Power Control in UWB Position-Location Networks

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***Abstract*— In this paper, we discuss the use of power control in Ultra-Wideband (UWB) Position-Location Networks (PoLoNets)[1], where the goal is to periodically estimate the locations of mobile nodes that move through a network of location-aware reference nodes. The reference nodes provide Time-of-Arrival (TOA) based range estimates that are used to estimate the locations of mobile nodes at regular intervals. We examine the accuracy of location-estimation through the Cramer-Rao lower bound (CRLB) and show that the localization accuracy fluctuates or "fades" as a mobile node moves through the network of reference nodes. The use of power control is shown to improve the robustness of location-estimates and provide higher localization accuracy. Two power control approaches are presented and compared with the empirically-obtained optimal solutions.**

I. Introduction

In outdoor environments, accurate position information can be obtained via GPS. However, there are many situations where GPS is either unreliable (e.g., indoor scenarios), or impractical (e.g., where GPS receivers are too bulky or expensive), requiring the development of other solutions. For instance, consider the command-and-control [1] of a firefighter operation, where multiple personnel are deployed in a building. For safety and efficiency purposes, it is desirable to establish an infrastructure for remote personnel-position tracking, which requires a position-location network (PoLoNet) that is independent of GPS. Other applications of such PoLoNets include inventory control, home automation, safety networks, tracking personal items, personnel monitoring, command and control in emergency situations and the guidance of robots in remote or hazardous locations.

Impulse-based ultra-wideband (UWB) or Impulse-Radio is an excellent physical layer solution for PoLoNets because of its robustness in harsh multipath environments, its ability to fuse accurate (on the order of tens of centimeters) position-location [2] with low-data rate communication [3] and its covertness for tactical applications.

In this work, we investigate the use of transmit power control schemes in *infrastructure-based* UWB PoLoNets. This implies the presence of stationary *reference nodes* in the area of interest whose locations are known *a priori*. This can be viewed as the end-result of the ad hoc deployment of reference nodes, that subsequently learn their locations based on location-aware anchors or a local relative coordinate system. As shown in Figure 1, these reference nodes aid the estimation of the locations of *mobile nodes* entering the area of interest via ranging and triangulation. A subset of the reference nodes provides TOA-based range estimates to the mobile node, which are used to estimate (triangulate) its location. The mobile nodes periodically update their location-estimate by "ranging" to reference nodes in order to maintain accuracy of location estimates. These location estimates can then be routed through the multi-hop network of reference nodes to a data-sink that monitors the locations of mobile nodes.

Localization accuracy [2] is an extremely important feature of UWB PoLonets, particularly in emergency-response applications. For instance, in a fire-fighter [1] position-tracking system, the knowledge of whether a firefighter is on one side of a door or the other could be critical. In this paper, we focus on the accuracy of location-estimates of mobile nodes and show that the localization accuracy "fades" as the mobile node moves through the network of reference nodes, due to variations in (i) connectivity with reference nodes, (ii) range-estimate variances and (iii) geometry. Our goal is to improve the localization accuracy of mobile nodes and make mobile node location-estimates more robust to fluctuations arising due to node mobility through the use of transmit power control. We demonstrate that power control algorithms based on location-estimates can be used to ensure a desired localization accuracy.

Power control algorithms for wireless and cellular communications have been thoroughly investigated [4], are typically designed to satisfy quality-of-service (QOS) requirements and reduce power-consumption. The design of UWB PoLoNets is a relatively new area [1] and the literature on the subject is limited [5], [6], [7]. Power control in order to maximize capacity of generic UWB networks was investigated in [8], [9]. However, in PoLoNets an important design goal is to maximize localization accuracy and to the best of our knowledge, power control in UWB PoLoNets from the perspectives of localization accuracy has not been investigated.

This paper is organized as follows: In section II, we describe the network architecture of UWB PoLoNets and discuss bounds used to model TOA-based ranging and location-estimation. In sections III and IV, we examine the variation of localization accuracy due to node mobility and impact of power control on the localization accuracy. A means of quantifying and estimating the "quality" of location-estimates is discussed in Section V. Two power control estimates based on the defined quality metric are discussed in detail in section VI. A comparison of these power control approaches with simulated optimal solutions obtained can be found in section VII. We conclude in section VIII.

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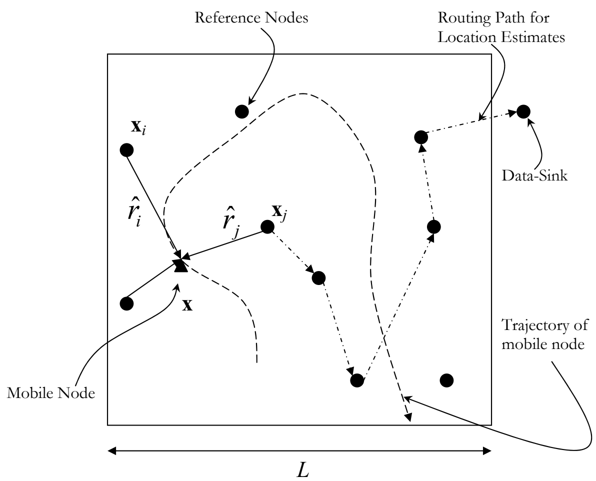

Fig. 1. Network Architecture: mobile nodes obtain range estimates using transmissions from reference nodes and triangulate their location.

II. Network architecture

We assume the existence of N_R reference nodes, whose (known) locations are $\boldsymbol{x}_i = [x_i \ y_i]^T$, $i = 1, 2, \cdots, N_R$, that are distributed over an $L \times L$ meter^2 area. In the absence of network-wide synchronization, the mechanism of TOA-ranging between two nodes is via a packet-handshake as discussed in [3], [6]. Ranges to reference nodes can be estimated based on the delays between the transmission and reception times of the packets comprising the handshake. Therefore, when a mobile node needs to estimate its current location, denoted by $\boldsymbol{x} = [x \ y]^T$, it initiates such a packet handshake with a certain transmit power P (this implies that the responding reference node completes the packet-handshake using the same transmit power P).

The effective received signal-to-noise-ratio (SNR) at reference node i, during a "range-request" broadcast by the mobile node is modeled by $\xi_i = K_P P R_i^{-\beta}$, where β is the path-loss exponent, $R_i = \|\boldsymbol{x} - \boldsymbol{x}_i\|$ is the distance between the nodes, and K_P is a constant that subsumes the effects of other physical layer parameters. If multiple uncoordinated mobile nodes are present, then the SNR would have to be replaced by signal-to-interference-and-noise ratio (SINR). In this work, for simplicity, we examine the case with one mobile node in a network of reference nodes. We assume that the minimum SNR required for a packet to be received successfully is given by $\xi_{\min}$, resulting in a maximum radius of coverage $R_{\max}(P) = \left(\frac{K_P P}{\xi_{\min}}\right)^{\frac{1}{\beta}}$ around $\boldsymbol{x}$. Let $N(P)$ represent the number of reference nodes present within a radius $R_{\max}(P)$ from the mobile node. Consequently, for a transmit power P and in the absence of collisions, a mobile node can obtain $m = N(P)$ estimates r_i of the true ranges R_i, $i = 1, 2, \cdots, m$, from the arrival times of the responses. As P increases, $N(P)$ either increases or remains the same, due to the increase in the transmission radius $R_{\max}(P)$.

We assume that the range estimation errors are zero-mean Gaussian random variables:

$$r_i = R_i + n_i, \ n_i \sim \mathcal{N}(0, \sigma_i^2), \ i = 1, 2, \cdots, m, \qquad (1)$$

which is a common assumption in the literature on position-location [10], [11]. The presence of non-line-of-sight (NLOS) links results in positive bias errors in UWB TOA-based ranging schemes [3], [12]. The impact of NLOS range errors on location-estimation is currently being investigated, but in this work, we restrict our attention to the case where all links are LOS.

The Cramer-Rao Lower Bound (CRLB) for the estimation of the TOA of a single-multipath component in AWGN [13] was shown to be inversely proportional to the effective SNR. The CRLB for TOA-based range estimation in a multipath environment [13] can be approximated by

$$\sigma_R^2 \approx \frac{K_R}{\xi_{\text{eff}}} = \frac{K_R R^\beta}{K_P P}, \qquad (2)$$

where K_R is a constant and ξ_{eff} is the effective SNR. From the above equation we see that the variance of TOA-based range estimates increases with range. Combining (1) and (2), our model for the estimated ranges ($i = 1, 2, \cdots, m = N(P)$) becomes

$$r_i \sim \mathcal{N}(R_i, \sigma_i^2), \ \sigma_i^2 = \frac{K_R R_i^\beta}{K_P P}. \qquad (3)$$

The CRLB for the estimation of a node's location given unbiased Gaussian range estimates $\{r_i\}$ from known locations $\{\boldsymbol{x}_i\}$, ($i = 1, 2, \cdots, m$) has been derived previously in [10], [11]. We define our measure of localization accuracy, the *localization error* of a location-estimate, as the sum of the variances of the x and y coordinates of the location estimate $\boldsymbol{x}$. The localization error of the minimum-variance unbiased estimator (MVUE) is given by

$$\Omega_{\boldsymbol{x},m} = \sigma_x^2 + \sigma_y^2 = \frac{\sum_{i=1}^m \frac{1}{\sigma_i^2}}{\sum_{i=1}^m \sum_{j=1,j>i}^m \frac{\sin^2(\alpha_i - \alpha_j)}{\sigma_i^2 \sigma_j^2}}, \qquad (4)$$

where α_i is the orientation (angle) of the ith reference node relative to the node whose location is being estimated: $\alpha_i = \measuredangle(\boldsymbol{x}_i - \boldsymbol{x})$. For our case, from (3) and (4)

$$\Omega_{\boldsymbol{x},m} = \frac{K_R \sum_{i=1}^m \frac{1}{R_i^\beta}}{K_P P \sum_{i=1}^m \sum_{j=1,j>i}^m \frac{\sin^2(\alpha_i - \alpha_j)}{R_i^\beta R_j^\beta}}. \qquad (5)$$

From (4), we observe that the localization error is a function of (i) the geometry of reference nodes (α_i, $i = 1, 2, \cdots, m$), (ii) the variances of the range estimates (σ_i^2, $i = 1, 2, \cdots, m$) and (iii) the number of range estimates (m).

A. Impact of Geometry

Suppose we define

$$\gamma_m \triangleq \sum_{i=1}^m \frac{1}{\sigma_i^2} > 0, \ \psi_m \triangleq \sum_{i=1}^m \sum_{j=1,j>i}^m \frac{\sin^2(\alpha_i - \alpha_j)}{\sigma_i^2 \sigma_j^2} \geq 0,$$

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then the *generalized Geometric Dilution of Precision* (GGDOP) is defined by

$$\Gamma_m \triangleq \frac{\psi_m}{\gamma_m^2}. \qquad (6)$$

The above definition of the GGDOP allows us to partially dissociate the impact of range variances and geometry on the localization error: for a fixed set of range variances $\{\sigma_i^2\}$, $i = 1, 2, \cdots, m$, this implies that increasing the GGDOP results in a decrease of the localization error:

$$\Omega_{\boldsymbol{x},m} = \frac{\gamma_m}{\psi_m} = \frac{1}{\gamma_m \Gamma_m}. \qquad (7)$$

The classical definition of the GDOP [11] is a special case of the above with $\sigma_i^2 = \sigma^2$, $\forall i$. For a fixed set of range estimate variances, as the GGDOP Γ_m increases, the localization error decreases. It must be noted that Γ_m is not purely a function of the geometry of reference nodes, but also depends on the relative range variances. However, if all range variances are all scaled by a factor while maintaining the same node orientations, the value of Γ_m remains unchanged. It can be shown that (see Appendix I) the GGDOP is bounded for all $\{\sigma_i^2\}$, $\{\alpha_i\}$: $0 \leq \Gamma_m \leq \frac{1}{4}$.

It is evident from (7) that when the reference nodes are collinear, i.e., $\alpha_i = \alpha$, $\forall i$, $\Gamma_m = 0$ and $\Omega_{\boldsymbol{x}} \to \infty$; when $\Gamma_m = \frac{1}{4}$, $\Omega_{\boldsymbol{x}} = \frac{4}{\sum_{i=1}^{m} \frac{1}{\sigma_i^2}}$. It is clear from (7) that for a fixed value of Γ_m, the localization error $\Omega_{\boldsymbol{x},m}$ decreases as the range variances $\{\sigma_i^2\}$ decrease.

B. Impact of Range Estimate Variances

It is intuitive that improving the accuracy of range estimates should improve the accuracy of the mobile's location estimate. Formally, suppose we have an initial geometric configuration of reference nodes $\{\alpha_i\}$, $i = 1, 2, \cdots, m$ with the corresponding range variances $\{\sigma_i^2\}$, $i = 1, 2, \cdots, m$. The decrease in *any* of the variances results in the reduction of the localization error. Specifically, if $\sigma_i'^2 = a\sigma_i^2$, $0 < a \leq 1$, the new localization error $\Omega'_{\boldsymbol{x}}$ satisfies

$$\Omega'_{\boldsymbol{x}} \leq \Omega_{\boldsymbol{x}}. \qquad (8)$$

The proof of the above result is provided in Appendix II.

C. Impact of Connectivity

The localization error of a mobile node depends upon its connectivity with reference nodes through $m = N(P)$. Suppose we have an initial geometric configuration of reference nodes $\{\alpha_i\}$ with range variances $\{\sigma_i^2\}$, $i = 1, 2, \cdots, m$. The introduction of an additional range-estimate from a node with orientation α_{m+1} and variance σ_{m+1}^2 results in a reduction of the localization error. Specifically,

$$\Omega_{\boldsymbol{x},m+1} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{\gamma_m}{2\sigma_{m+1}^2} - \frac{\sqrt{\gamma_m^2 - 4\gamma_m \cos(2\alpha_{m+1} - 2\nu)}}{2\sigma_{m+1}^2}} \leq \Omega_{\boldsymbol{x},m}, \qquad (9)$$

where ν is given by (18). Equality holds when $\alpha_1 = \alpha_2 = \cdots = \alpha_m = \alpha_{m+1}$. The proof is given in Appendix III. Therefore, except when all the reference nodes are collinear, enhancing the extent of connectivity with reference nodes, which increases the number of available range estimates, always improves localization accuracy.

III. "FADING" OF LOCALIZATION ACCURACY

As shown in (7), the localization error $\Omega_{\boldsymbol{x}}$ is dependent on the geometry of reference nodes (through Γ_m), connectivity with localized reference nodes (through m) and range estimate variances (which depend on R_i). It is evident that as a mobile node moves through the network of reference nodes, all three quantities can vary with time and therefore, the localization error of a mobile node is a function of time. Figure 2 illustrates an example of the variation of a mobile node's localization error as the mobile node moves through an area containing randomly distributed reference nodes. The fluctuation of the localization error is analogous to the spatial fading of received signal power in wireless propagation channels, although spatial fading is due to multipath while localization fading is due to changes is geometry of reference nodes, range estimate variances and connectivity.

It was shown in (9) that increasing the number of range estimates from reference nodes decreases the localization error. Therefore, one way of reducing the fluctuation of localization error due to changes in connectivity is to initially deploy reference nodes with a high node density. An alternative means of improving localization accuracy and making it robust to node mobility is through the use of power control. We see that increasing the transmit power P (i) increases $R_{\max}$, improving connectivity and resulting in the availability of a larger number of range estimates $m = N(P)$, and (ii) from (3), reduces the range-estimate variances.

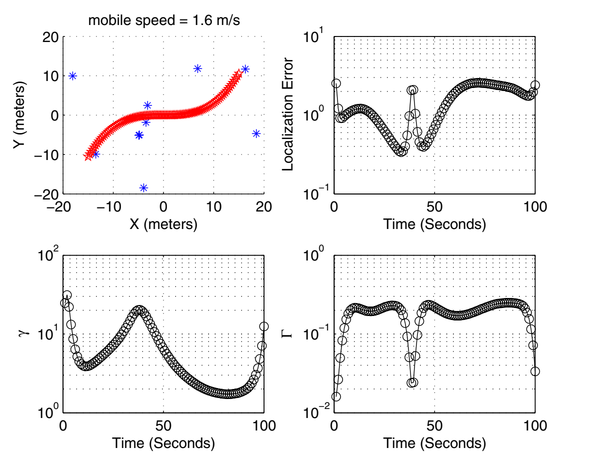

Fig. 2. "Spatial Fading" of Localization Accuracy. In this example, $\beta = 2$, $P = 10$ mW, $N = 10$, $L = 40$, $\xi_{\min} = 20$ dB, $v = 1.6$ meters/second. We see that the reference node geometry (determined by Γ) plays an important role in determining the localization accuracy.

It must be pointed out that the short-term fading in localization accuracy can be partially mitigated by tracking algorithms such as Kalman filtering. However, in PoLoNets

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with a low density of reference nodes, limited connectivity with reference nodes — which results in poor localization accuracy — can span longer durations of time, thereby limiting the effectiveness of tracking algorithms. In such situations, we can resort to power-control to improve connectivity and reduce localization error. In the following section, we quantify the decrease in localization error with an increase in transmit power. It must be noted that although the investigation of power control algorithms that follows is performed with UWB PoLoNets in mind, the development is not specific to the UWB physical layer and be applied to generic PoLoNets.

IV. Effect of Increasing Transmit Power

Let P be the initial transmit power. Given $m = N(P)$ range estimates from reference nodes, the mobile node estimates its location with the localization error given by (4). Suppose the transmit power is increased by a factor $\chi > 0$. Then the new transmit power is given by $P' = P\,(1+\chi)$. Let the new number of responding reference nodes be $N(P') = m + n \geq m$. From (3), the new range-estimate variances are given by

$$\sigma_i'^2 = \begin{cases} \frac{\sigma_i^2}{1+\chi}, & i = 1, 2, \cdots, m. \\ \frac{K_R R_i^\beta}{K_P P'}, & i = m+1, m+2, \cdots, m+n. \end{cases} \quad (10)$$

The improvement in terms of localization error is defined as $D = \Omega_{\boldsymbol{x}}(P) - \Omega_{\boldsymbol{x}}(P')$. The expression for D in terms of χ is derived in Appendix IV and is given by (21). It is clear from (21) that $D > 0$ if $P' > P$ which implies that the localization error decreases with an increase in transmit power. As stated previously, the decrease in the localization error can be attributed to two effects of increasing the transmit power: improved connectivity resulting a larger number of range estimates ($m + n \geq m$,) and reduced range variances ($\sigma_i'^2 < \sigma_i^2$).

V. Quality of Location Estimates

We define the *Quality of Localization* (QOL) $Q_{\boldsymbol{x}}$ of a location-estimate $\boldsymbol{x}$ as the *reciprocal of the localization error*:

$$Q_{\boldsymbol{x}}(P) = \frac{1}{\Omega_{\boldsymbol{x}}} = \frac{K_P P \sum_{i=1}^{m} \sum_{j=1,j>i}^{m} \frac{\sin^2(\alpha_i - \alpha_j)}{R_i^\beta R_j^\beta}}{K_R \sum_{i=1}^{m} \frac{1}{R_i^\beta}}. \quad (11)$$

The larger the value of the QOL, the higher the localization accuracy. It is evident that in an infinite field of reference nodes as $P \to \infty$, $Q_{\boldsymbol{x}}(P) \to \infty$. We would like to find the transmit power P with which a given target location-estimate quality Q_0 is obtained. We can therefore set up an objective function to be minimized as

$$Z(P) = \left(Q_{\boldsymbol{x}}(P) - Q_0\right)^2. \quad (12)$$

It can be verified from (11) that the transmit power $P = P^\star$ that minimizes this objective function is given by the solution of the following equation:

$$P^\star = \frac{Q_0 K_R \sum_{i=1}^{N(P^\star)} \frac{1}{R_i^\beta}}{K_P \sum_{i=1}^{N(P^\star)} \sum_{j=1,j>i}^{N(P^\star)} \frac{\sin^2(\alpha_i - \alpha_j)}{R_i^\beta R_j^\beta}}. \quad (13)$$

However, this equation cannot directly be used to solve for $P^\star$, since we do not know a priori the values of $N(P)$, $\{R_i\}$ or $\{\alpha_i\}$ for a specific mobile location in the network. In our case, the value of $P^\star$ can be obtained via simulation in order to serve as a benchmark for the power control schemes proposed in this paper.

A. Quality Metric

In order to implement power control based on the quality of location estimates, it is essential for a mobile node to be able to assess the quality of its location-estimate. Suppose an initial power level P_0 was used and a location estimate $\hat{\boldsymbol{x}}$ was obtained (e.g., using a least-squares estimator). Assuming the knowledge of the constants K_P and K_R via calibration, the range estimates r_i, and the reference node coordinates $\boldsymbol{x}_i$, $i = 1, 2, \cdots, m$, the quality of the location estimate can be computed. The angles $\hat{\alpha}_i$ can be estimated using $\hat{\alpha}_i = \measuredangle(\boldsymbol{x}_i - \hat{\boldsymbol{x}})$. The variances of the range estimates can be estimated using

$$\hat{\sigma}_i^2 = \frac{K_R r_i^\beta}{K_P P_0}. \quad (14)$$

Therefore, the QOL estimate can be evaluated using

$$\hat{Q}_{\hat{\boldsymbol{x}}} = \frac{\sum_{i=1}^{m} \sum_{j=1,j>i}^{m} \frac{\sin^2(\hat{\alpha}_i - \hat{\alpha}_j)}{\hat{\sigma}_i^2 \hat{\sigma}_j^2}}{\sum_{i=1}^{m} \frac{1}{\hat{\sigma}_i^2}}. \quad (15)$$

A mobile node can adapt its transmit-power based on the QOL estimates as described below.

VI. Power-Adaptation based on quality of Location-Estimates

In this section, we describe two power control algorithms for UWB PoLoNets based on location-estimates in order to obtain a target QOL Q_0. The first algorithm uses an adaptive power-update based on the estimated QOL, whereas the second is a two-step procedure that tries to compute the transmit power required to obtain the target localization accuracy. We assume that mobile node displacements are small in the time-span over which power control algorithms are implemented.

A. QOL-based Iteration

In a mobile environment, it may be necessary to update the location-estimate of the mobile node periodically. In such a case, the transmit power can be updated at each attempt to compute the location-estimate, based on the quality of the previous location estimate. Consider a simple iterative power control algorithm, where the kth iteration is of the form

$$P_{k+1} = P_k + \Delta_P \left(Q_0 - \hat{Q}_{\hat{\boldsymbol{x}}}(P_k)\right), \quad (16)$$

where Δ_P represents the step-size of the power increment or decrement. It is clear from the above equation that if $\hat{Q}_{\hat{\boldsymbol{x}}}(P_k) < Q_0$, then the transmit power in the subsequent iteration is increased; if $\hat{Q}_{\hat{\boldsymbol{x}}}(P_k) > Q_0$, then the transmit power is decreased. Based on the framework of power control convergence analysis provided in [4], it can be shown that the above power control scheme converges to the optimal solution, provided the value of Δ_P is sufficiently small.

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B. Two-Step Solution

In applications where a location-estimate is computed at irregular intervals or based on a requirement, an iterative scheme such as that described above may not be suitable. In such a case, we would like a non-iterative solution to directly compute the required transmit power. Let P_0 be the initial transmit power with the corresponding QOL estimate $\hat{Q}_{\hat{\boldsymbol{x}}}(P_0)$. From (21), the unknown increment χ can be computed (as shown in Appendix IV) setting the improvement $D = \frac{1}{\hat{Q}_{\hat{\boldsymbol{x}}}(P_0)} - \frac{1}{Q_0}$, if the term C in (21) is known. However, C includes ranges from nodes that *would* be responding *after* the transmit power is increased by a factor χ. Since this information is not known when χ is computed, we assume a worst-case geometry for nodes that would be introduced if the transmit power were increased, given by (22), leading to $C = 0$. The details of the computation of χ can be found in Appendix IV.

Since the computed value of χ assumes a worst-case geometry, the transmit power $P' = P_0(1 + \chi)$ is, with high probability, larger than the optimal transmit power $P^\star$ that ensures a quality metric Q_0. Therefore, the second step in the solution is to decrease the transmit power to a level that ensures $\hat{Q}_{\hat{\boldsymbol{x}}} = Q_0$. This ensures that if the location-estimate is updated subsequently, the initial transmit power used is close to the optimal transmit power. In this case, since the locations and ranges for all the nodes are known, we can selectively exclude the largest ranges and retain the range estimates that ensure $|\hat{Q}_{\hat{\boldsymbol{x}}} - Q_0|$ is minimized. The power control algorithm is defined explicitly as follows:

(1) Start with a transmit power P_0.

(2) Using transmit power P_0, obtain range estimates r_i, $i = 1, 2, \cdots, m$ and reference node coordinates $\boldsymbol{x}_i$, $i = 1, 2, \cdots, m$. Compute an estimate of the mobile node's location $\hat{\boldsymbol{x}}$. Using this location estimate, estimate the angles $\hat{\alpha}_i = \measuredangle(\boldsymbol{x}_i - \hat{\boldsymbol{x}})$. Compute the range variances using (14). Compute the QOL the location-estimate $\hat{\boldsymbol{x}}$ using (15).

(3) If $\hat{Q}_{\hat{\boldsymbol{x}}} > Q_0$, then arrange the available range estimates in increasing order. Compute the QOL estimate from (15) with the first $m = k$ ranges, ending with the full set of ranges. Find the value of $k = k^\star$ for which the quality metric $\hat{Q}_{\hat{\boldsymbol{x}}}(k)$ is closest to Q_0. Compute the transmit power corresponding to the $k^\star$th range using $P = \frac{r_{k^\star}^\beta \xi_{\min}}{K_P}$. Stop.

(4) Else, if $\hat{Q}_{\hat{\boldsymbol{x}}} < Q_0$, compute χ using (23), where we replace $\{\sigma_i^2\}$ with $\{\hat{\sigma}_i^2\}$, $\{\alpha_i\}$ with $\{\hat{\alpha}_i\}$, and D with $\frac{1}{Q_0} - \frac{1}{\hat{Q}_{\hat{\boldsymbol{x}}}}$. Next, go to step (3).

It must be noted that in general, the transmit power obtained using step (3) will not be equal to the optimal transmit power because (i) there may not exist a $k^\star$ such that $\hat{Q}_{\hat{\boldsymbol{x}}}(k^\star) = Q_0$. In such a case, the final $k^\star = \text{argmin}_k |\hat{Q}_{\hat{\boldsymbol{x}}}(k) - Q_0|$, and (ii) the transmit-power is computed using $P = \frac{r_{k^\star}^\beta \xi_{\min}}{K_P}$, which uses a range-estimate and not the true range.

C. Multiple uncoordinated mobile nodes

The power-control algorithms discussed above are applicable to the case of a single mobile node in a network of reference nodes. The transmit power is varied from an initial value until the desired localization accuracy is attained. In the presence of multiple uncoordinated mobile nodes, increasing (decreasing) the transmit power of a given mobile node also increases (decreases) the interference observed at other mobile and reference nodes. From (2), increasing the transmit power decreases the SINR seen at other nodes thereby degrading the quality of range estimates and consequently the localization error. In such scenarios, stochastic power control algorithms such as those discussed in [4] can be applied, where the objective is to find the vector of mobile transmit powers that minimizes the localization errors of the mobile nodes. The stochastic convergence of such algorithms is currently being investigated.

VII. Results and Further Work

Figure 3 compares the performance of the two power control schemes with the optimal solution $P^\star$ obtained through (13) for a given mobile location. For different values of N_R and K_R (which controls the range-variance through (3)), we observe that the transmit power obtained by both schemes closely matches the optimal transmit power $P^\star$ in all cases. The value of L was fixed at 10 meters, and the number of randomly distributed reference nodes N_R was increased, thereby increasing the effective node density. It is observed that the transmit power required to attain a QOL $Q_0 = 100\ m^{-2} \Rightarrow \Omega_0 = 0.01\ m^2$ decreases as the node density increases, which is expected from (9), since for a given transmit power, the number of available range estimates m increases as we increase the node density. Also, as K_R increases, the range estimate variances increase and we require a higher transmit power to attain the target localization error $\Omega_0 = 1/Q_0$.

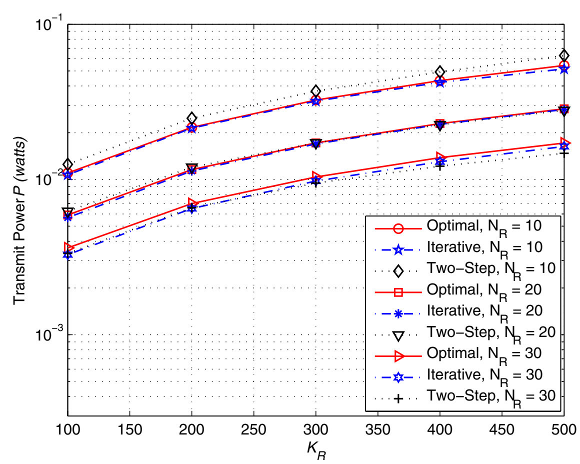

Fig. 3. Comparison of the two power control schemes with the optimal solution: the final transmit power is shown versus K_R and the number of reference nodes N_R. In this simulation, L = 10 meters, $\beta = 2$, $\xi_{min} = 20$ dB, $Q_0 = 100\ m^{-2}$.

Figure 4 shows an example of the impact of using iterative power control on a mobile node's localization accuracy as it moves through a network of $N_R = 15$ reference nodes that are

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randomly scattered over a 1600 m^2 area. The mobile node is assumed to move at a speed of $v_m = 1.6\ m/s$ and $N_{iter} = 10$ power control iterations are performed per second. We observe that the use of power control not only reduces the fluctuation of localization accuracy, but also ensures lower localization error, i.e., a higher localization accuracy. This however, comes at the price of using a higher transmit power, which would lead to higher interference levels when several mobile nodes are present, which impacts ranging performance.

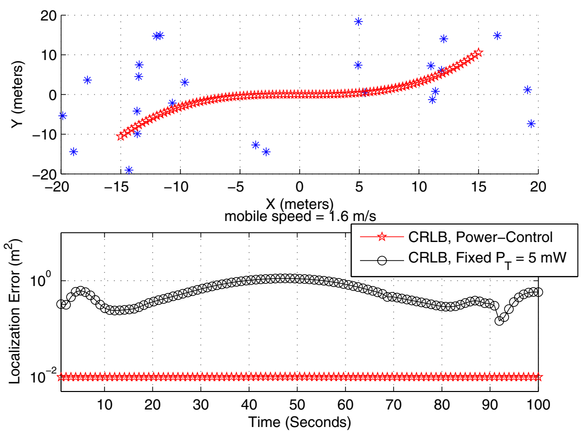

Fig. 4. (a) Trajectory of Mobile Nodes (b) Reduction in "Spatial Fading" of localization error using power control. The parameters used for this simulation were: $P_0 = 5$mW, $\beta = 2$, $N = 25$, $L = 40$, $\xi_{\min} = 20$ dB, $v_m = 1.6$ m/s.

Power control schemes for the case of several mobile nodes are currently being investigated, particularly from stochastic convergence perspectives. This paper provides a basic framework for the development of power control algorithms in the relatively new field of UWB PoLoNet design. Several assumptions were made in order to simplify analysis: e.g., the CRLB was used to compute the localization error, whereas in practice, location estimators do not attain this lower bound. The impact of NLOS links and techniques for its mitigation also need to be analyzed. The design of joint tracking and power-control algorithms that are robust to node mobility warrants further investigation.

VIII. Conclusions

In this paper, we look at the use of power control schemes in infrastructure-based UWB Position-Location Networks (PoLoNets). The localization accuracy of a mobile node is shown to be determined by connectivity with reference nodes, range-estimate variances and geometry of mobile nodes relative to the reference nodes. This results in large-scale fluctuation of localization accuracy as the mobile node traverses the network. Two ameliorative power control schemes are discussed: a iterative scheme based on the quality of location-estimates and a two-step analytical solution. We demonstrate that the two approaches approximate the empirically-obtained optimal solution, and significantly improve the robustness of location-estimates.

Appendix I: Bounds on Γ_m

The generalized GDOP is defined by

$$\Gamma_m = \frac{\psi_m}{\gamma_m^2} = \frac{\sum_{i=1}^m \sum_{j=1,j>i}^m \frac{\sin^2(\alpha_i-\alpha_j)}{\sigma_i^2\sigma_j^2}}{\left(\sum_{i=1}^m \frac{1}{\sigma_i^2}\right)^2} = \frac{1}{4} - \frac{\left(\sum_{i=1}^m \frac{\cos 2\alpha_i}{\sigma_i^2}\right)^2 + \left(\sum_{i=1}^m \frac{\sin 2\alpha_i}{\sigma_i^2}\right)^2}{4\left(\sum_{i=1}^m \frac{1}{\sigma_i^2}\right)^2}.$$

When $\alpha_i = \alpha_j$, we see that we obtain the lower bound, $\Gamma_m = 0$. The upper limit $\Gamma_m = \frac{1}{4}$ is achieved when $\sum_{i=1}^m \frac{\cos 2\alpha_i}{\sigma_i^2} = 0$ and $\sum_{i=1}^m \frac{\sin 2\alpha_i}{\sigma_i^2} = 0$. A specific case of the above result is discussed in [10].

Appendix II: Proof of Inequality (8)

Without loss of generality, let the mth range estimate be reduced by a factor a, $0 < a \le 1$. Then

$$\Omega'_{\boldsymbol{x}} = \frac{\gamma_{m-1} + \frac{1}{a\sigma_m^2}}{\psi_{m-1} + \frac{1}{a\sigma_m^2}\zeta_{m-1}}, \quad \zeta_{m-1} = \sum_{k=1}^{m-1} \frac{\sin^2(\alpha_k - \alpha_m)}{\sigma_k^2}.$$

After some manipulation, it can be shown that

$$\zeta_{m-1} = \frac{\gamma_{m-1} - \sqrt{\gamma_{m-1}^2 - 4\psi_{m-1}}\cos(2\alpha_m - 2\nu)}{2} \ge \frac{\gamma_{m-1} - \sqrt{\gamma_{m-1}^2 - 4\psi_{m-1}}}{2}, \qquad (17)$$

where the angle ν is defined as

$$\nu = \frac{1}{2}\arctan\left(\frac{\sum_{i=1}^{m-1}\left(\frac{\sin 2\alpha_i}{\sigma_i^2}\right)}{\sum_{i=1}^{m-1}\left(\frac{\cos 2\alpha_i}{\sigma_i^2}\right)}\right). \qquad (18)$$

The difference between $\Omega_{\boldsymbol{x}}$ and $\Omega'_{\boldsymbol{x}}$ is then given by

$$\begin{aligned}\Omega_{\boldsymbol{x}} - \Omega'_{\boldsymbol{x}} &= \frac{(1-a)\sigma_m^2\left(\gamma_{m-1}\zeta_{m-1} - \psi_{m-1}\right)}{\left(\sigma_m^2\psi_{m-1} + \zeta_{m-1}\right)\left(a\sigma_m^2\psi_{m-1} + \zeta_{m-1}\right)} \\ &\ge \frac{(1-a)\sigma_m^2\left(\gamma_{m-1}\left(\frac{\gamma_{m-1}}{2} - \frac{\sqrt{\gamma_{m-1}^2 - 4\psi_{m-1}}}{2}\right) - \psi_{m-1}\right)}{\left(\sigma_m^2\psi_{m-1} + \zeta_{m-1}\right)\left(a\sigma_m^2\psi_{m-1} + \zeta_{m-1}\right)} \\ &= \frac{\frac{(1-a)\sigma_m^2\gamma_{m-1}^2}{4}\left(1 - \sqrt{1 - \frac{4\psi_{m-1}}{\gamma_{m-1}^2}}\right)^2}{\left(\sigma_m^2\psi_{m-1} + \zeta_{m-1}\right)\left(a\sigma_m^2\psi_{m-1} + \zeta_{m-1}\right)} \ge 0.\end{aligned}$$

Since $\psi_{m-1} = \Gamma_{m-1}\gamma_{m-1}^2 \le \frac{\gamma_{m-1}^2}{4}$ and $0 < a \le 1$, and all other quantities are positive. Therefore $\Omega_{\boldsymbol{x}} \ge \Omega'_{\boldsymbol{x}}$.

Appendix III. Proof of Inequality (9)

The localization error with $(m+1)$ range estimates

$$\Omega_{\boldsymbol{x},m+1} = \frac{\sum_{i=1}^{m+1} \frac{1}{\sigma_i^2}}{\sum_{i=1}^{m+1}\sum_{j=1,j>i}^{m+1} \frac{\sin^2(\alpha_i-\alpha_j)}{\sigma_i^2\sigma_j^2}} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{1}{\sigma_{m+1}^2}\sum_{i=1}^m \frac{\sin^2(\alpha_i - \alpha_{m+1})}{\sigma_i^2}} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{\zeta_m}{\sigma_{m+1}^2}} \qquad (19)$$

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where $\zeta_m = \sum_{i=1}^{m} \frac{\sin^2(\alpha_i - \alpha_{m+1})}{\sigma_i^2}$. From (17) and (19),

$$\Omega_{\boldsymbol{x},m+1} = \frac{\gamma_m + \frac{1}{\sigma_{m+1}^2}}{\psi_m + \frac{\gamma_m}{2\sigma_{m+1}^2} - \frac{\sqrt{\gamma_m^2 - 4\psi_m}\cos(2\alpha_{m+1} - 2\nu)}{2\sigma_{m+1}^2}}, \quad (20)$$

where the angle ν is defined in (18). From (4) and (20)

$$\begin{aligned} &\Omega_{\boldsymbol{x},m} - \Omega_{\boldsymbol{x},m+1} \\ &= \frac{\frac{\gamma_m^2}{2} - \frac{\gamma_m \sqrt{\gamma_m^2 - 4\psi_m}\cos(2\alpha_{m+1} - 2\nu)}{2} - \psi_m}{\psi_m \left(\sigma_{m+1}^2 \psi_m + \frac{\gamma_m}{2} - \frac{\sqrt{\gamma_m^2 - 4\psi_m}\cos(2\alpha_{m+1} - 2\nu)}{2}\right)} \\ &= \frac{\frac{\left(\sqrt{1-4\Gamma_m} - \cos(2\alpha_{m+1} - 2\nu)\right)^2 + \sin^2(2\alpha_{m+1} - 2\nu)}{2}}{\gamma_m \left(\sigma_{m+1}^2 \psi_m + \frac{\gamma_m}{2} - \frac{\sqrt{\gamma_m^2 - 4\psi_m}\cos(2\alpha_{m+1} - 2\nu)}{2}\right)}. \end{aligned}$$

Since the numerator and denominator are always positive, $\Omega_{\boldsymbol{x},m} \geq \Omega_{\boldsymbol{x},m+1}$.

Appendix IV. Variation of Ω_x with Transmit Power

Let us define the following terms:

$$\begin{aligned} g_m &= \sqrt{1 - 4\Gamma_m}, \ \gamma_n = \sum_{i=m+1}^{m+n} \frac{1}{\sigma_i^2} = \gamma_{m+n} - \gamma_m, \\ \psi_n &= \sum_{i=m+1}^{m+n} \sum_{j=m+1, j>i}^{m+n} \frac{\sin^2(\alpha_i - \alpha_j)}{\sigma_i^2 \sigma_j^2}, \ \Gamma_n = \frac{\psi_n}{\gamma_n^2}. \end{aligned}$$

We have

$$\Omega(P') = \frac{\sum_{i=1}^{m} \frac{1}{\sigma_i'^2}}{\sum_{i=1}^{m} \sum_{j=1,j>i}^{m} \frac{\sin^2(\alpha_i - \alpha_j^2)}{\sigma_i'^2 \sigma_j'^2}} = \frac{\gamma_{m+n}}{(1+\chi)\,\psi_{m+n}}.$$

The denominator term is simplified using

$$\psi_{m+n} = \sum_{i=1}^{m+n} \sum_{j=1,j>i}^{m+n} \frac{\sin^2(\alpha_i - \alpha_j)}{\sigma_i^2 \sigma_j^2} = \psi_m + \Delta\psi,$$

where

$$\Delta\psi = \gamma_m \left[\frac{(1-g_m)\gamma_n}{2} + g_m \sum_{i=m+1}^{n} \frac{\sin^2(\alpha_i - \nu)}{\sigma_i^2} \right] + \Gamma_n \gamma_n^2.$$

Using the above expressions, after some manipulation, the improvement in terms of localization error due to the increase in the transmit power can be written as

$$\Omega_{\boldsymbol{x}}(P') - \Omega_{\boldsymbol{x}}(P) = \frac{\frac{\chi A}{1+\chi} + \frac{B\gamma_n}{(1+\chi)}[B + \chi] + C}{A\,(A + B\gamma_n)} \geq 0, \quad (21)$$

where $A = \Gamma_m \gamma_m, \ B = \frac{(1-g_m)}{2}$ and

$$C = g_m \sum_{i=m+1}^{m+n} \frac{\sin^2(\alpha_i - \nu)}{\sigma_i^2} + \frac{\Gamma_n \gamma_n^2}{\gamma_m}.$$

The worst-case geometry for the introduced reference nodes is given by:

$$\alpha_i = \nu, \ i = m+1, m+2, \cdots, m+n. \quad (22)$$

This implies $\Gamma_n = 0$ and therefore, $C = 0$ in (21). Here, due to lack of knowledge of n or γ_n, we assume that the nodes are uniformly distributed with a density ρ and $\beta = 2$. Since $n = N(P') - N(P)$ here depends on χ, assuming n is the average number of new nodes that transmit range response packets after the increase in transmit power, we can write:

$$n = N(P') - N(P) \approx K_N P \chi,$$

where $N(P) = \lfloor \rho\pi R_{\max}^2 \rfloor \approx \rho\pi \left(\frac{\xi_{\min}}{K_P P}\right)^{\frac{2}{\beta}} = K_N P^{\frac{2}{\beta}}$. Additionally, since we do not know the variances σ_i^2, $i = m+1, \cdots, m+n$, we assume they are a factor η larger than σ_m^2. This leads to $\gamma_n = \frac{n}{\eta\sigma_m^2} \approx \frac{K_N P_0 \chi}{\eta\sigma_m^2} = E\chi$, where $E = \frac{K_N P_0}{\eta\sigma_m^2}$. Therefore, substituting $D = \Omega_{\boldsymbol{x}}(P') - \Omega_{\boldsymbol{x}}(P) = \frac{1}{Q_{\boldsymbol{x}}(P')} - \frac{1}{Q_{\boldsymbol{x}}(P)}$ into (21), we can solve for χ:

$$\begin{aligned} \chi &= \frac{b \pm \sqrt{b^2 - 4ac}}{2a}, \ a = BE(DA - 1), \quad (23) \\ b &= (A + B^2 E) - DA(A + BE), \ c = DA^2. \end{aligned}$$

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