region formed by the NLOS constraints (Figure 9(b)). In the latter sub-case, we simply pick the intersection point that lies within the feasible region. In the first two sub-cases, the ambiguity remains.

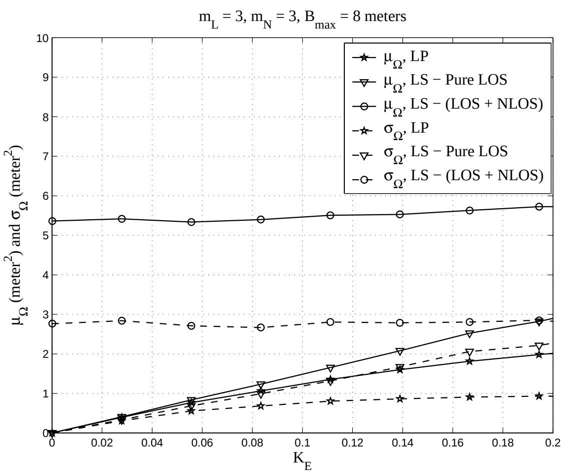


Figure 10: The mean μ_Ω and the standard deviation σ_Ω of the localization error Ω are plotted versus K_E. Here, $m_L = 3$, $m_N = 3$ and $B_{\max} = 8$ meters.

5. SIMULATION RESULTS

In this section, we present simulation results that demonstrate that the proposed approach mitigates the effect of NLOS bias errors and utilizes the NLOS range information to improve sensor localization accuracy. In the following discussion, the anchor nodes are randomly distributed over an $L \times L$ area where $L = 10$ meters. The unknown location of the sensor node is $\boldsymbol{x} = [5\ 5]^T(meters)$. We compare the performance of three location-estimation approaches in terms of the mean and standard deviation of the localization error Ω: (i) the proposed LP approach, (ii) the LS estimator, utilizing only LOS range estimates, while discarding the NLOS range estimates ("Pure-LOS"), and (iii) the LS estimator, utilizing both LOS and NLOS range estimates, without the mitigation of NLOS bias errors ("LOS+NLOS").

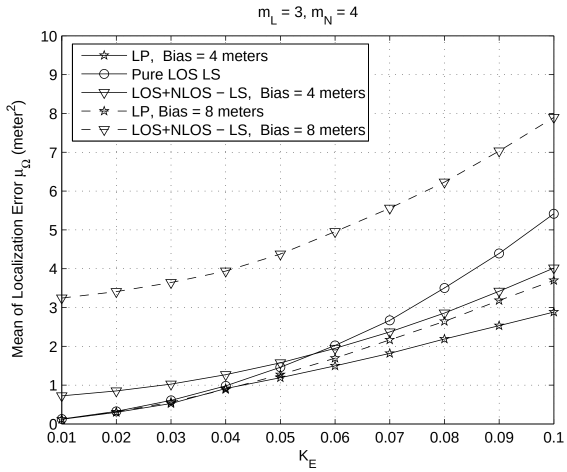


Figure 11: Mean μ_Ω of the Localization Error Ω, $m_L = 3$, $m_N = 4$. The maximum bias $B_{\max}$ is increased from 4 meters to 8 meters.

For these three methods, the values of Ω are computed for a large number of realizations of the measurement noise, bias errors and anchor locations. The mean μ_Ω and the standard deviation σ_Ω are shown in Figure 10, for different values of the proportionality constant K_E defined in (2). In this simulation, $m_L = 3$, $m_N = 3$ and $B_{\max} = 8$ meters. As expected, in all three cases, sensor localization accuracy degrades as the variance of the range estimates increases (i.e., K_E increases). The proposed LP approach outperforms the other two schemes in terms of both the mean and standard deviation of the localization error, and therefore, on the average, produces more accurate sensor location estimates. In general, it was observed that for all three estimation procedures, μ_Ω and σ_Ω follow similar trends in terms of their variation with K_E:

$$\mu_{\Omega,LP} < \mu_{\Omega,LS-Pure\ LOS} < \mu_{\Omega,LS-(LOS+NLOS)}$$
$$\sigma_{\Omega,LP} < \sigma_{\Omega,LS-Pure\ LOS} < \sigma_{\Omega,LS-(LOS+NLOS)}.$$

The variation of the mean localization error μ_Ω with K_E, while respectively increasing the maximum bias $B_{\max}$, and the number of NLOS range estimates m_N, is shown in Figures 11 and 12 for a specific set of anchor locations. We observe that (a) the LP approach performs better than both the LS approaches and is less sensitive to increase in bias errors, and (b) the performance of the LP approach improves as m_N increases, whereas the opposite effect is observed with the LS estimator that utilizes LOS and NLOS range estimates without bias error mitigation. The former effect is due to the fact that the NLOS ranges do not contribute to the objective function of the linear program, while the latter is because additional NLOS range estimates reduce the size of the feasible region, thereby reducing the maximum possible values of the localization error. The LP approach once again outperforms the LS case that utilizes only LOS range estimates.

6. CONCLUSIONS

In this paper, we described a novel linear-programming approach to the problem of sensor localization in NLOS en-

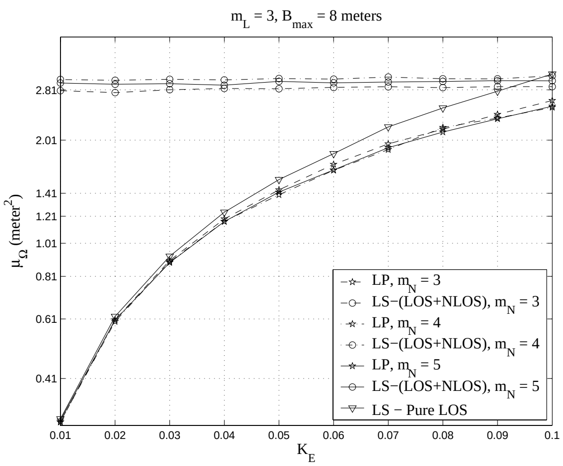


Figure 12: Mean μ_Ω of the Localization Error Ω, $m_L = 3$, $B_{\max} = 8$ meters. The number of NLOS range estimates is varied for $m_N = 3$ to $m_N = 5$.

vironments. The main motivation for the development of this method was that in typical indoor sensor networks, it is likely that we would be required to compute a sensor's location using a mixture of LOS and NLOS range estimates. Using the LOS range estimates to define the objective function, and the NLOS range estimates to restrict the feasible region for the linear program, we showed that NLOS range information can be used to improve sensor localization accuracy without incurring performance degradation due to bias errors. This approach was shown to perform as well as the LS estimator when only LOS range estimates are provided, and better than the LS estimator when a mixture of LOS and NLOS range estimates is provided. Relative to LS estimation, it was found that the proposed approach is less sensitive to increase in NLOS bias errors and that increasing the number of NLOS range estimates improves sensor localization accuracy. Further, the proposed approach can be applied to a general three-dimensional location-estimation problem.

7. REFERENCES

[1] J. M. Rabaey, M. J. Ammer, J. L. da Silva, D. Patel, and S. Roundy, "Picoradio supports ad hoc ultra-low power wireless networking," *IEEE Comput.*, vol. 33, pp. 42–48, July 2000.

[2] N. Patwari, A. O. Hero, M. Perkins, N. S. Correal, and R. J. O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 51, pp. 2137–2148, Aug. 2003.

[3] R. Fleming and C. Kushner, "Low-power, miniature, distributed position location and communication devices using ultra-wideband, nonsinusoidal communication technology," tech. rep., July 1995. Aetherwire Inc. Semi-Annual Tech. Rep., ARPA Contract J-FBI-94-058.

[4] R. L. Moses, D. Krishnamurthy, and R. Patterson, "An auto-calibration method for unattended ground sensors," in *ICASSP 2002*, pp. 2941–2944, May 2002.

[5] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, R. L. Moses, and N. S. Correal, "Locating the nodes," *IEEE Signal Processing Magazine*, vol. 22, pp. 54– 69, July 2005.

[6] J.-Y. Lee and R. A. Scholtz, "Ranging in a dense multipath environment using an UWB radio link," *IEEE Journal on Selected Areas in Communications*, vol. 20, pp. 1677–1683, December 2002.

[7] T. Silventoinen and M. Rantalainen, "Mobile station emergency locating in GSM," in *1996 IEEE International Conference on Personal Wireless Communications*, pp. 232 –238, Feb. 19-21 1996.

[8] M. P. Wylie and J. Holtzman, "The Non-Line of Sight Problem in Mobile Location Estimation," in *5th IEEE International Conference on Universal Personal Communications*, vol. 2, pp. 827 – 831, 1996.

[9] J. Borras, P. Hatrack, and N. B. Mandayam, "Decision theoretic framework for NLOS identification," in *1998 Vehicular Technology Conference, (VTC '98)*, vol. 2, pp. 1583 – 1587, 18th-21st May 1998.

[10] Y. Qi and H. Kobayashi, "Cramer-Rao lower bound for geolocation in non-line-of-sight environment," in *Proceedings of ICASSP 2002*, vol. 3, pp. III–2473 – III–2476, May 2001.

[11] S. M. Kay, *Fundamentals of Statistical Processing, Volume I : Estimation Theory.* 1993. 2nd Edition, Prentice-Hall Inc.

[12] J. J. Caffery, "A new approach to the geometry of TOA location," in *2000 IEEE Vehicular Technology Conference*, vol. 4, pp. 1943 – 1949, September 2000.

[13] L. Doherty, K. S. J. Pister, and L. E. Ghaoui, "Convex position estimation in wireless sensor networks," in *Proceedings of INFOCOM 2001*, vol. 3, pp. 1655–1663, April 2001.

[14] X. Wang, Z. Wang, and R. O'Dea, "A TOA-based location algorithm reducing the errors due to non-line-of-sight (NLOS) propagation," *IEEE Transactions on Vehicular Technology*, vol. 52, pp. 112–116, January 2003.

[15] E. G. Larsson, "Cramer-Rao bound analysis of distributed positioning in sensor networks," *IEEE Signal Processing Letters*, vol. 11, pp. 334–337, March 2004.

[16] P.-C. Chen, "A non-line-of-sight error mitigation algorithm in location estimation ," in *Wireless Communications and Networking Conference 1999*, vol. 1, pp. 316–320, Sept. 21-24th 1999.

[17] C. Chang and A. Sahai, "Estimation bounds for localization," in *Proceedings of IEEE SECON 2004*, pp. 415 – 424, 4-7 Oct. 2004.

[18] Y. Qi and H. Kobayashi, "On relation among time delay and signal strength based geolocation methods," in *Proceedings of IEEE GLOBECOM '03*, vol. 7, pp. 4079–4083, December 2003.

[19] M. A. Spirito, "On the accuracy of cellular mobile station location estimation," *IEEE Transactions on Vehicular Technology*, vol. 50, pp. 674–685, May 2001.

[20] M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali, *Linear Programming and Network Flows.* John Wiley and Sons, New York, 1995.